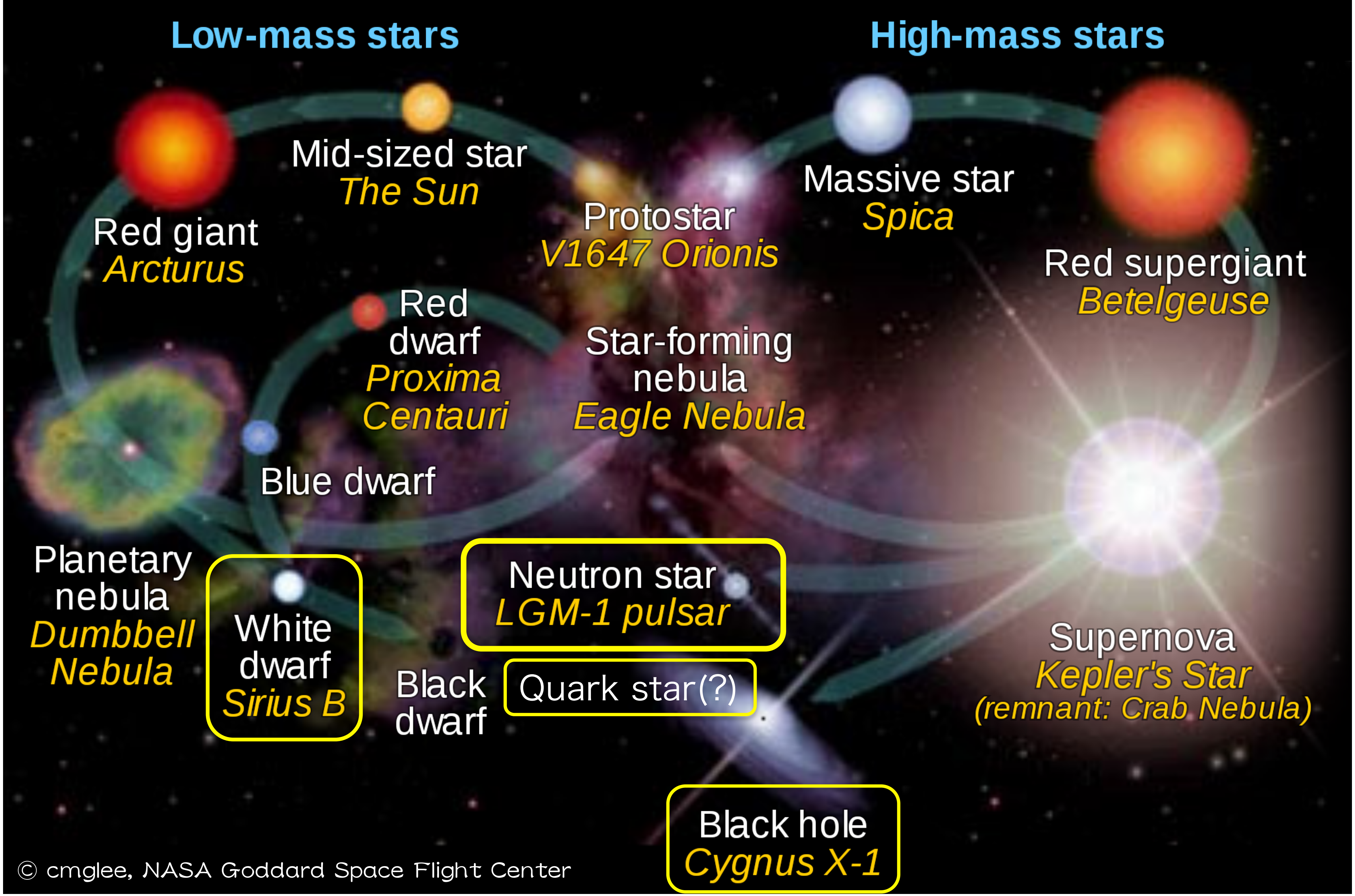
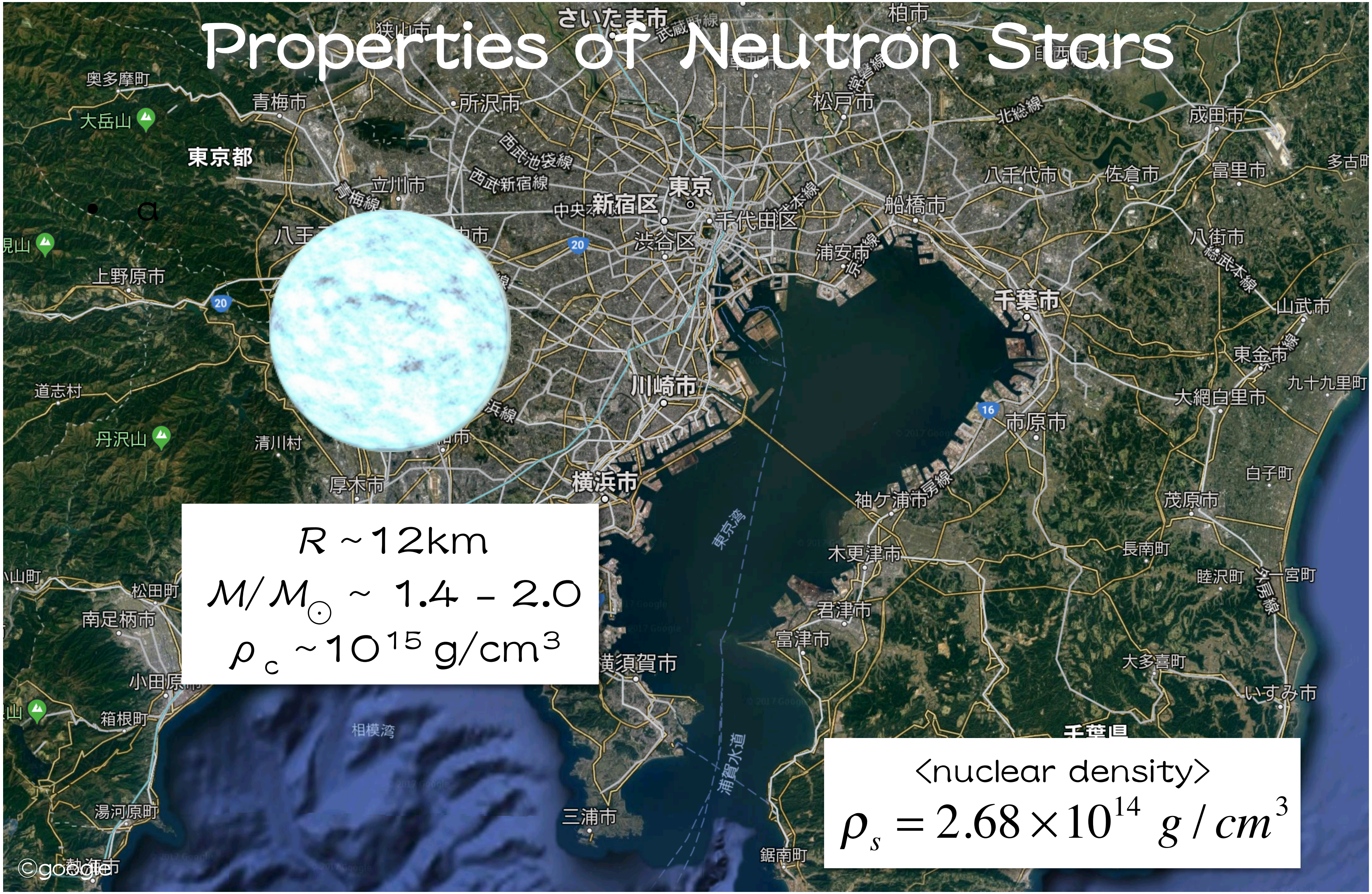
コンパクト天体と重力理論

Hajime SOTANI (NAOJ)

Star life cycles



Properties of Neutron Stars



$R \sim 12\text{km}$

$M/M_{\odot} \sim 1.4 - 2.0$

$\rho_c \sim 10^{15}\ \text{g/cm}^3$

<nuclear density>

$$\rho_s = 2.68 \times 10^{14}\ g/cm^3$$

motivations for considering NSs

- understanding physics for a high density region
  - equation of state (EOS)
  - associated with nuclear/particle physics
- existence of high magnetic fields (such as magnetars)
  - 10^{14} - 10^{15} G at stellar surface
  - stronger than a critical field strength of QED (~4×10^{13}G)
- probing the theory of gravity
  - no observations and experiments exist for showing defects of GR
  - even in strong field-regime?
  - alternative theory of gravity??
- suitable "laboratory" for physics in extreme states

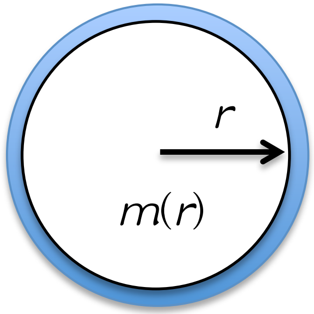
How to construct NSs

- Tolman-Oppenheimer-Volkoff (TOV) equation gives density profile of the spherically symmetric equilibrium of cold NSs <u>in GR</u>.

$$\frac{dP}{dr} = -\rho \frac{Gm}{r^2} \left(1 + \frac{4\pi r^3 P}{mc^2}\right)\left(1 + \frac{P}{\rho c^2}\right)\left(1 - \frac{2Gm}{c^2 r}\right)^{-1}$$

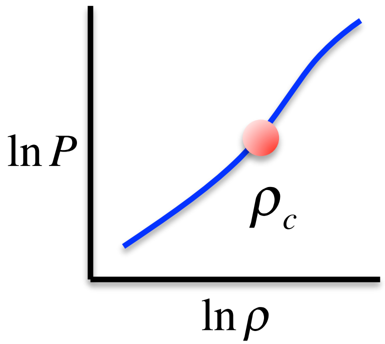
relativistic correction

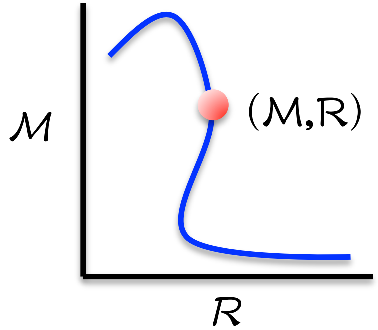
$$\frac{dm}{dr} = 4\pi r^2 \rho$$



$P = P(\rho)$ equation of state (EOS)

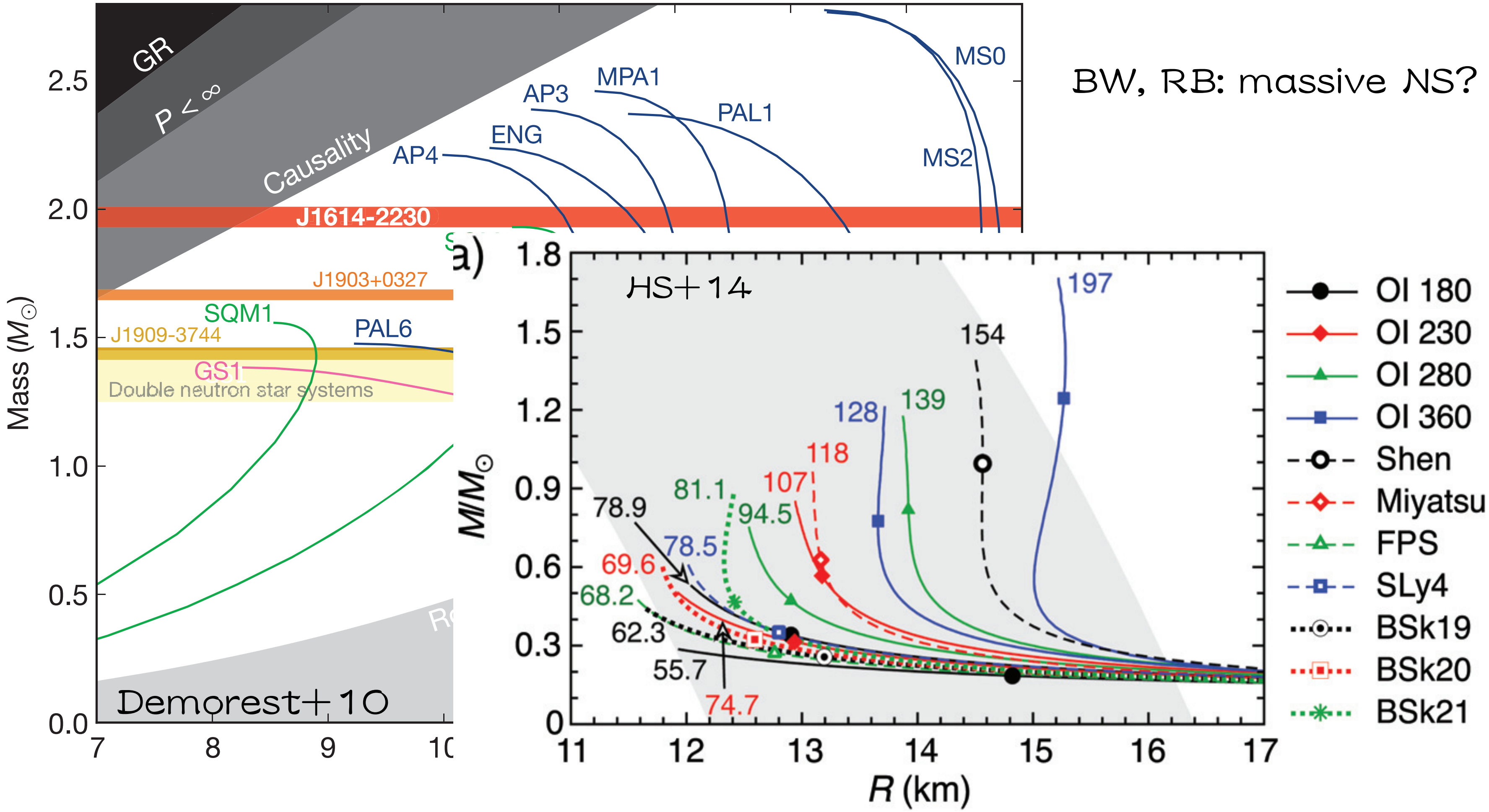
$$R = r \quad @P = 0$$
$$M = m(R)$$





EOS is essential to construct the stellar model, but still uncertain.

too many EOSs suggested...



BW, RB: massive NS?

NS observations can make a constraint on EOS!!

saturation

- radius of an atomic nucleus with mass number A

$$R / A^{1/3} \simeq r_0 \approx 1.2 \times 10^{-13} cm$$

- binding energy

$$E(A) / A \simeq 8\ MeV$$

which are independent of atomic nuclei.

- density of atomic nuclei

$$\rho \approx \frac{M}{R^3} \approx \frac{mA}{r_0^3 A} = \frac{m}{r_0^3} \equiv \rho_s$$

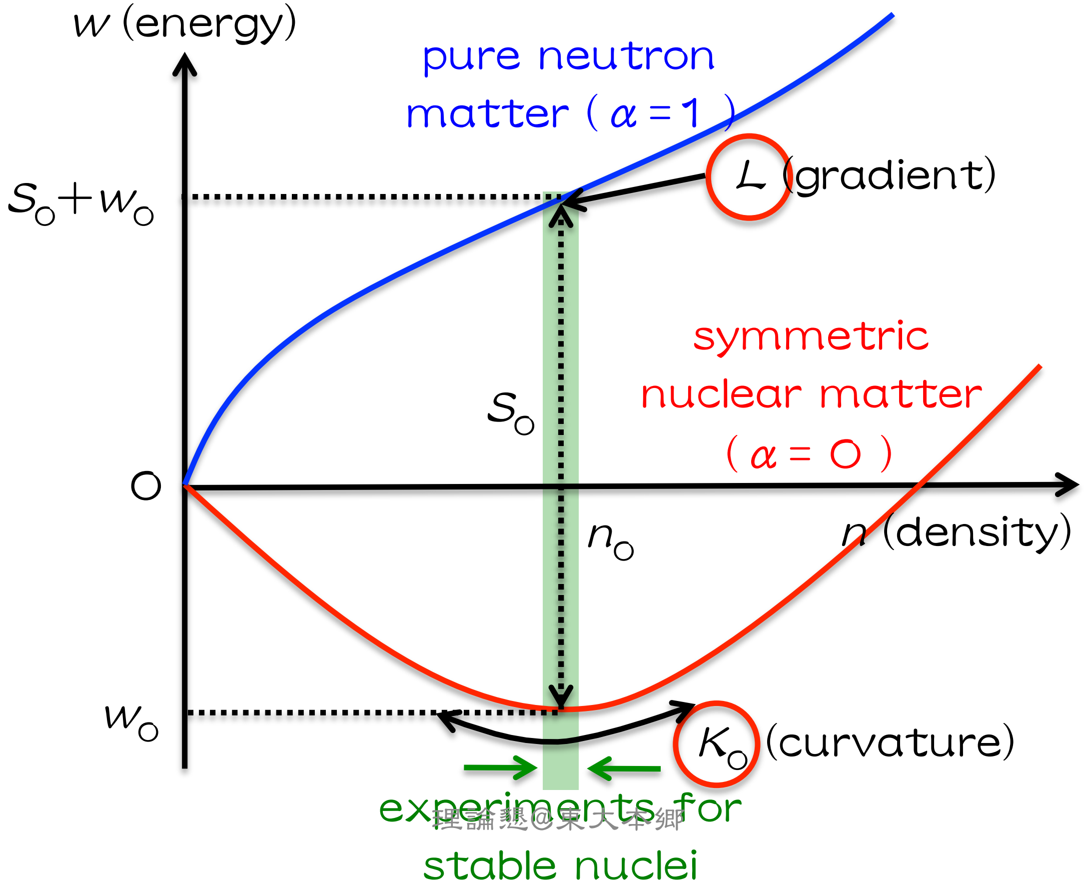
saturation density = 2.68×10^{14} g/cm^3
(baryon number density = 0.16fm^{-3})

EOS near the saturation point

- Bulk energy per nucleon near the saturation point of symmetric nuclear matter at zero temperature;

incompressibility, symmetry parameter

$$w = w_0 + \frac{K_0}{18n_0^2}(n-n_0)^2 + \left[S_0 + \frac{L}{3n_0}(n-n_0)\right]\alpha^2$$



$$\alpha = \frac{n_n - n_p}{n}$$

$$P(n) = -n^2 \frac{dw}{dn}$$

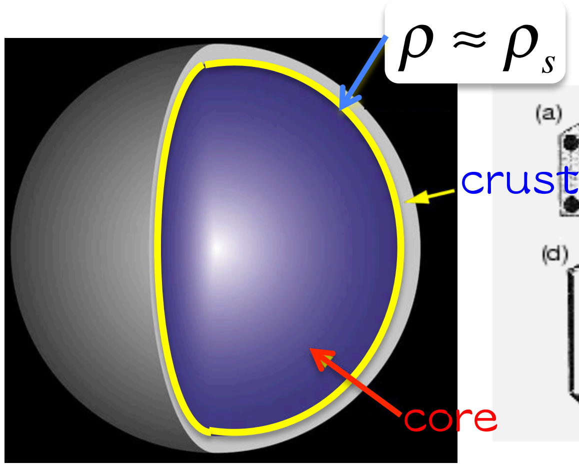
neutron stars

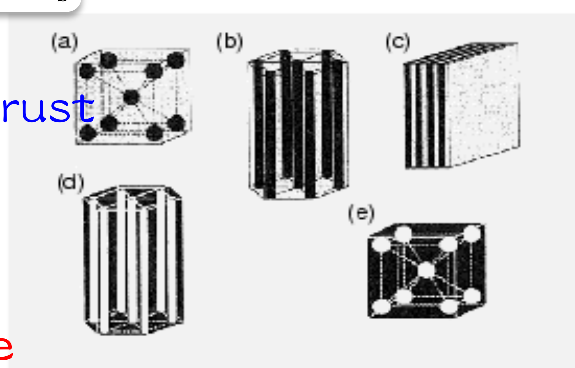
- Structure of NS
  - solid layer (crust)
  - nonuniform structure (pasta)
  - fluid core (uniform matter)
- Crust thickness ≲ 1km

$$\frac{\Delta R}{R} \simeq 2.1 \times 10^{-2} \frac{R}{M}\left(1 - \frac{2M}{R}\right)$$

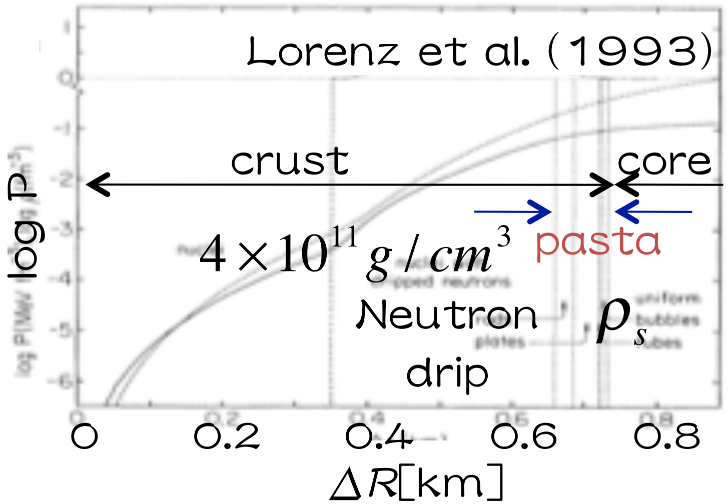
- Determination of EOS for high density (core) region could be quite difficult on Earth
- Constraint on EOS via observations of neutron stars
  - stellar mass and radius
  - stellar oscillations (& emitted GWs)

"(GW) asteroseismology"

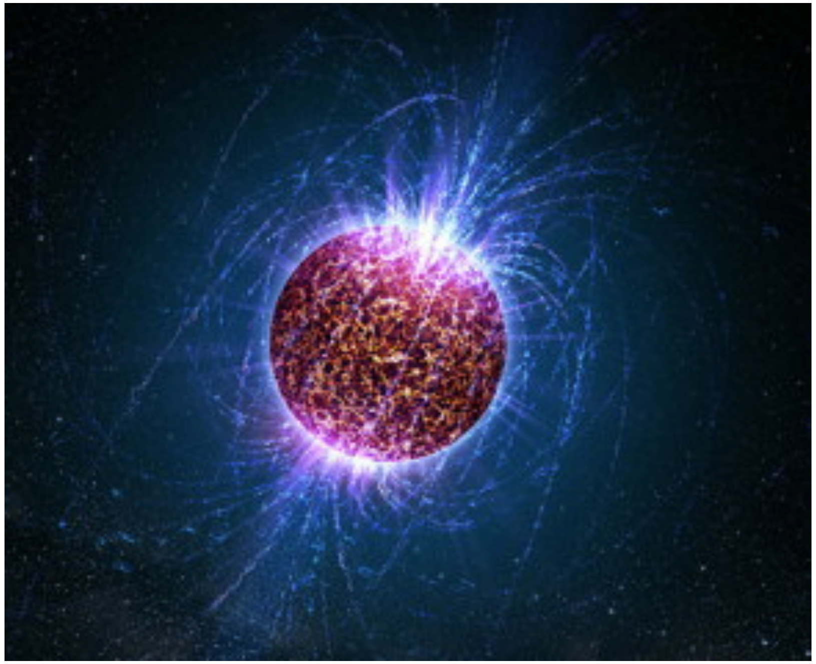




Oyamatsu (1993)



NSs - EOS



(1) TOV equation

(2) equation of state

- model
- nuclear interaction
- composition

- physics in NS crust
- low-mass NSs

physics in NS (core)

high density region

constraints from the terrestrial nuclear experiments

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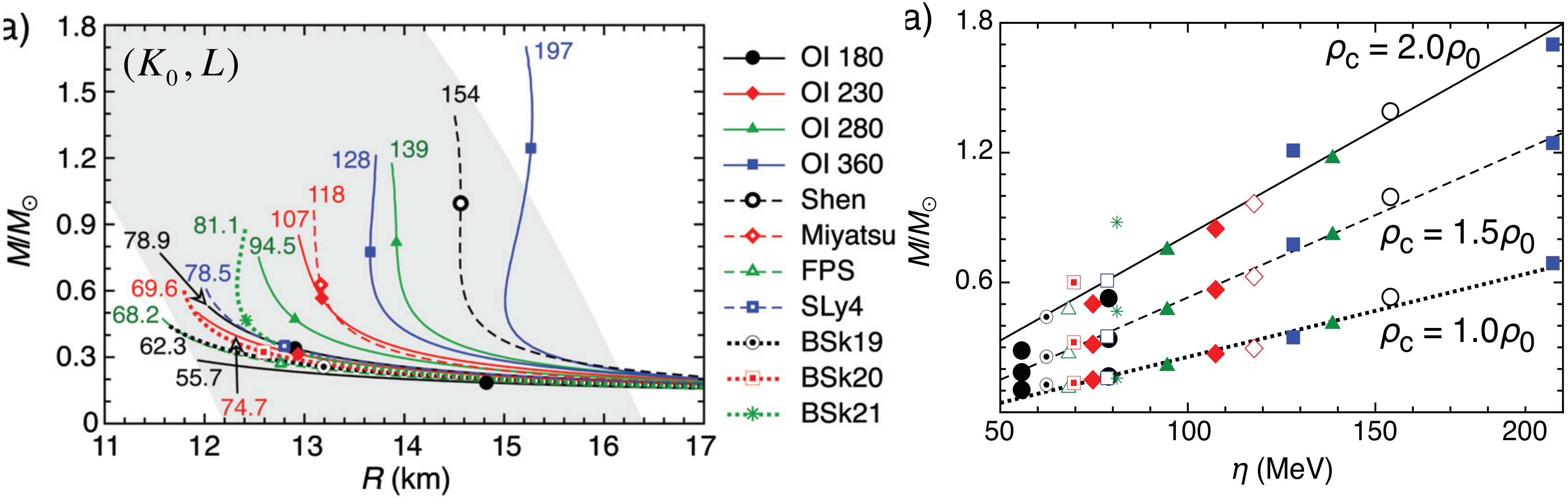
properties around the saturation density

Low-mass NSs (HS+14)



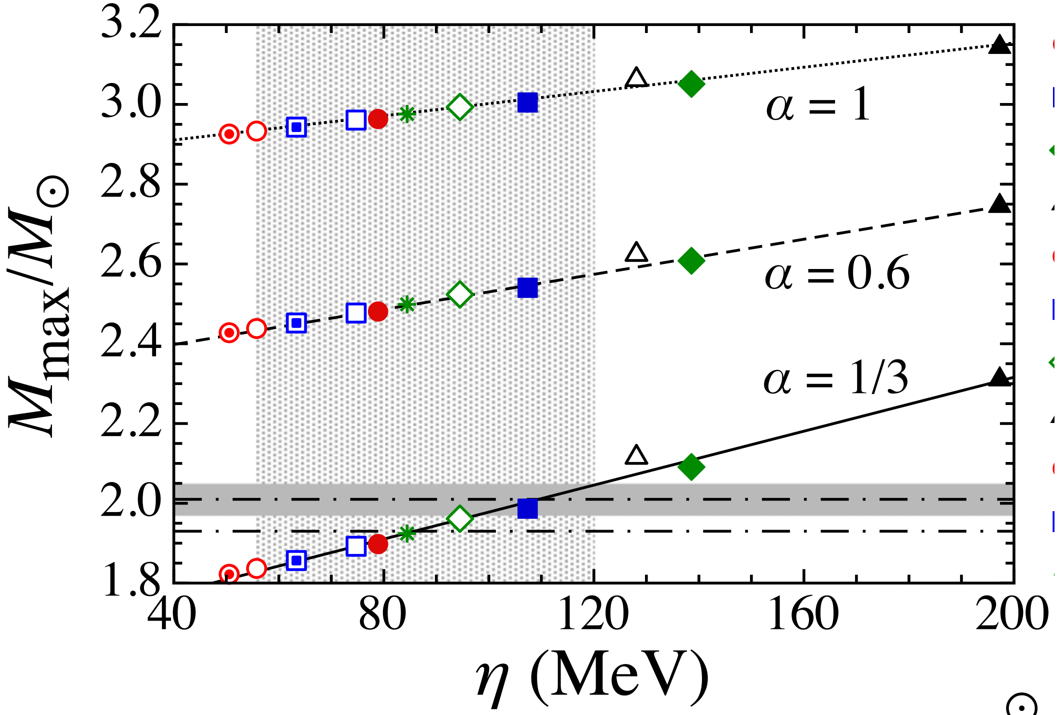
$$\eta = (K_0 L^2)^{1/3}$$

$$u_c = \rho_c / \rho_0$$

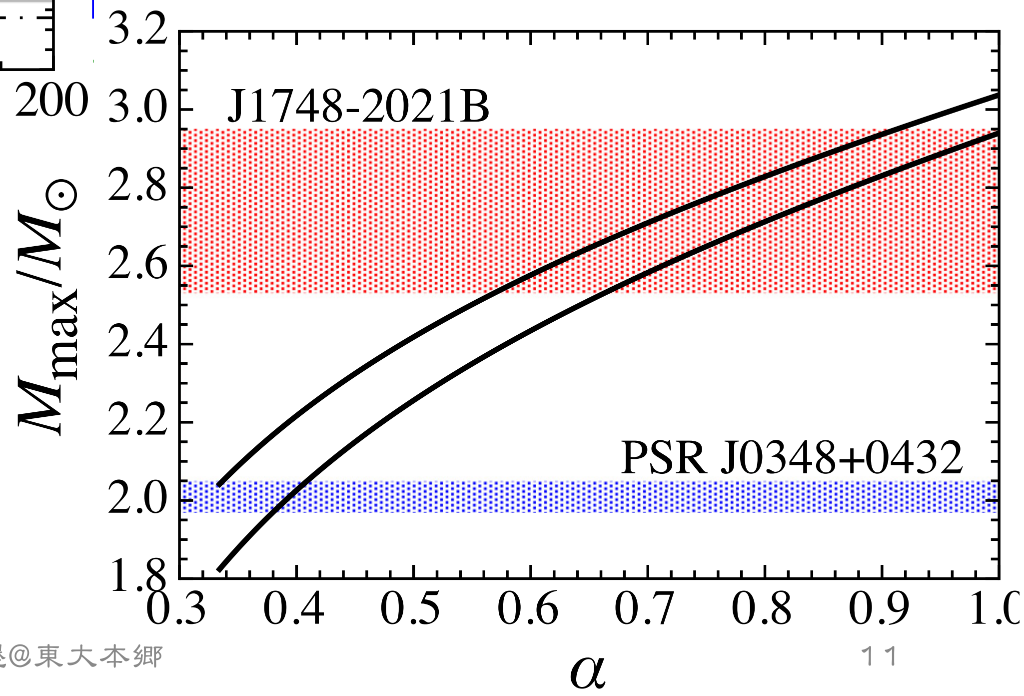
$$\frac{M}{M_\odot} = 0.371 - 0.820 u_c + 0.279 u_c^2 - (0.593 - 1.25 u_c + 0.235 u_c^2)\left(\frac{\eta}{100\,\mathrm{MeV}}\right)$$

$$z = 0.00859 - 0.0619 u_c + 0.0255 u_c^2 - (0.0429 - 0.108 u_c + 0.0120 u_c^2)\left(\frac{\eta}{100\,\mathrm{MeV}}\right)$$

maximum mass of NS

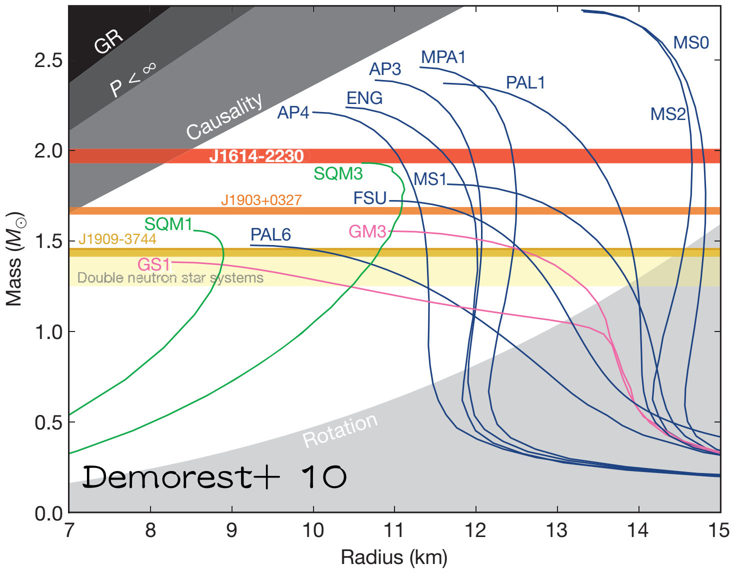


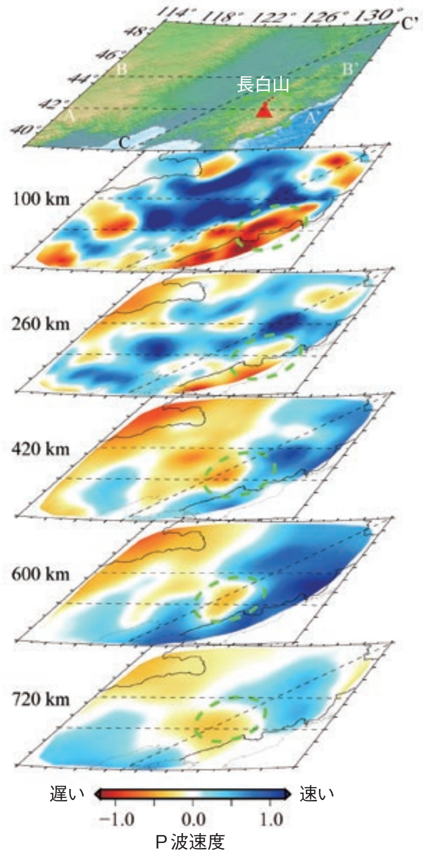
$\alpha = v_s^{\ 2}$



how to obtain the properties of NSs

- direct observations of NSs
  - mass
  - radius
  - compactness
- (GW) asteroseismology
  - using oscillation frequencies
  - similar to seismology and helioseismology





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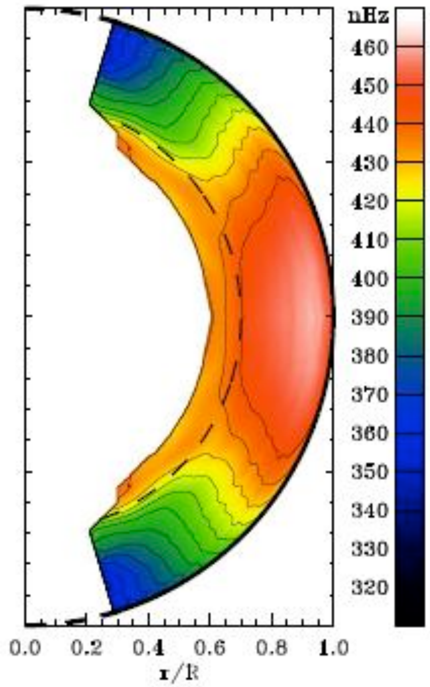

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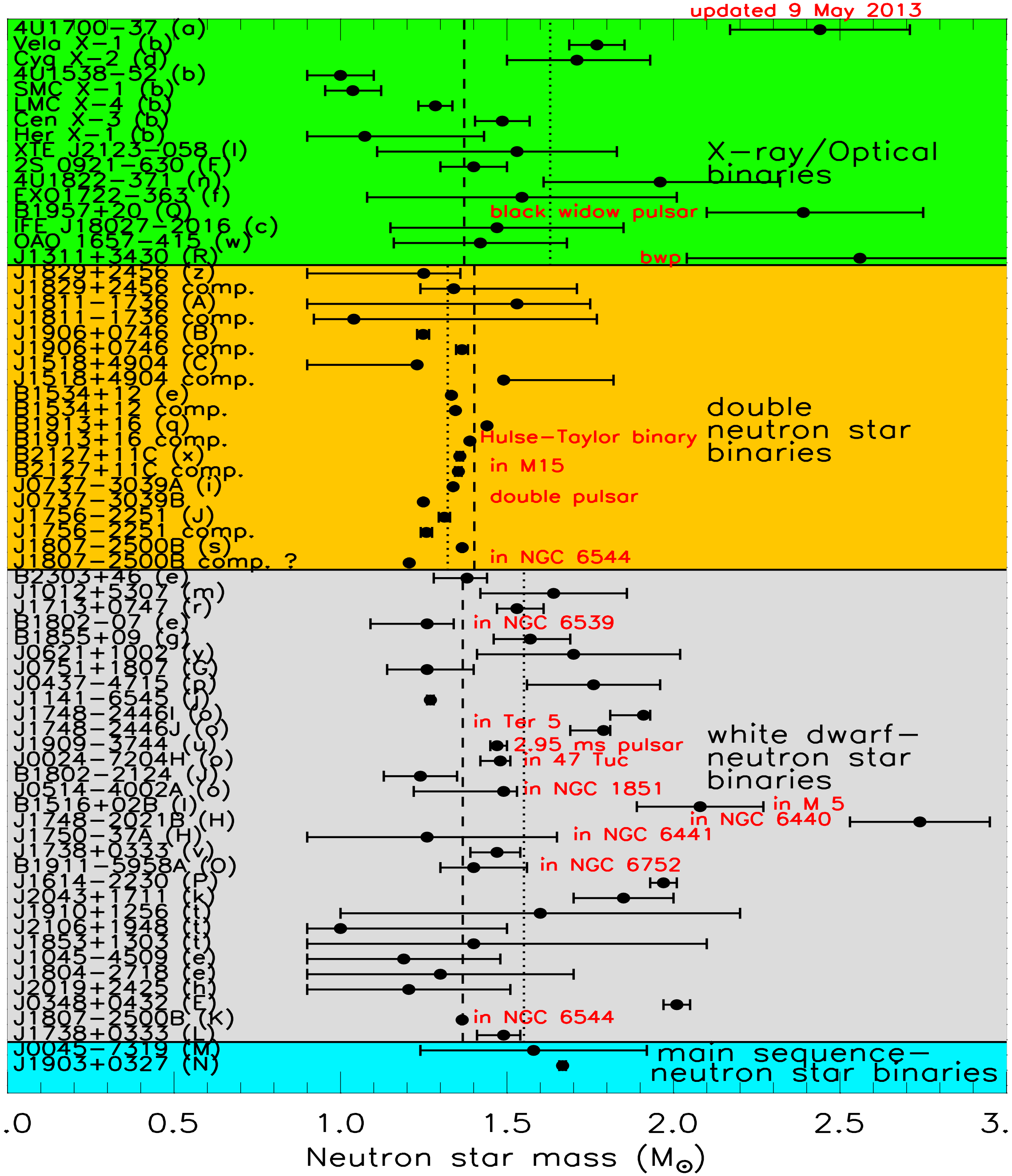
mass of NSs



vanKerkwijk 2010

Romani et al. 2012

Although simple average mass of w.d. companions is 0.23 $M_\odot$ larger, weighted average is 0.04 $M_\odot$ smaller

Demorest et al. 2010

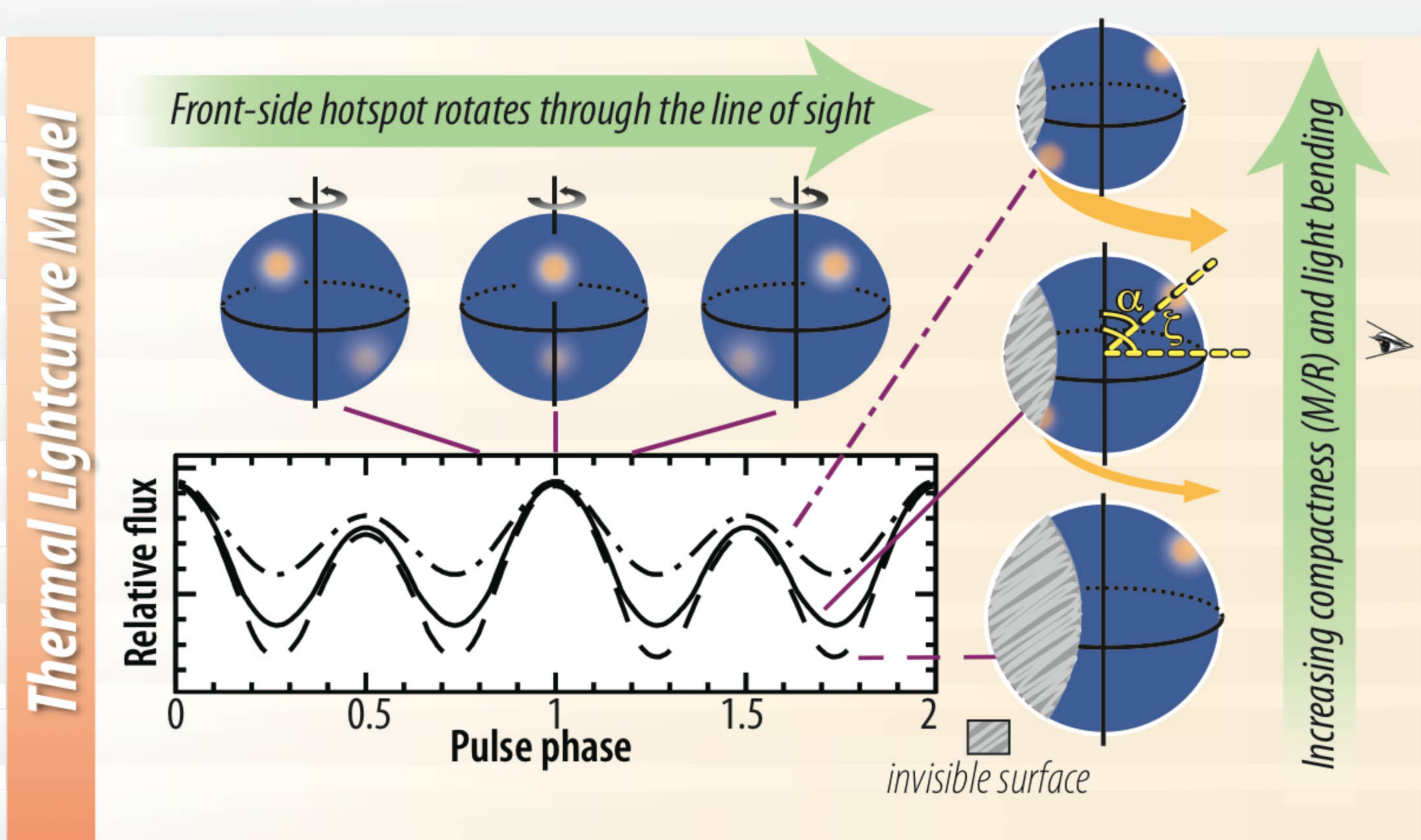
Antoniadis et al. 2013

Champion et al. 2008

Lattimer 2013

NICER (Neutron star Interior Composition ExploreR)

- pulse profile from a pulsar → M/R (compactness)

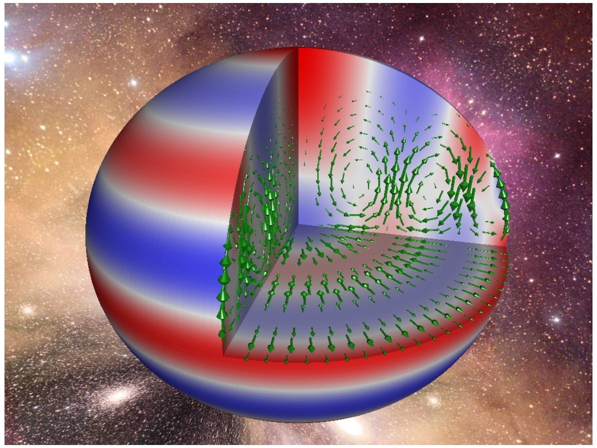


https://heasarc.gsfc.nasa.gov/docs/nicer/nicer_about.html

Oscillations & Instabilities

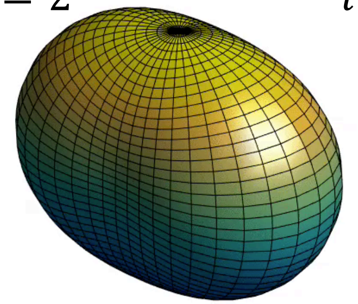
The most promising strategy for constraining the physics of neutron stars involves observing their "ringing" (oscillation modes)

- **f-mode :** scales with average density
- **p-modes:** probes the sound speed through out the star
- **g-modes :** sensitive to thermal/composition gradients
- **w-modes:** oscillations of spacetime itself.
- **s-modes:** Shear waves in the crust (*t*-modes)
- **Alfvèn modes:** due to magnetic field
- **i-modes:** inertial modes associated with rotation (r-mode)

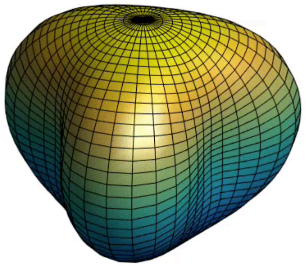


Typically **SMALL AMPLITUDE** oscillations –> weak emission of GWs
UNLESS
they become **unstable due to rotation** (r-mode & f-mode)

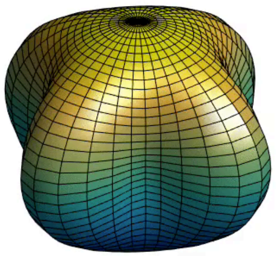
$l = 2, m = 2$



$l = 3, m = 3$



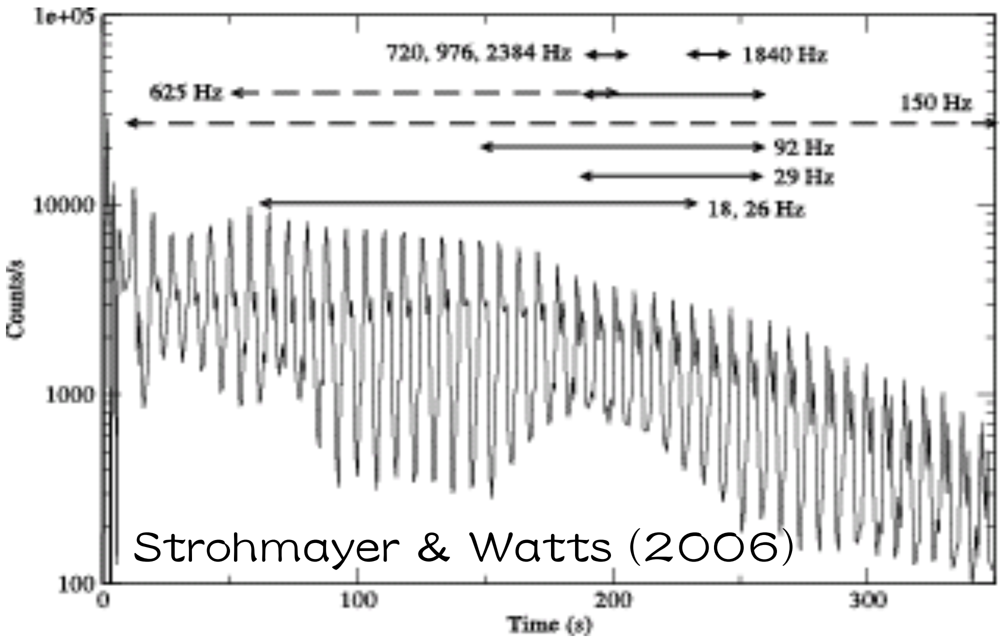
$l = 4, m = 4$



slide by Kokkotas

QPOs in SGRs

- Quasi-periodic oscillations (QPOs) in afterglow of giant flares from soft-gamma repeaters (SGRs)
  – SGR 0526-66 (5th/3/1979) : 43 Hz
  – SGR 1900+14 (27th/8/1998) : 28, 54, 84, 155 Hz
  – SGR 1806-20 (27th/12/2004) : 18, 26, 30, 92.5, 150, 626.5, 1837 Hz
  (Barat+ 1983, Israel+ 05, Strohmayer & Watts 05, Watts & Strohmayer 06)
  – additional QPO in SGR 1806-20 is found : 57Hz (Huppenkothen + 2014)



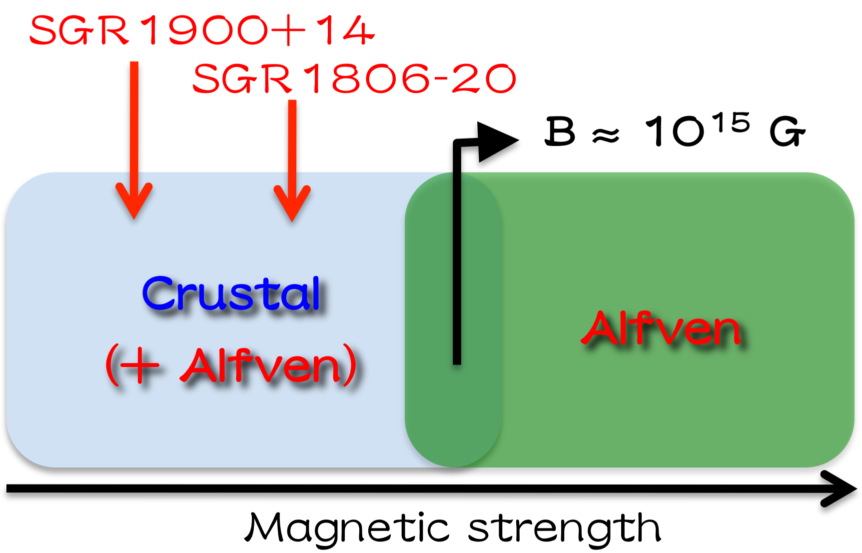
- Crustal torsional oscillation ?
- Magnetic oscillations ?

advantage for crustal oscillations

- magnetic configuration inside NSs are still uncertain
- EOSs for core region are unfixed yet

- to avoid such uncertainties, we focus on the crustal torsional oscillations without effects of magnetic field
  - fluid core: zero shear modulus ---> no shear oscillations
  - torsional oscillations localize only in crust region

- magnetic effect on torsional oscillations
  - frequencies can become larger (HS+07, Gabler+12,13)

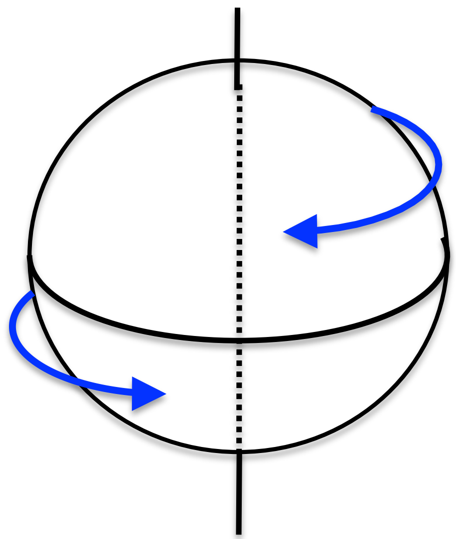


torsional oscillations

- axial parity oscillations
  - incompressible
  - no density perturbations (less associated with GWs)
- in Newtonian case (Hansen & Cioff 1980)

$$ {}_\ell t_0 \sim \frac{\sqrt{\ell(\ell+1)\mu/\rho}}{2\pi R} \sim 16\sqrt{\ell(\ell+1)}\ \mathrm{Hz} \qquad {}_\ell t_n \sim \frac{\sqrt{\mu/\rho}}{2\Delta r} \sim 500 \times n\ \mathrm{Hz} $$

  - μ : shear modulus
  - frequencies ∝ shear velocity $v_s = \sqrt{\mu/\rho}$
  - overtones depend on crust thickness
- one can consider torsional oscillations independently of core EOS



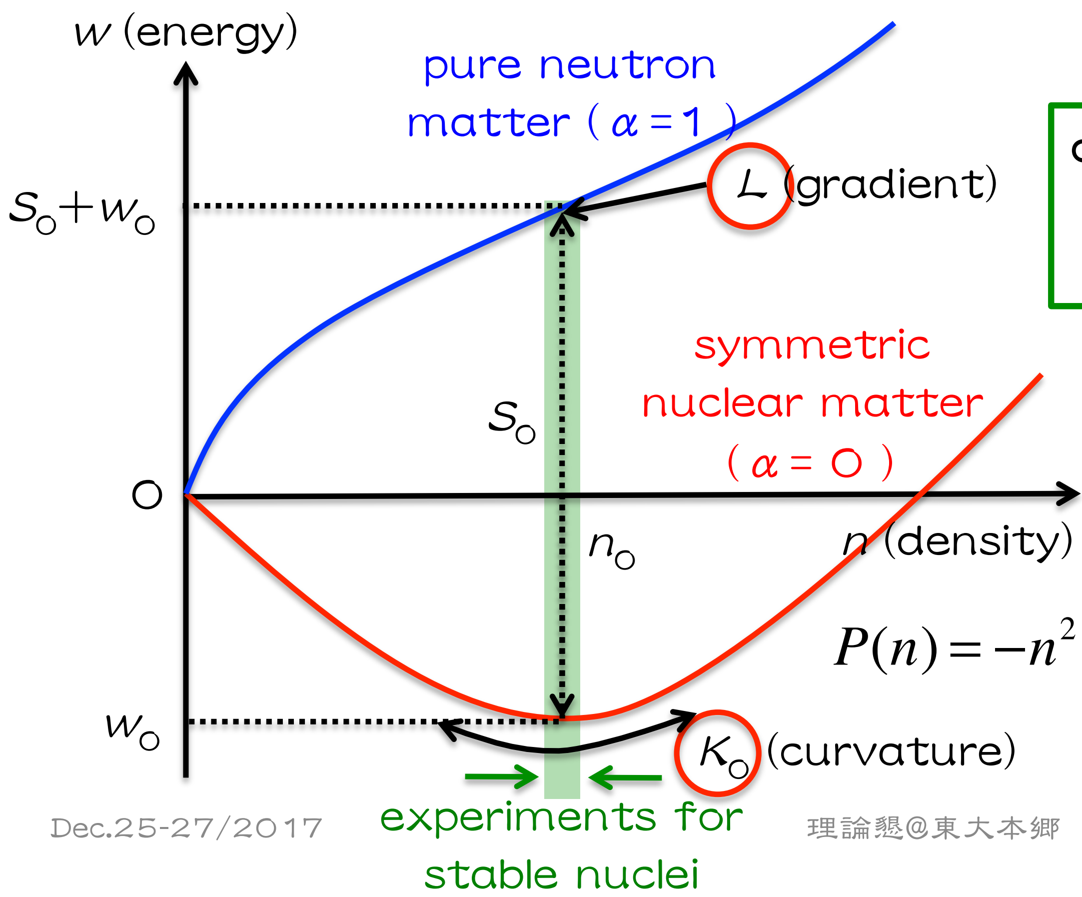
EOS near the saturation point

- Bulk energy per nucleon near the saturation point of symmetric nuclear matter at zero temperature;

incompressibility   symmetry parameter

$$w = w_0 + \frac{K_0}{18n_0^2}(n-n_0)^2 + \left[S_0 + \frac{L}{3n_0}(n-n_0)\right]\alpha^2$$

$$\alpha = \frac{n_n - n_p}{n}$$



$$P(n) = -n^2 \frac{dw}{dn}$$

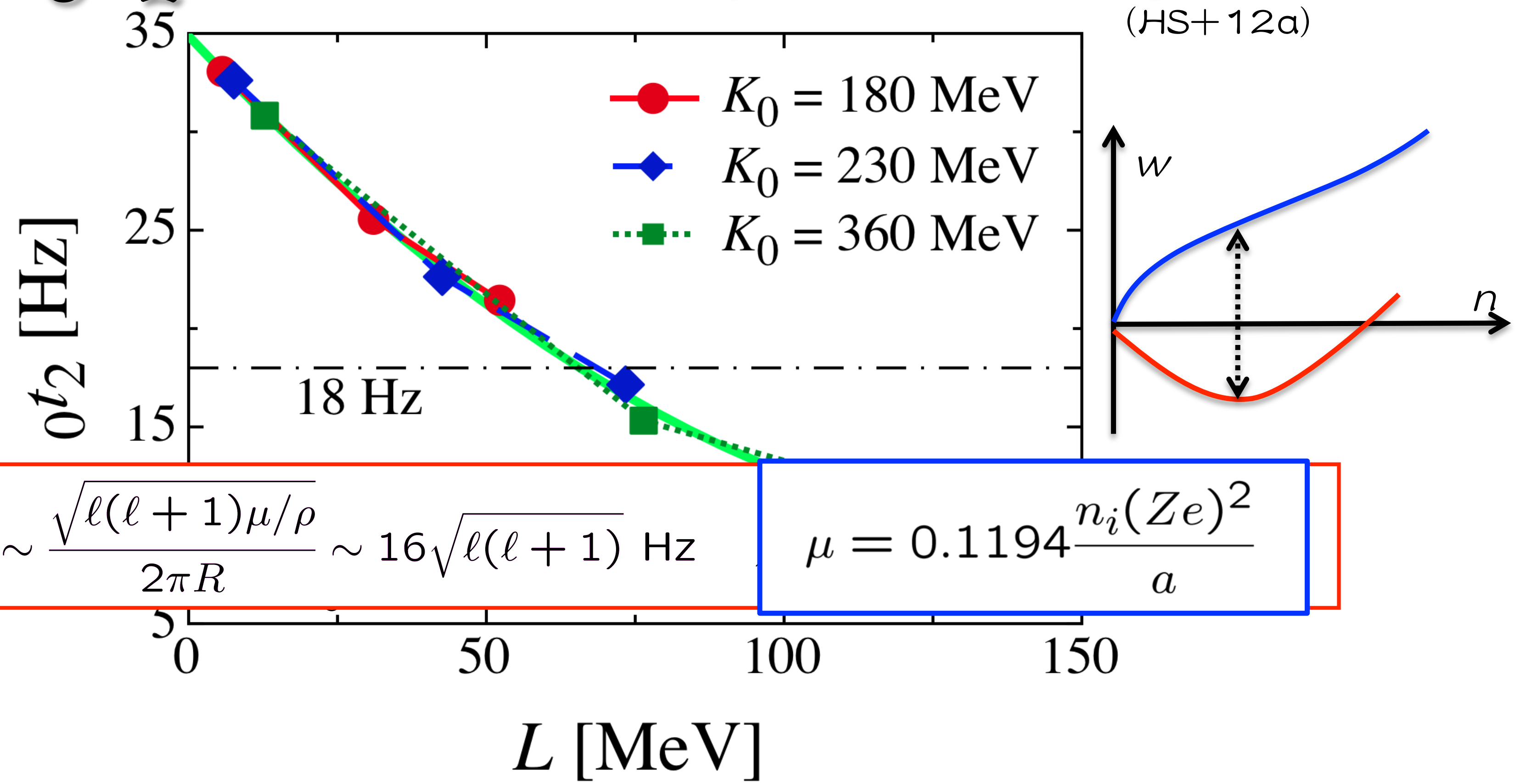
current constraints on K_0 & L
- K_0 = 230 ± 40 MeV (Khan+13)
- 40 ≲ L ≲ 80 MeV (Li+ 13)

for bcc lattice

$$\mu = 0.1194\frac{n_i(Ze)^2}{a}$$

n_i : ion number density
Z : charge of nuclei
a : Wigner-Seitz radius
(Strohmayer+ 91)

${}_0t_2$ without superfluidity



$${}_\ell t_0 \sim \frac{\sqrt{\ell(\ell+1)\mu/\rho}}{2\pi R} \sim 16\sqrt{\ell(\ell+1)}\ \text{Hz}$$

$$\mu = 0.1194\frac{n_i(Ze)^2}{a}$$

- ${}_0t_2$ is almost independent of the value of K_0
- For R=10~14 km and $M/M_\odot$=1.4~1.8, similar dependence on K_0
- Focus on L dependence of ${}_0t_2$

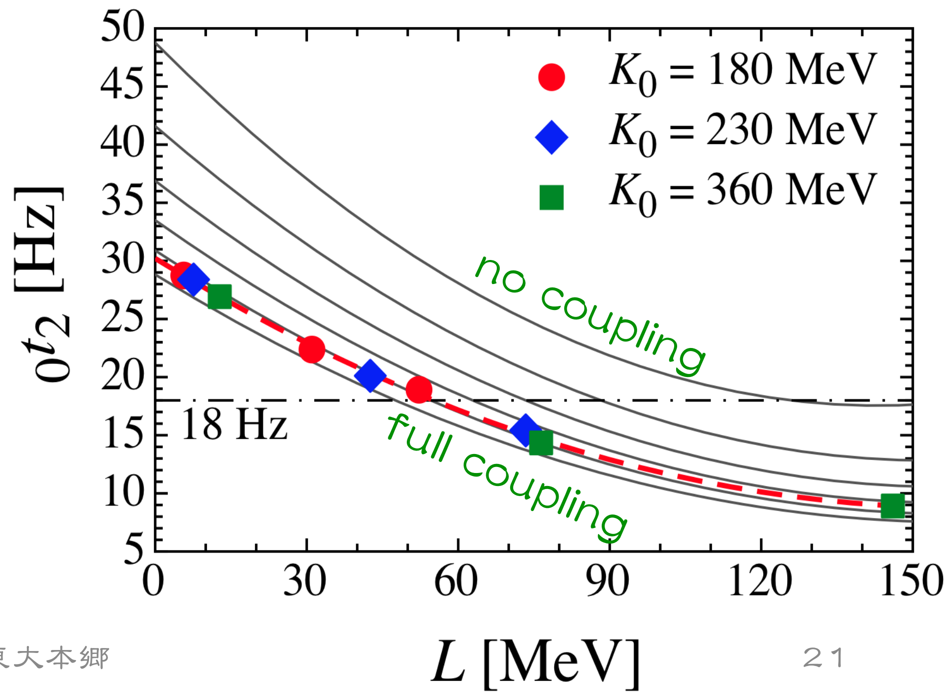
Effect of superfluidity

(HS+12b)

- For $\rho \gtrsim 4 \times 10^{11}$ g cm^{-3}, neutron could drip from nuclei
- Effective enthalpy affecting on the shear oscillations could be reduced
  - shear speed ($v_s^2 \sim \mu / H$) increases due to the effect of superfluidity

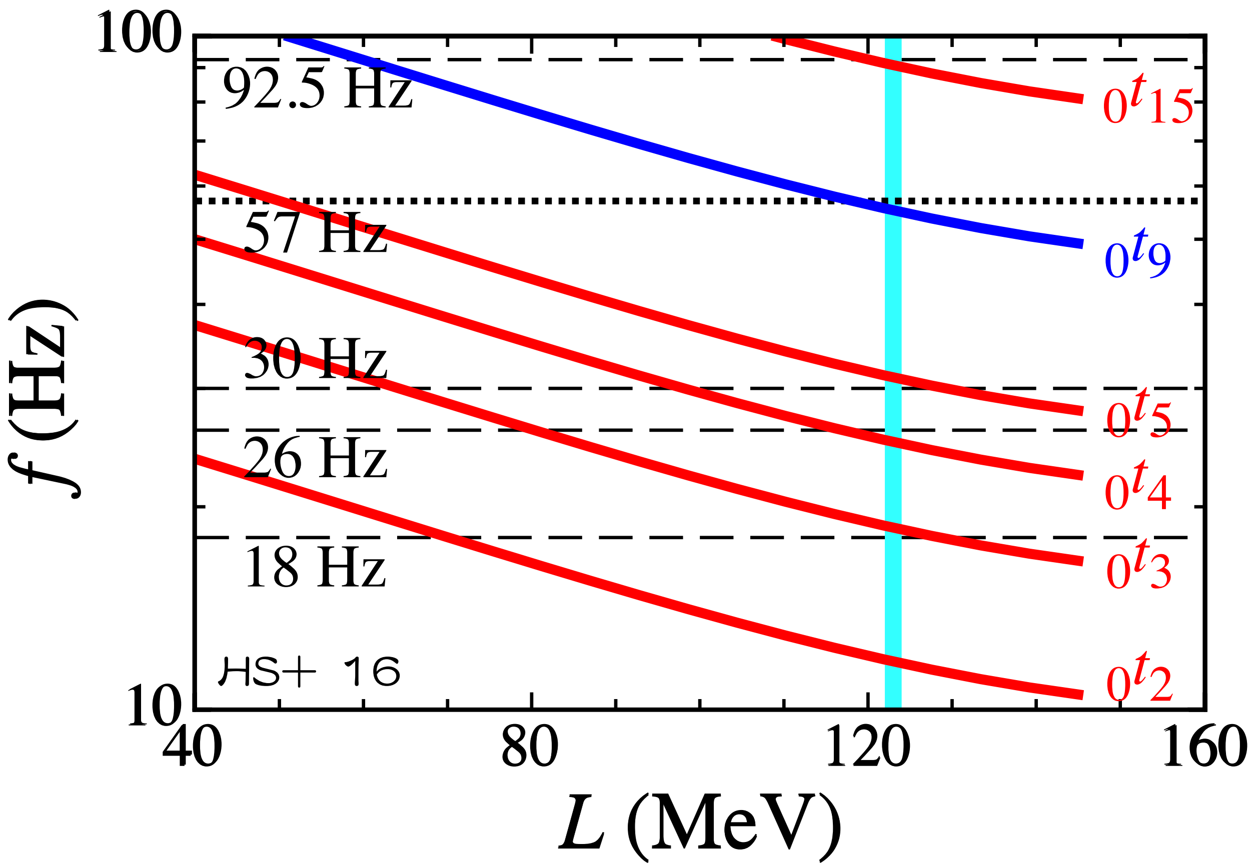
$$\mathcal{Y}'' + \left[\left(\frac{4}{r} + \Phi' - \Lambda'\right) + \frac{\mu'}{\mu}\right]\mathcal{Y}' + \left[\frac{\epsilon + p}{\mu}\omega^2 e^{-2\Phi} - \frac{(\ell+2)(\ell-1)}{r^2}\right]e^{2\Lambda}\mathcal{Y} = 0.$$

- ${}_0t_l$ could also increase due to the effect of superfluidity
- While, the fraction of superfluid neutron in dripped neutron is still unknown…
  - Chamel (2012): superfluid neutron are not so much (~10-30%?)



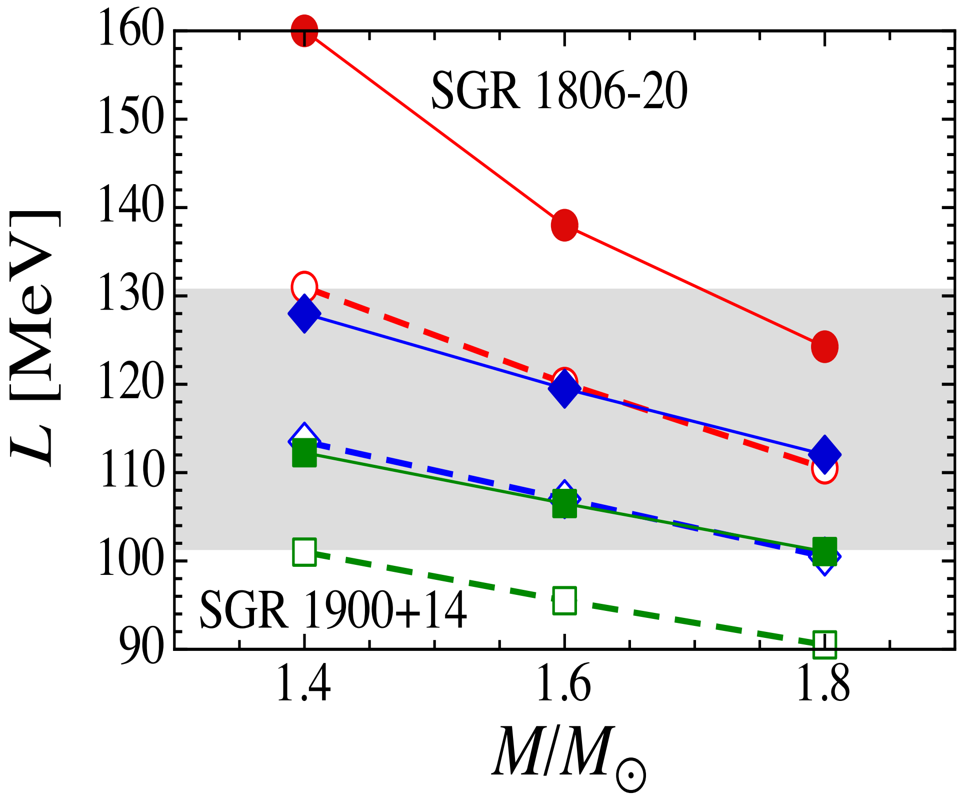
Identifications of SGR 1806-20

- for R = 12 km and M = 1.4$M_\odot$
- discovery of new QPO from SGR 1806-20, which is 57Hz

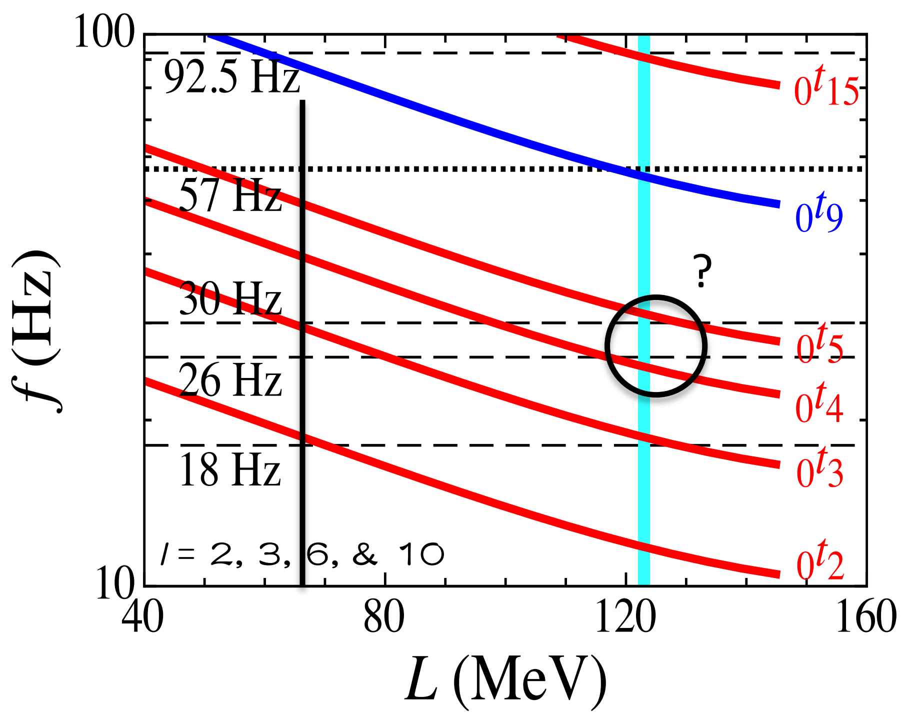


constraint on L via QPO frequencies

1) all QPOs come from crustal torsional oscillations (HS+13a)



for M=1.4M$_\odot$ & R=12km

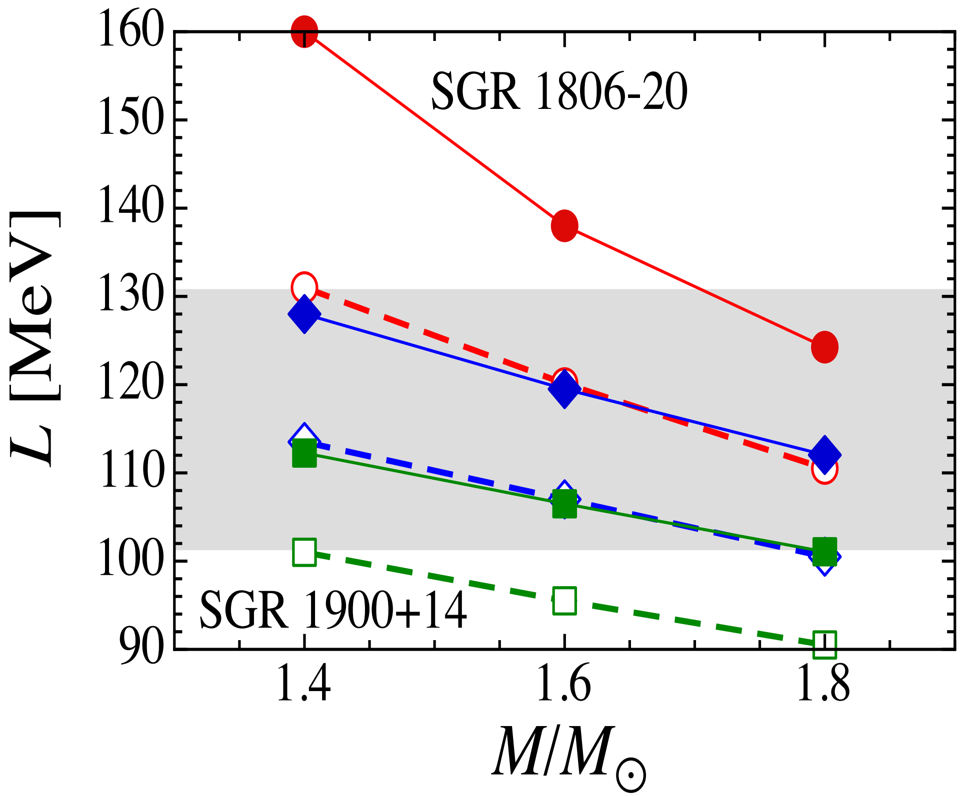


➡ $101.1 \leq L \leq 131.0$ MeV

cf) L = 40 ~ 80 MeV ??

constraint on L via QPO frequencies

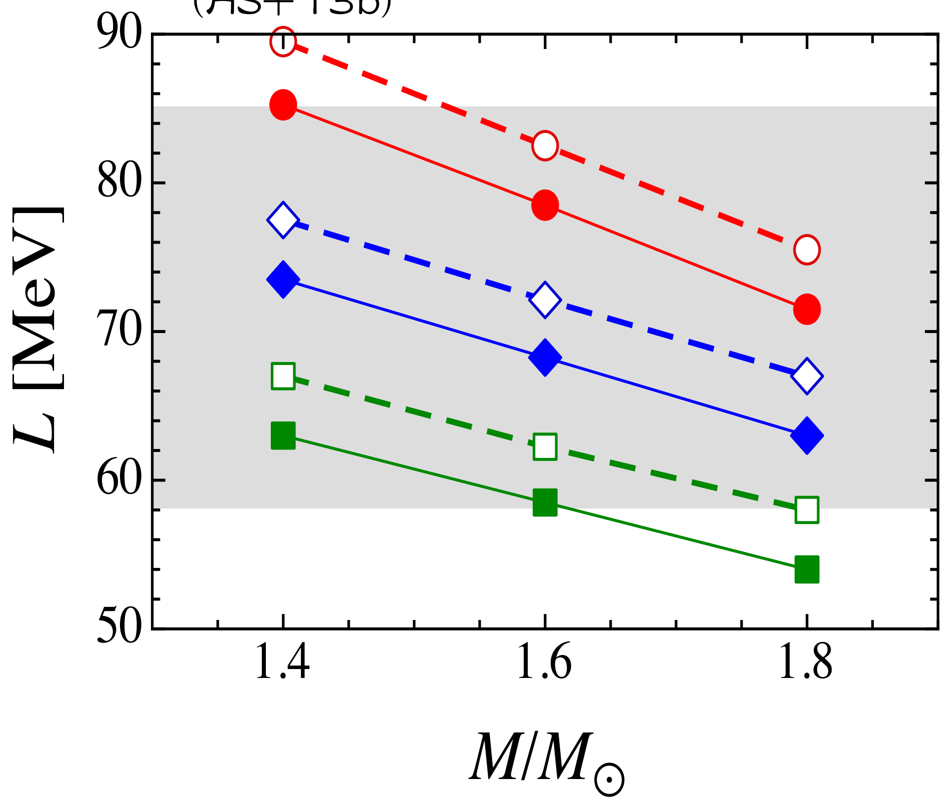
1) all QPOs come from crustal torsional oscillations (HS+13a)



➡ $101.1 \le L \le 131.0$ MeV

cf) L = 40 ~ 80 MeV ??

2) QPOs except for 26Hz come from crustal torsional oscillations (HS+13b)

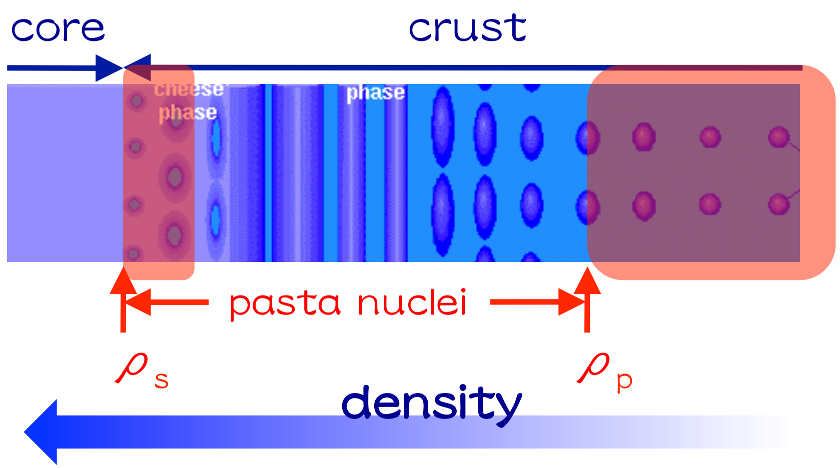


➡ $58.0 \le L \le 85.3$ MeV

need to prepare another oscillation mechanism to explain 26 Hz QPO !

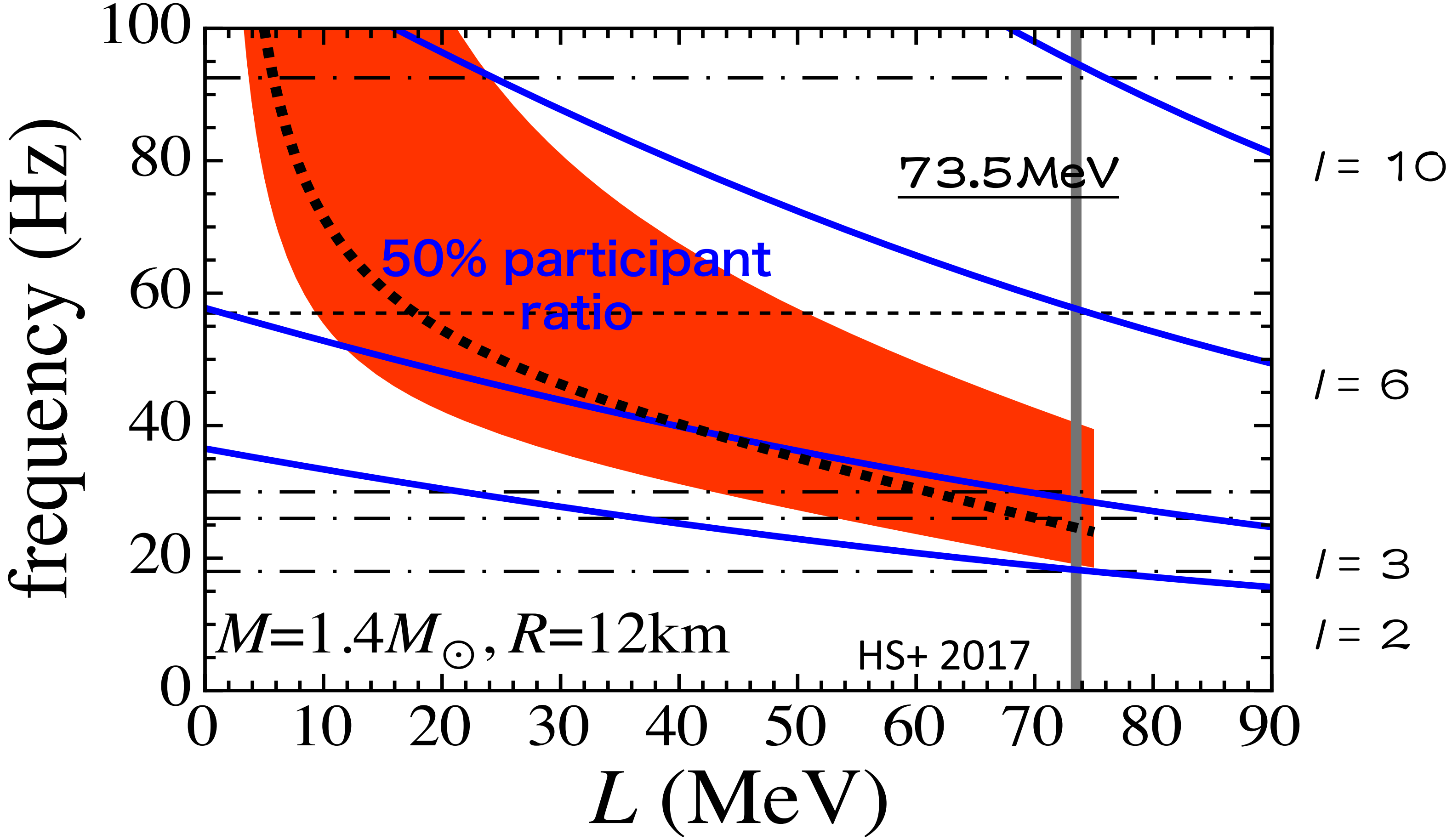
as a possibility of 26Hz...

- we consider the oscillations in the pasta structure
- shear modulus in pasta phase
  - slab phase: shear is the 3rd order of displacement (Landau)
    → in the linear perturbation, oscillations in slab are negligible
  - two *independent* oscillations can be excited in different regions:
    - ① oscillations in spherical and cylindrical nuclei
    - ② oscillations in bubble and cylindrical-hole nuclei
  - as a first step, we consider only oscillations in bubble phase
    - for $L \gtrsim 75$ MeV, bubble structure disappears



$$\mu = 0.1194 \frac{n_i (Ze)^2}{a}$$

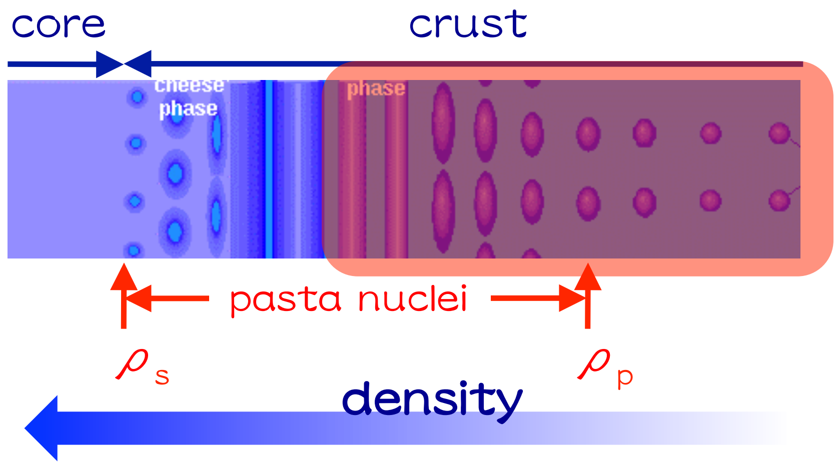
comparison with QPOs

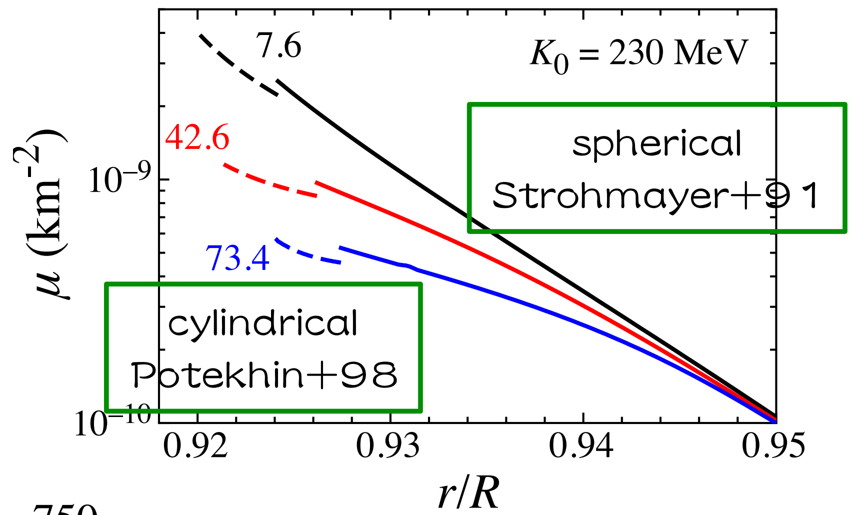


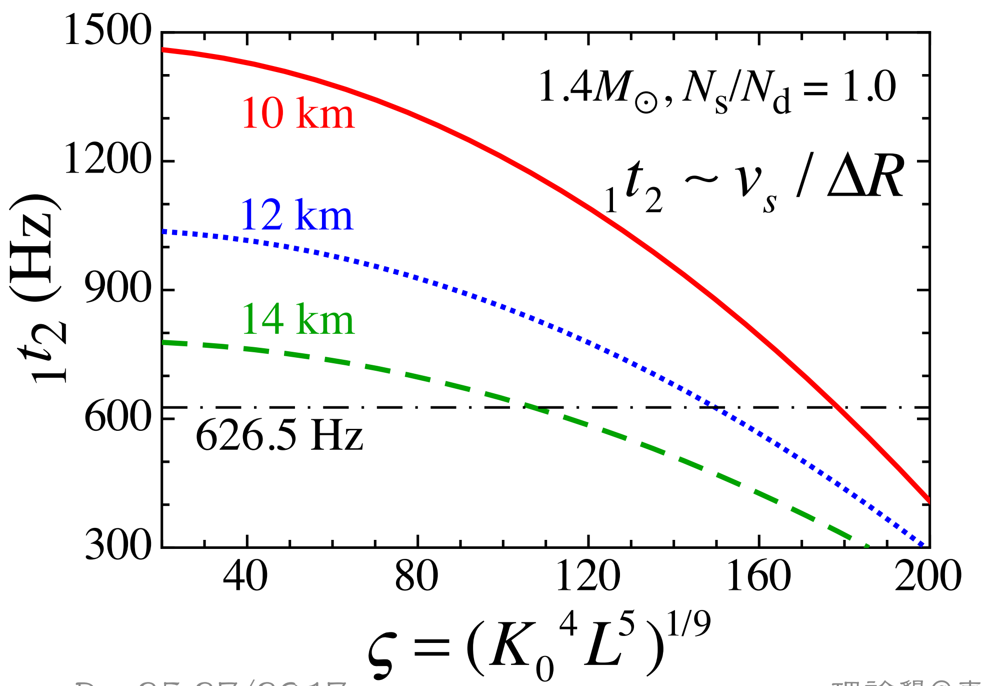
- Oscillation in bubble might be possible to correspond to 26Hz QPO, depending on the entrainment rate.
- Observational evidence for showing the existence of bubble phase!?

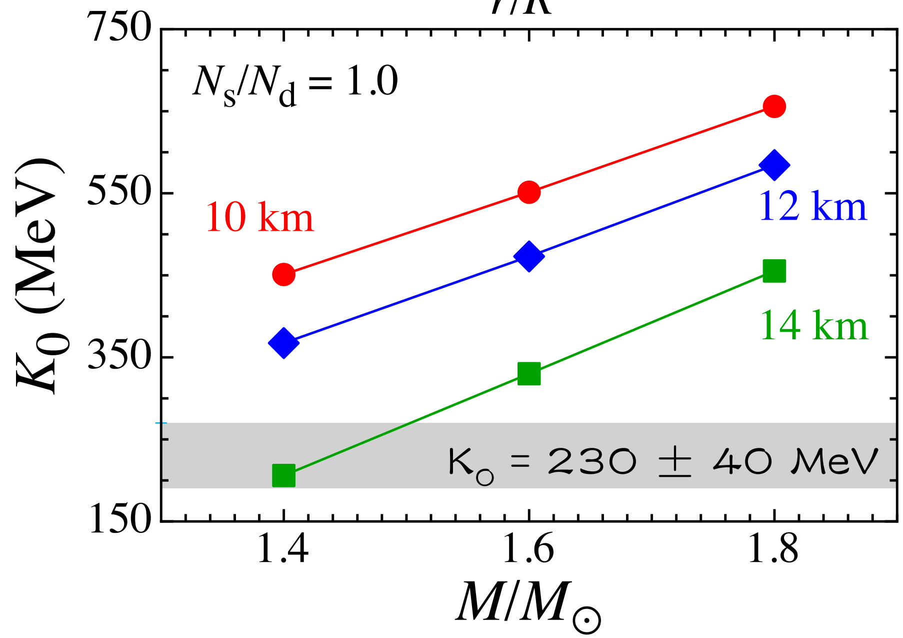
effect of pasta structure

HS+ in prep.

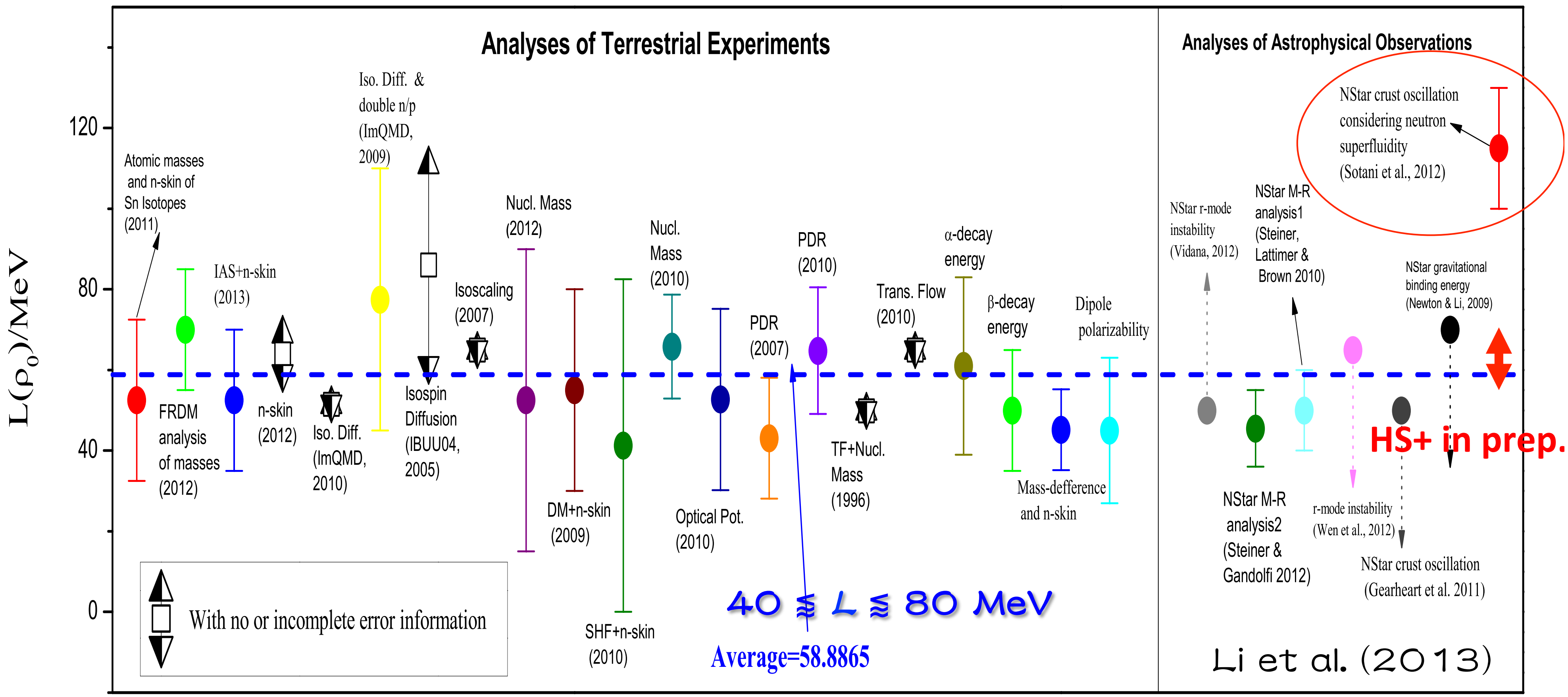








constraints on L



- 26Hz : bubble ($_0t_2$), 626.5Hz : spherical + cylindrical ($_1t_2$)
  → SGR1806-20 should be relatively low mass NS ($M \sim 1.3 M_\odot$, R~13km??)
  → L ~ 58-70MeV

Oscillations & Instabilities

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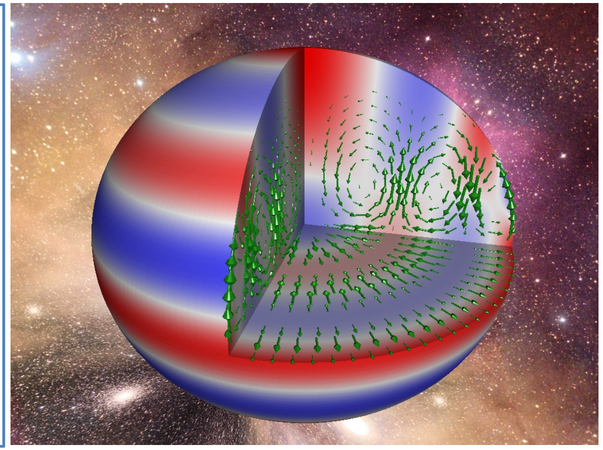

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UNLESS
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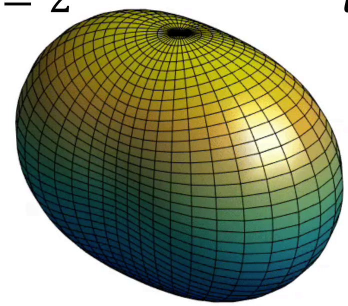

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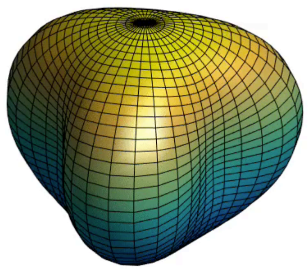
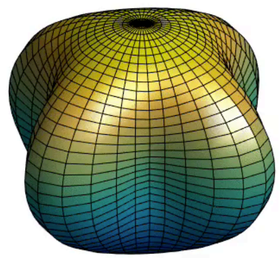


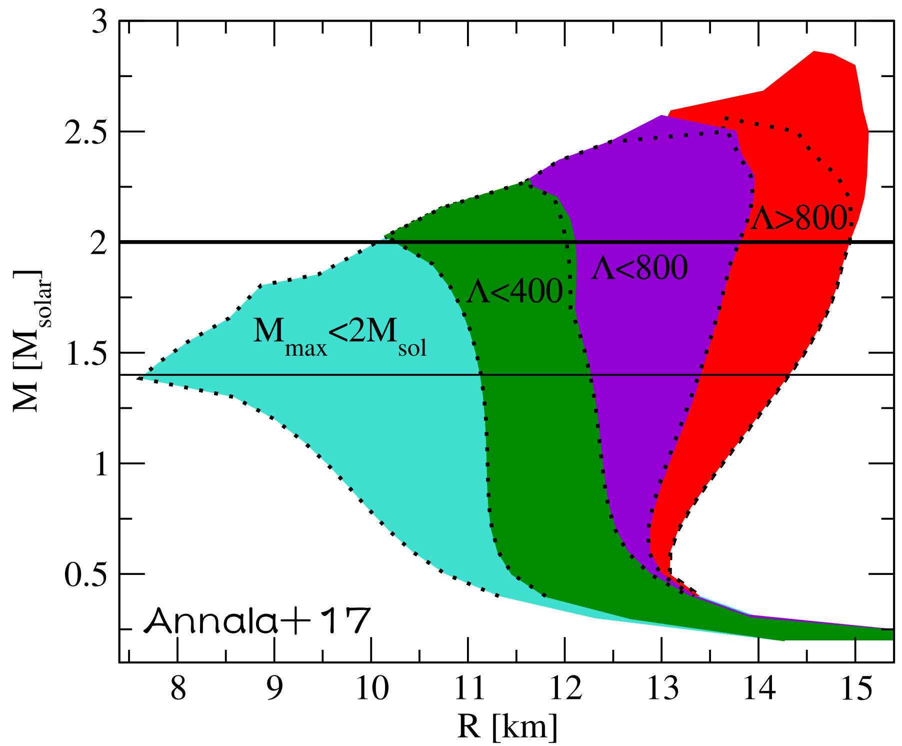
$l = 4, m = 4$



slide by Kokkotas

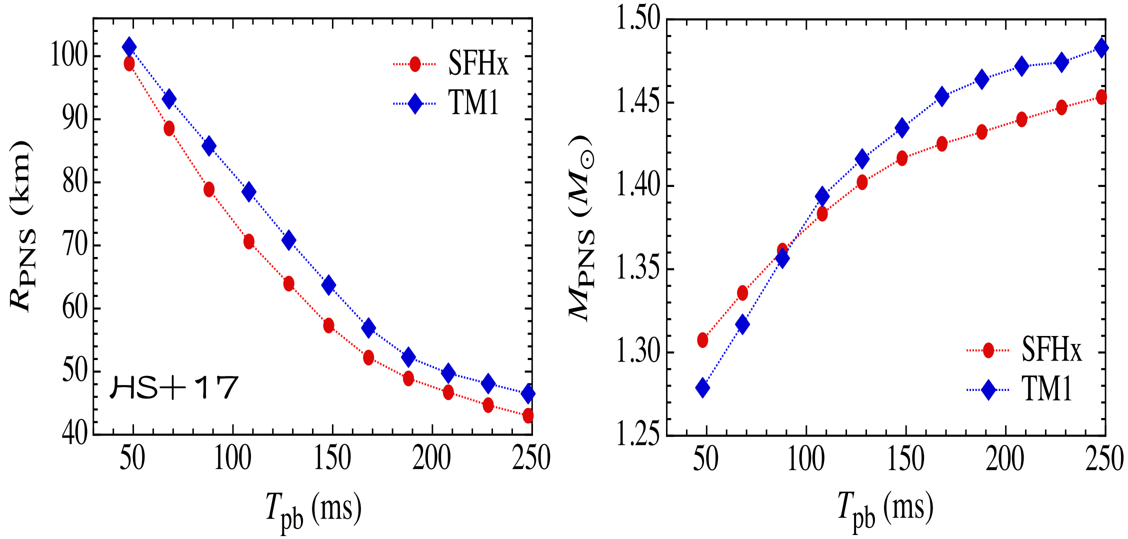
Dawn of GW astronomy era

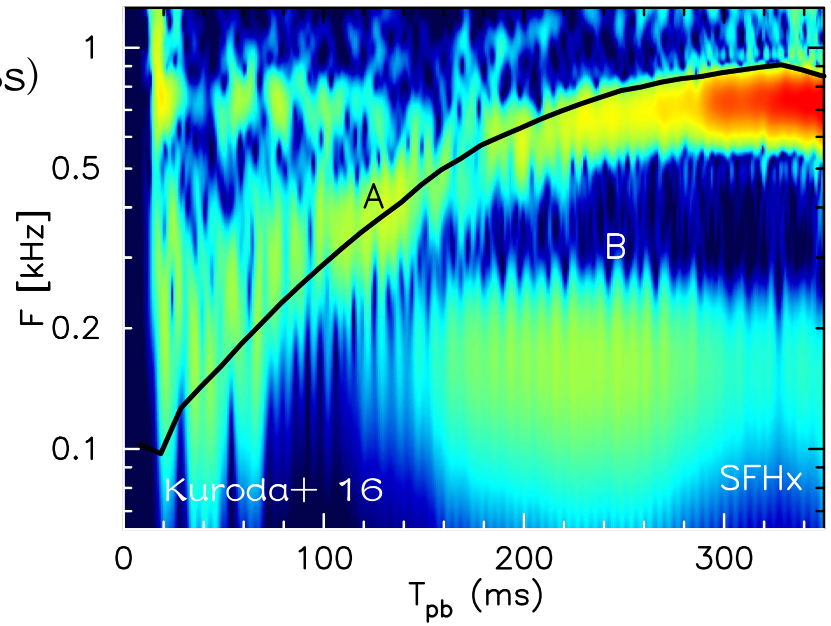
- tidal deformability : $\lambda = -Q_{ij}/E_{ij}$
  - Q_{ij} : response of quadrupole moment
  - E_{ij} : external tidal field
  - stiffer EOS : less compactness NS (or large radius with fixed mass) → larger λ
- Λ: dimensionless quantity of λ
  - $\lambda = \frac{C^5}{G}\Lambda R^5$ & $C = \frac{GM}{c^2 R}$
  - $\Lambda < 800$ (GW170817)



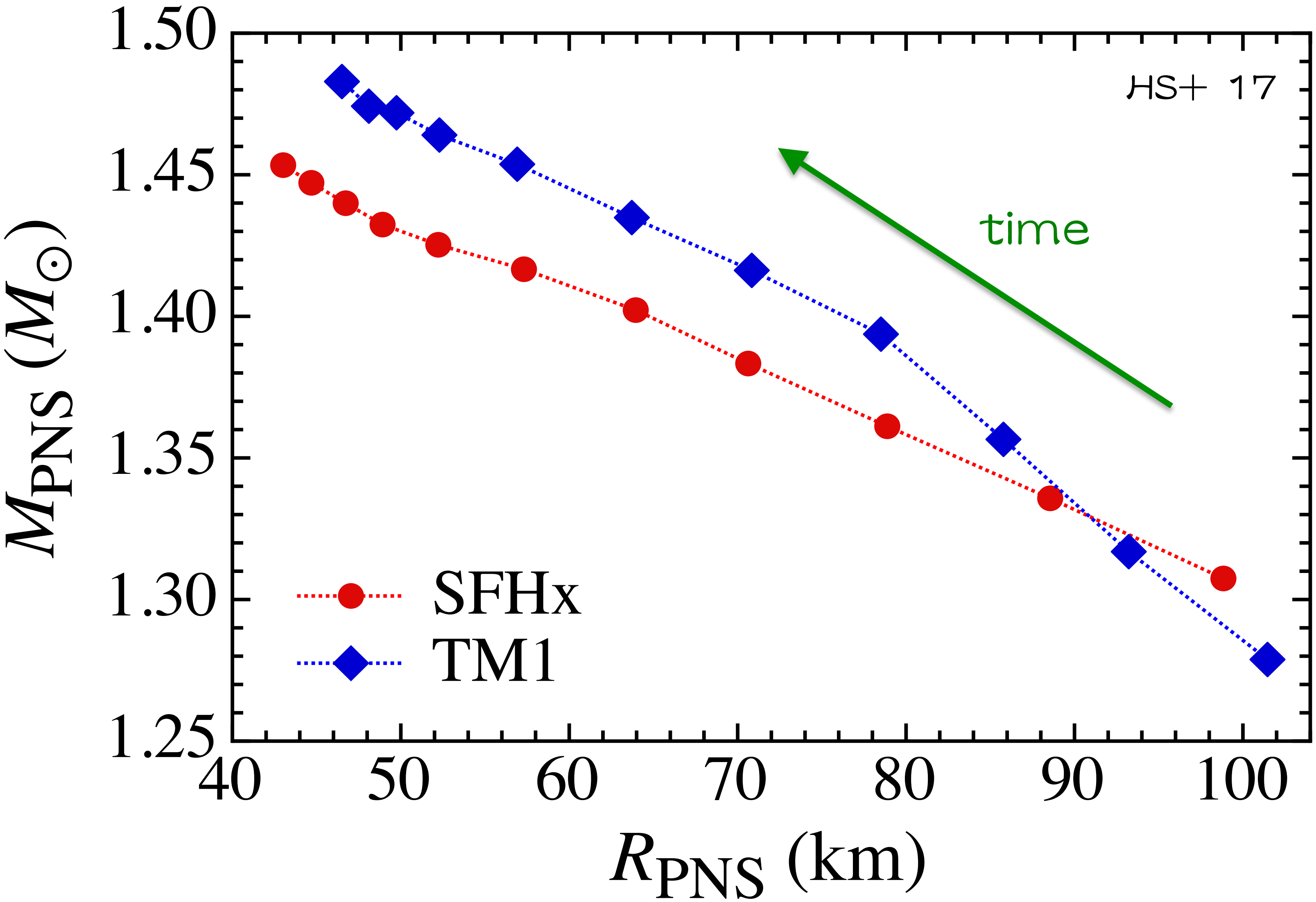
second candidate of GW source

- supernovae
  - event rate : ~1/100 yr in our galaxy
  - compered to binary merger, system is relatively spherically symmetric
    - less energy of gravitational waves
- numerical simulation shows specific GW frequencies
  - still difficult to extract GW signal, especially from protoneutron stars (PNSs)

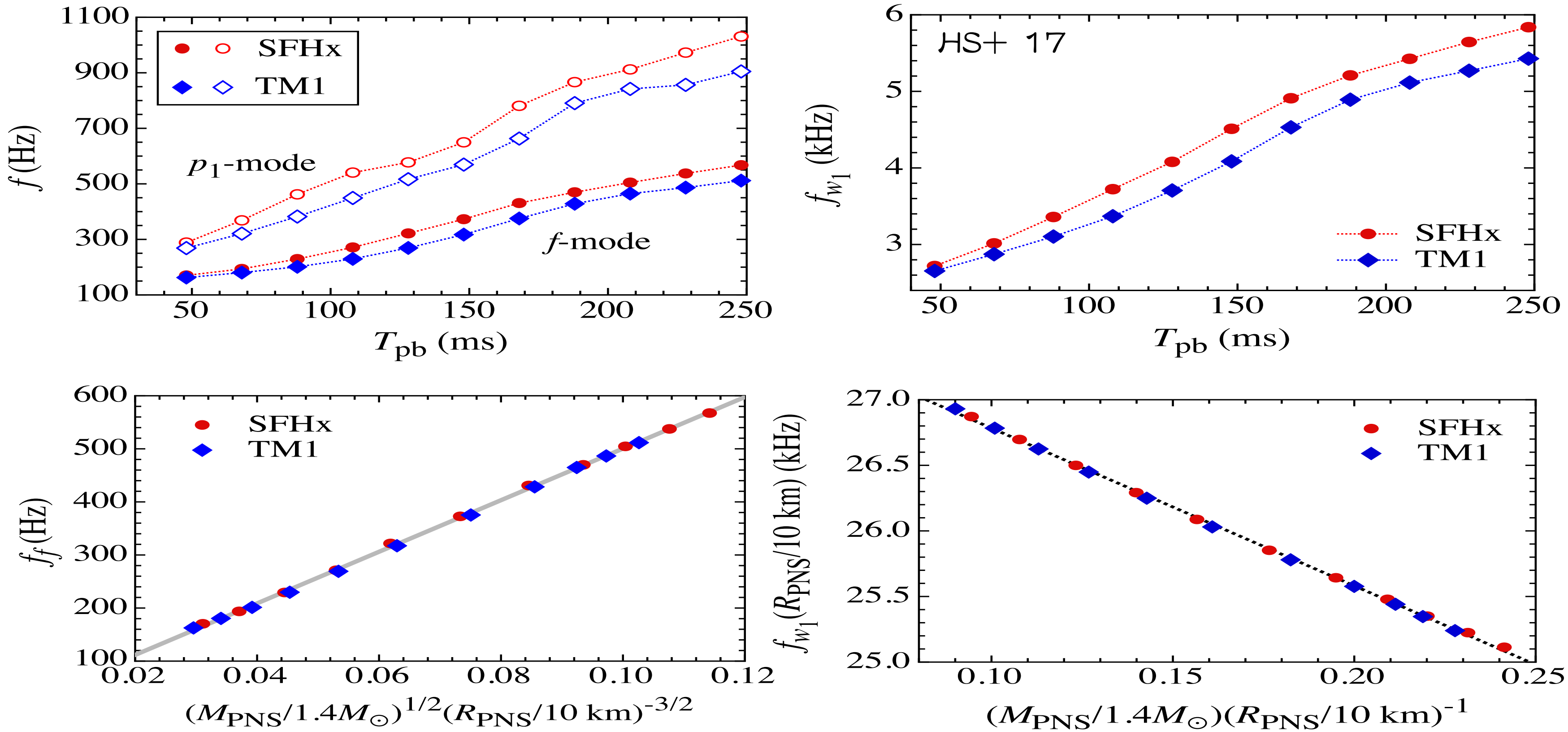




M_{PNS} & R_{PNS} evolution



evolutions of f- & w_1-modes

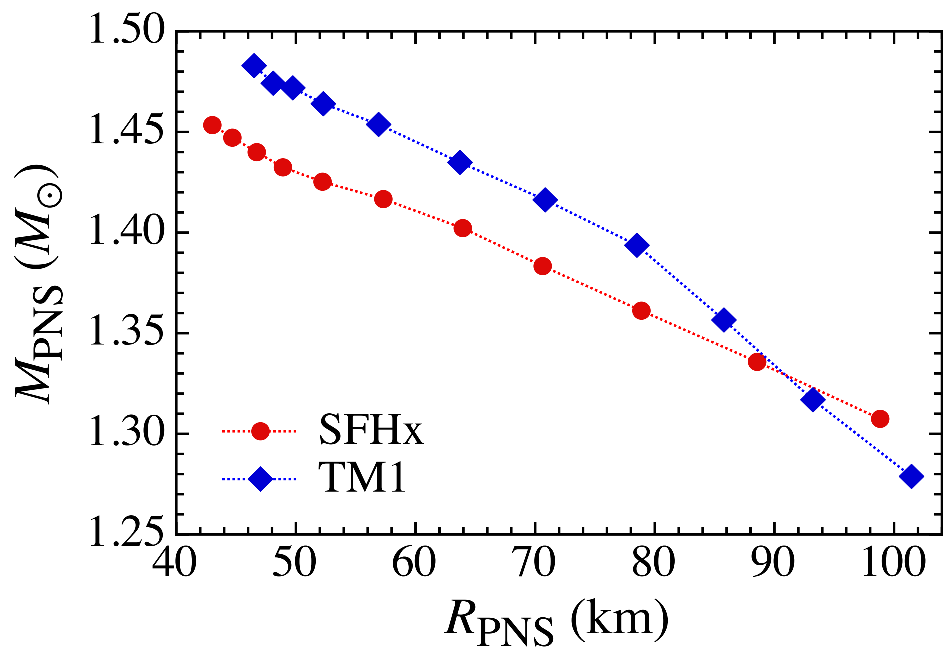


$$f_f^{(PNS)}(\text{Hz}) \approx 14.48 + 4859\left(\frac{M_{PNS}}{1.4\ M_\odot}\right)^{1/2}\left(\frac{R_{PNS}}{10\text{ km}}\right)^{-3/2}$$

$$f_{w_1}^{(PNS)}(\text{kHz}) \approx \left[27.99 - 12.02\left(\frac{M_{PNS}}{1.4\ M_\odot}\right)\left(\frac{R_{PNS}}{10\text{ km}}\right)^{-1}\right] \times \left(\frac{R_{PNS}}{10\text{ km}}\right)^{-1}$$

determination of EOS

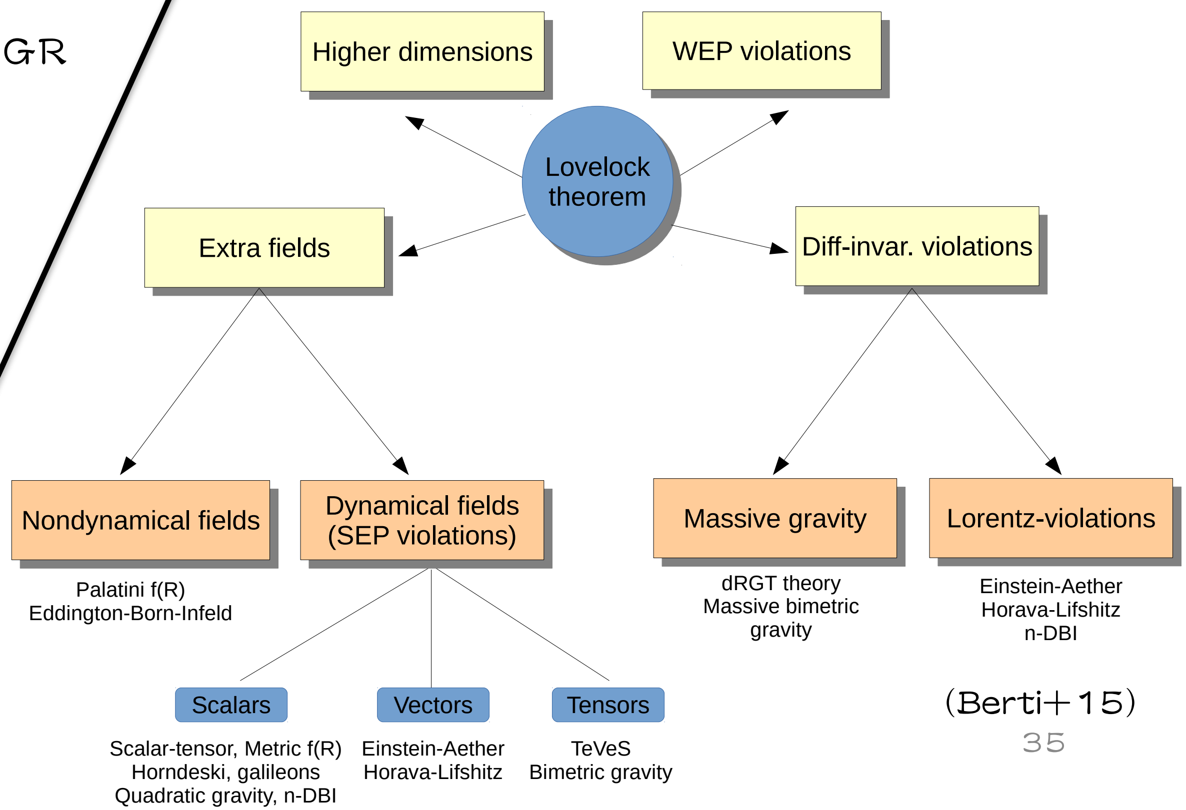
- with f- & w_1-modes GW observations, one can get two independent properties, M_{PNS}/R_{PNS}^3 & M_{PNS}/R_{PNS} at each time after core bounce
- one can determine (M_{PNS}, R_{PNS}) at each time after core bounce
  → determination of the EOS
- unlike cold NS cases, in principle one can determine the EOS even with ONE GW event !



alternative theories of gravity

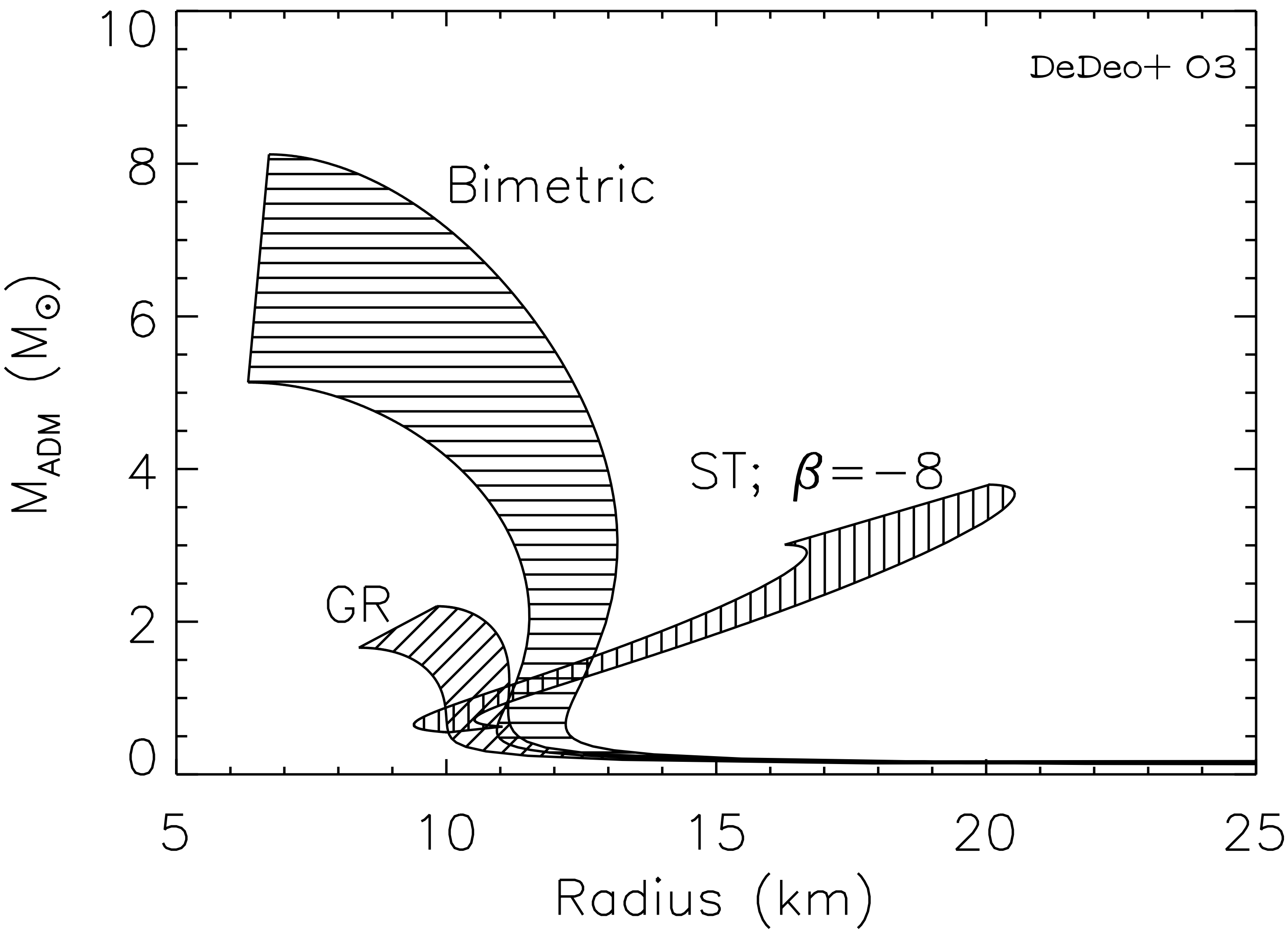
- most of tests of GR : weak field gravity (except for binary pulsar)
  - gravity in the range of 10^{-6} m-10^{11}m (~1 AU) is probed
  - GR may be modified at both low and high energies
- Lovelock's theorem (Lovelock 71):
  - *In four spacetime dimensions the only divergence-free symmetric rank-2 tensor constructed solely from the metric $g_{\mu\nu}$ and its derivatives up to second differential order, and preserving diffeomorphism invariance, is the Einstein tensor plus a cosmological term* (Berti+15).
  - possibility for modification of GR
    - additional field(s)
    - higher dimensions
    - diff-invar. violations
    - WEP violations
      Eotvos parameter (Earth;Be-Ti) ~10^{-13} (Schlamminger+08)

- Lorentz-invariance
- gravity should be mediated by a massless spin-2 field

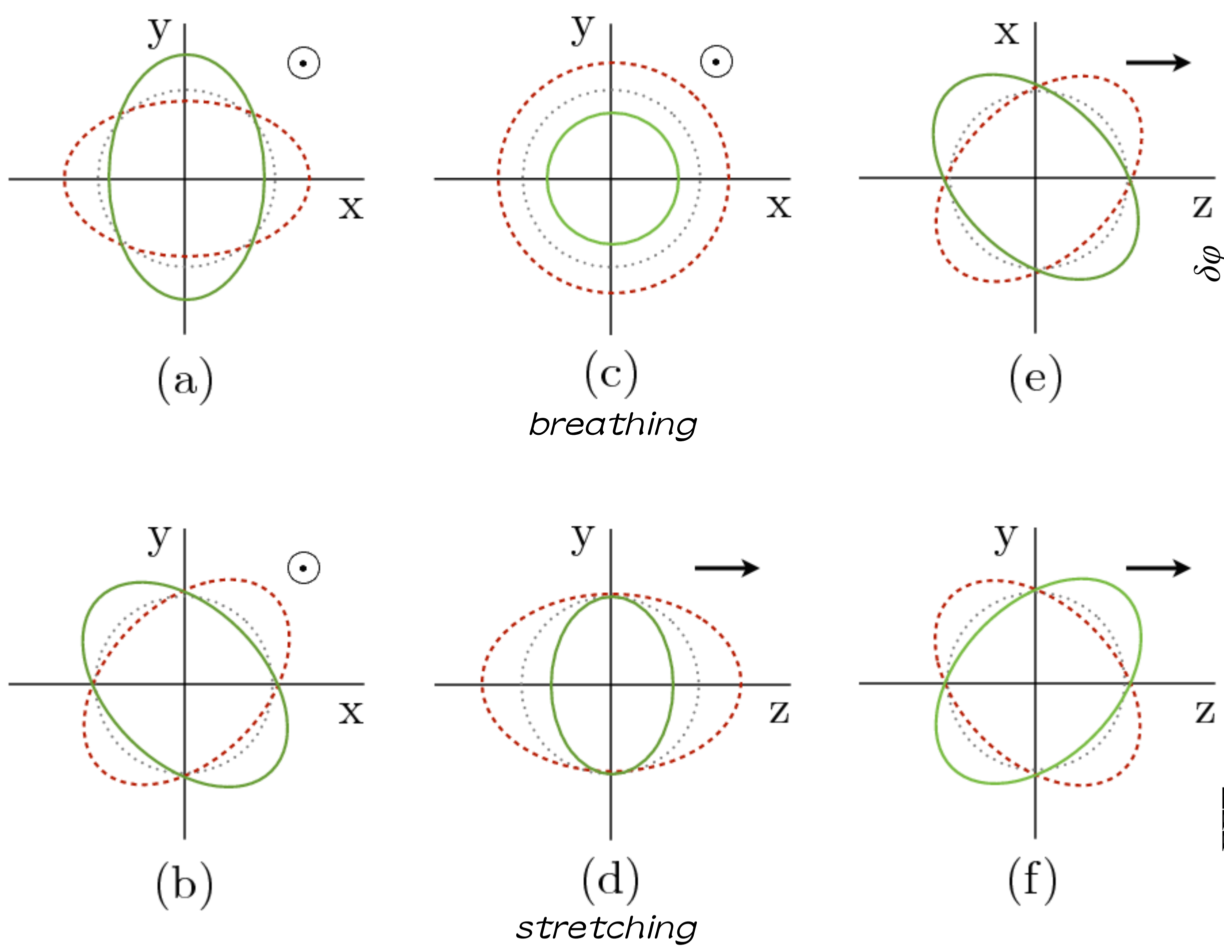


(Berti+15)

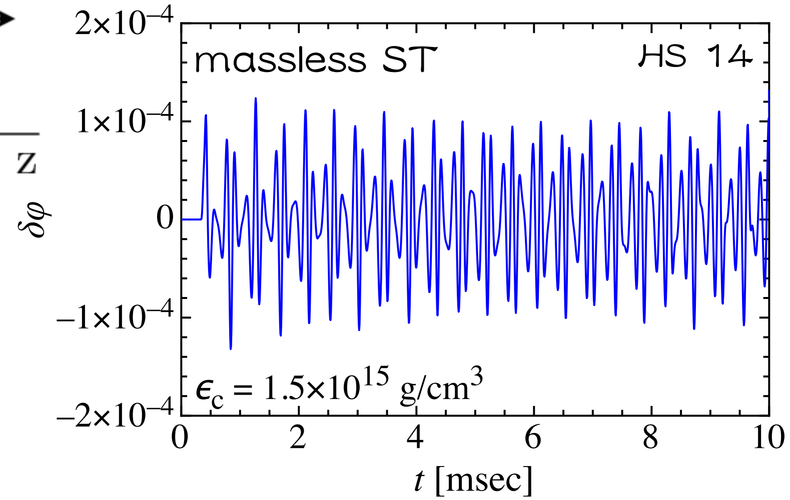
EOS vs. theory of gravity

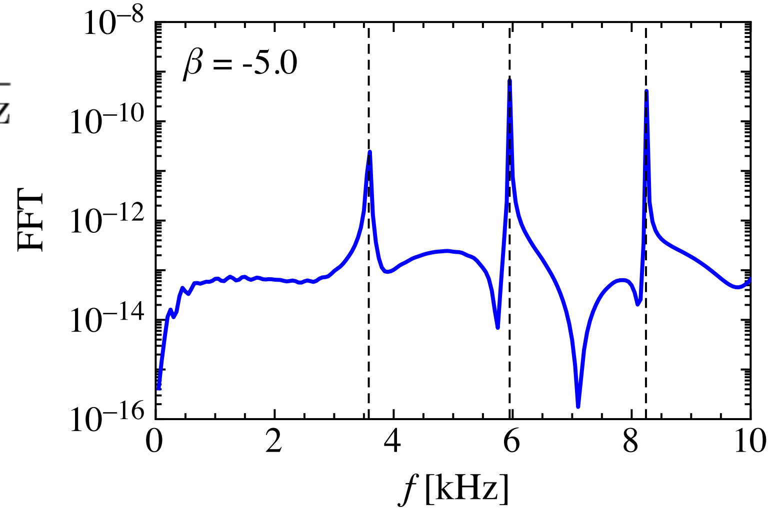


gravitational-wave polarization



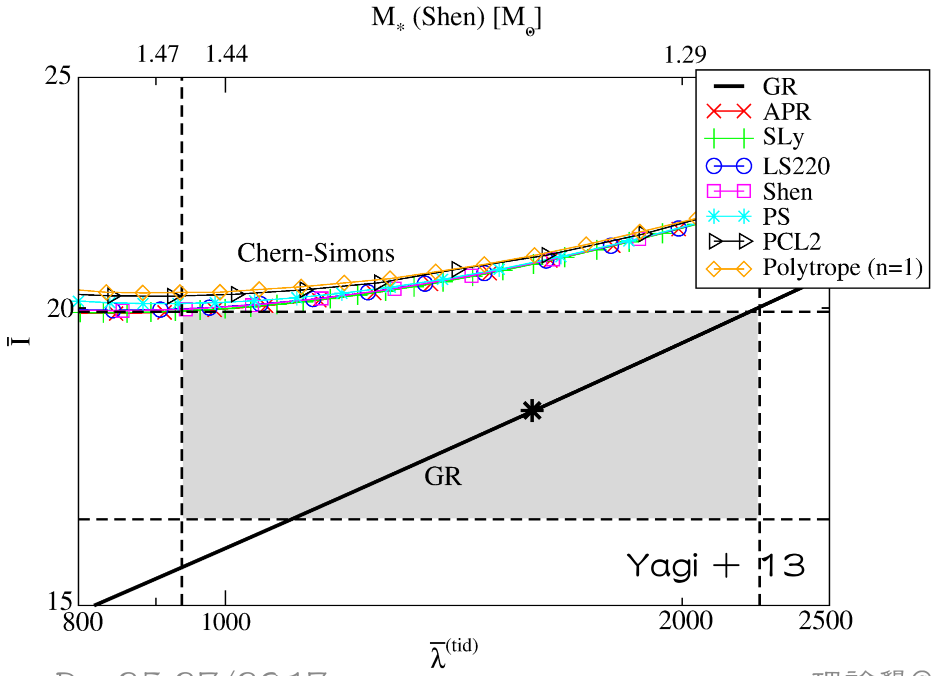
Mirshekari 13

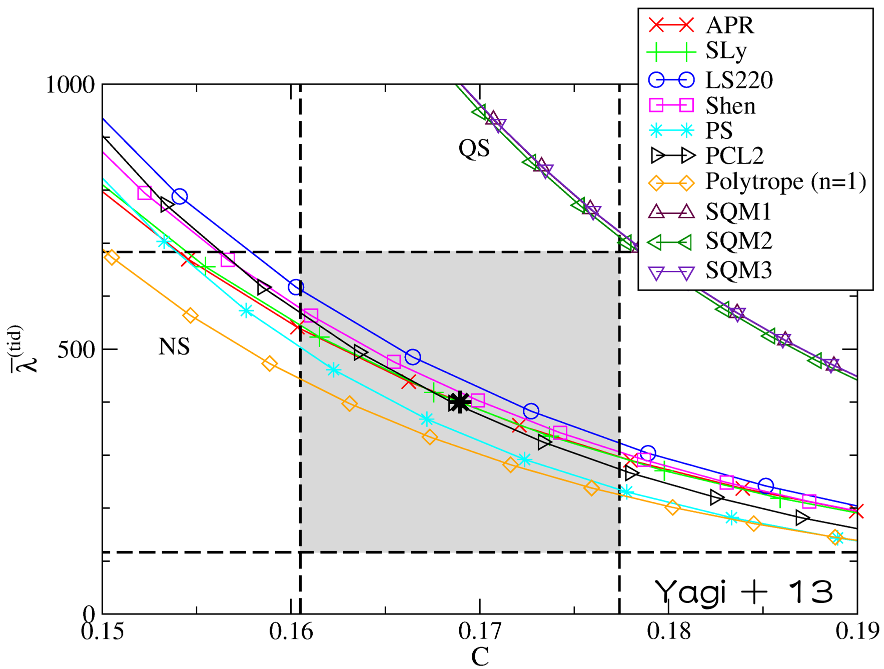




I-Love-Q

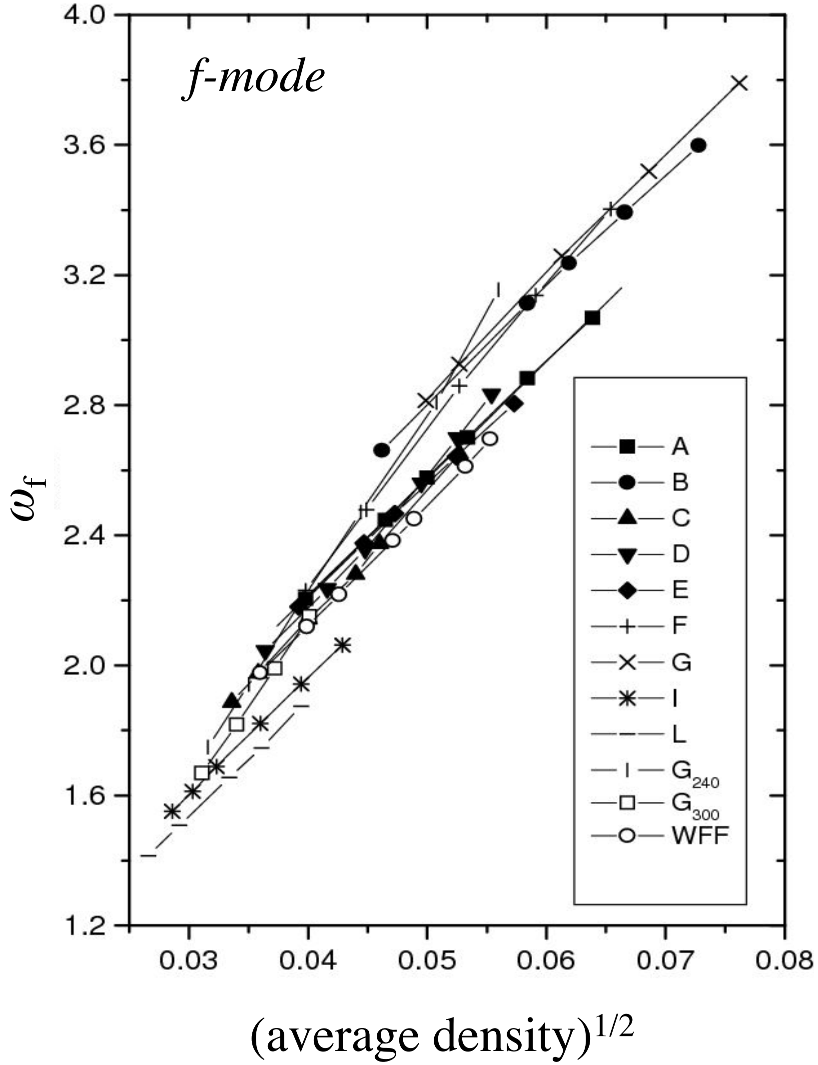
- universal relations between moment of inertia, the Love number, and the quadrupole moment (Yagi & Yunes 13).
  - moment of inertia : $I = J/\Omega \rightarrow \bar{I} = I/M^3$
  - (spin-induced) quadrupole moment : $Q \rightarrow \bar{Q} = -Q/(M^3\chi^2), \quad \chi \equiv J/M^2$
  - tidal Love number : $\lambda = -Q_{ij}/E_{ij} \rightarrow \bar{\lambda} = \lambda/M^5$

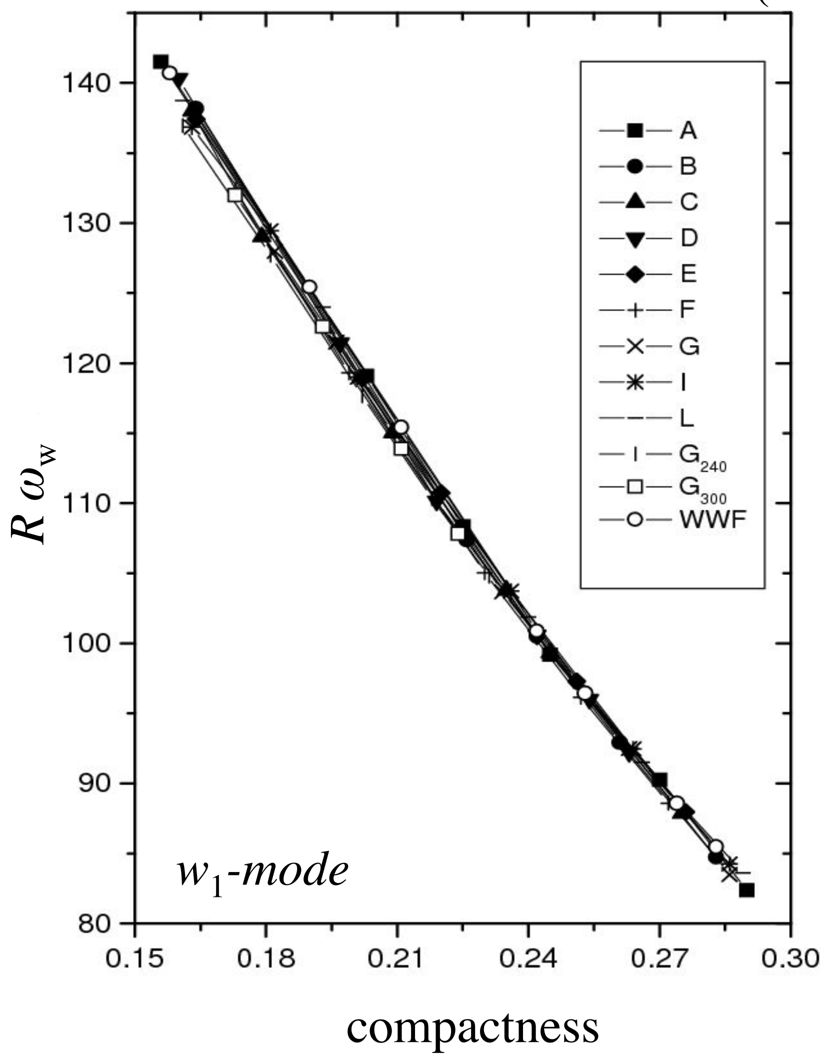




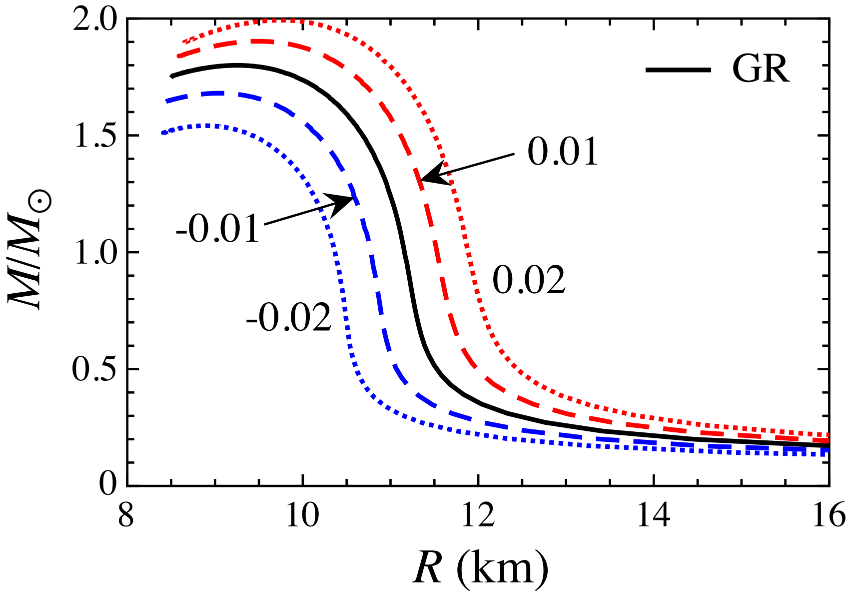
f- & w1-modes GW from NS

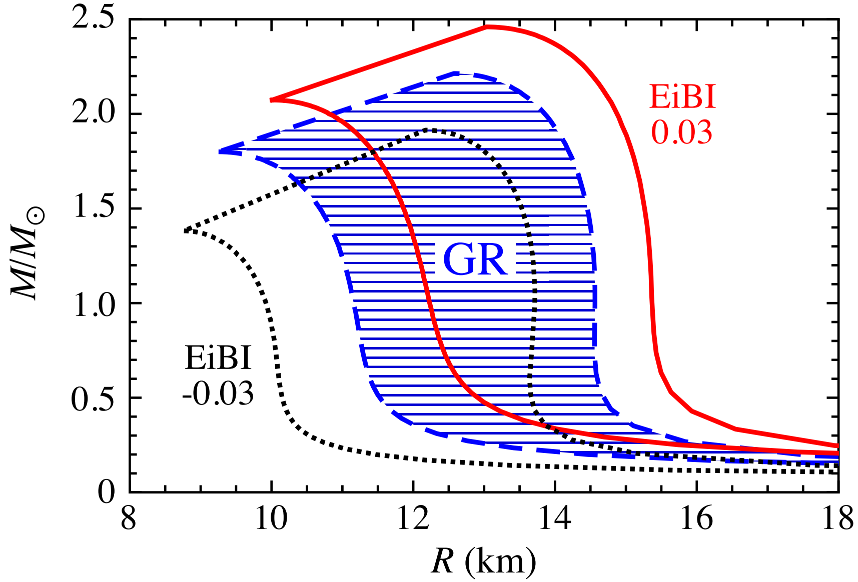
Andersson & Kokkotas (1998)

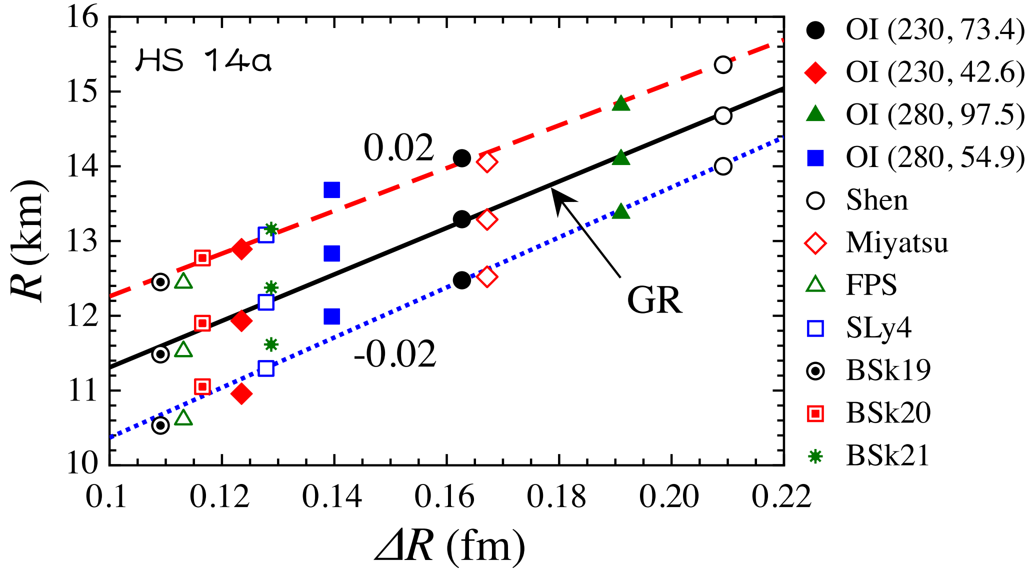




another possibility (EiBI)







R for $0.5M_\odot$ NS - n-thickness of ^{208}Pb

conclusion

- Neutron stars are a suitable candidate for probing physics in extreme states.
  - EOS for a high density region
  - strong magnetic field
  - theory of gravity
- Observations of M, R, & f can help us to understand NS physics
  - asteroseismology
  - Now, it is becoming to adopt GWs as a tool of astronomy
- Many alternative theories of gravity have been proposed
  - observation of GW polarization
  - a kind of universal relation