BH perturbations & gauge dof in the near-horizon limit

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Contents of the Talk

- Introduction
- Problems in 2nd-order perturbations
- Singular behavior of gauge dof
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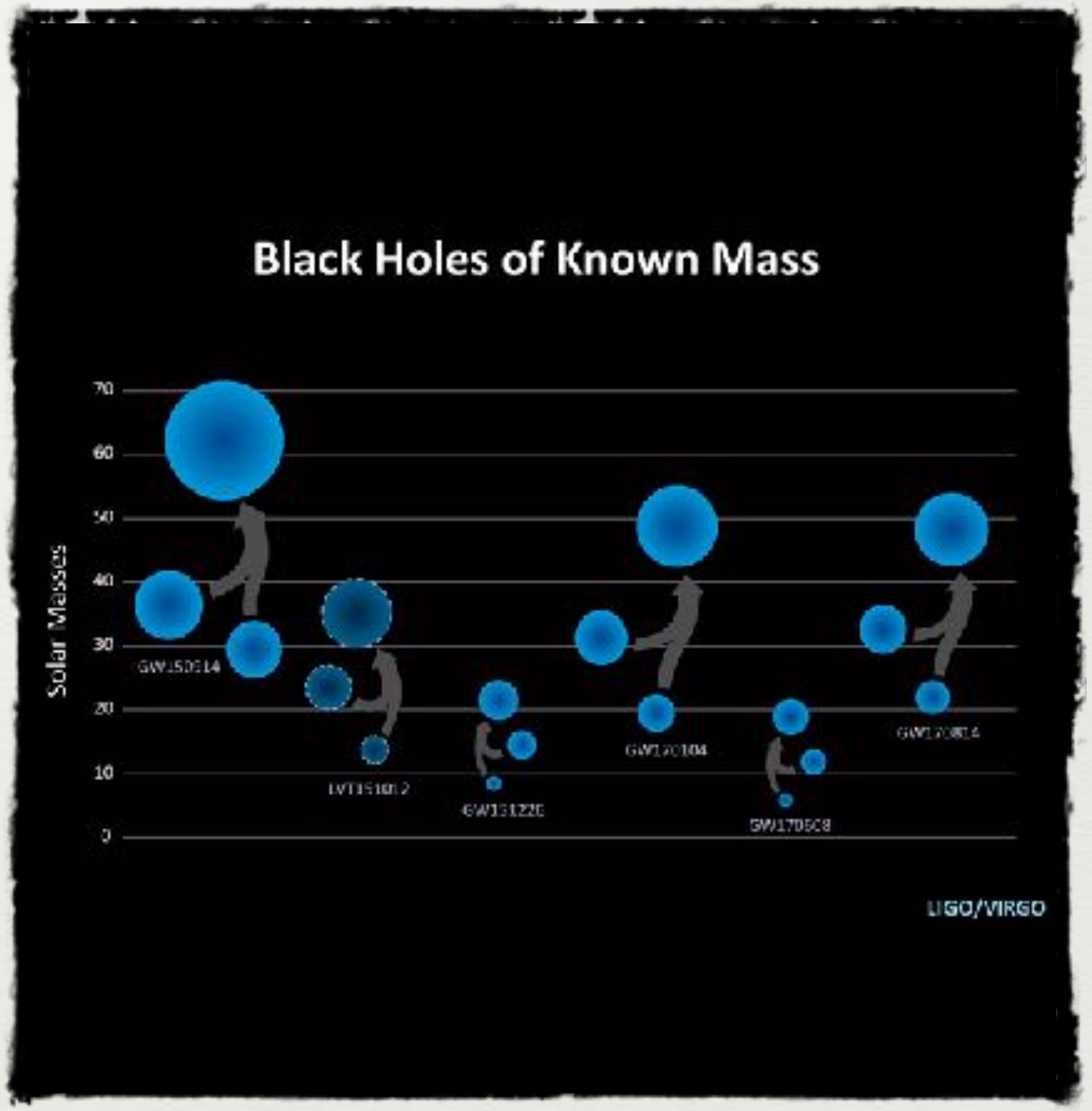
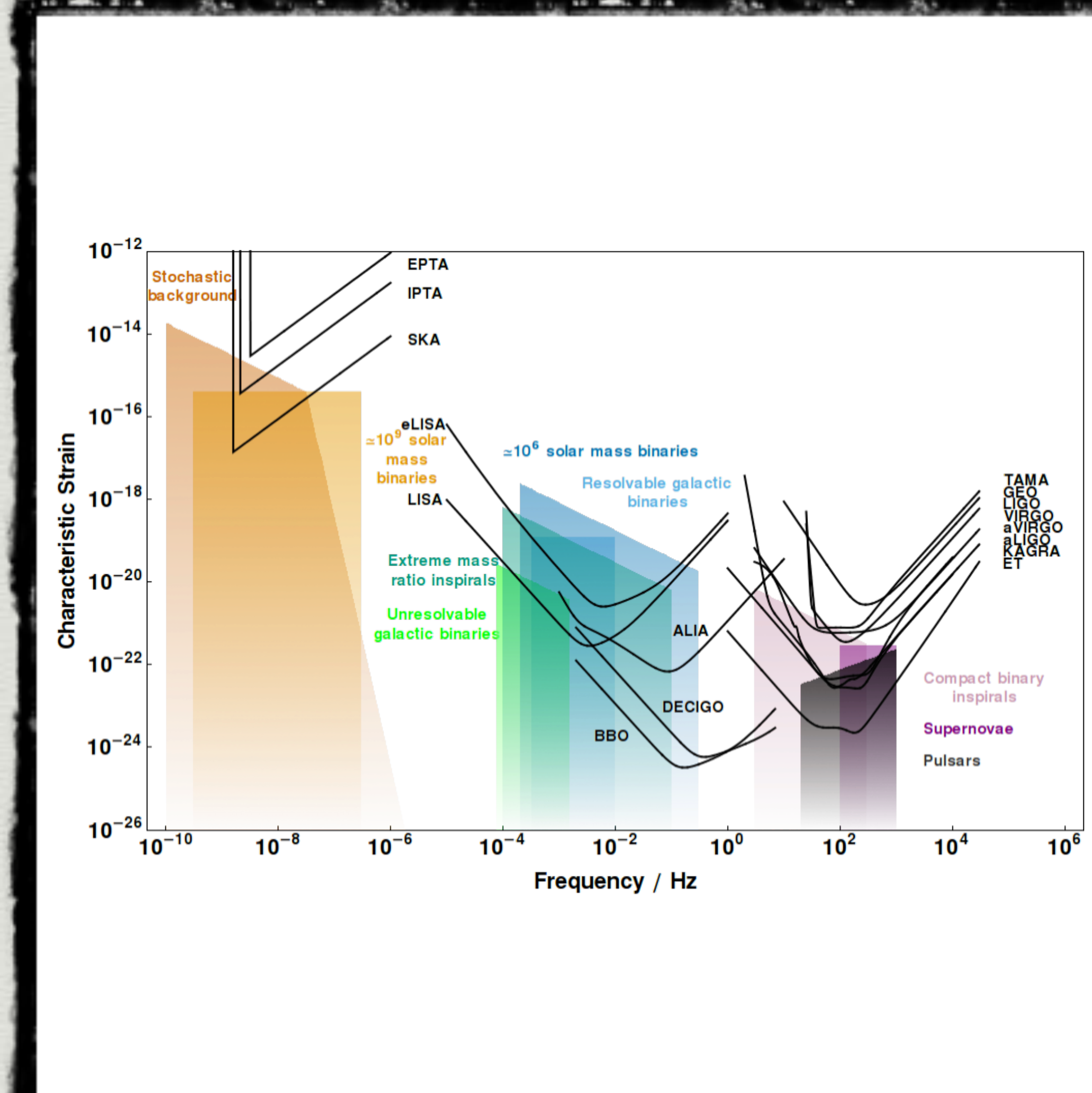
Detections of GWs from Binaries

- 6(+1) events of BBH/BNS merger
- New window of physics (test of GR, EoS of NS, GW propagation, early universe…)

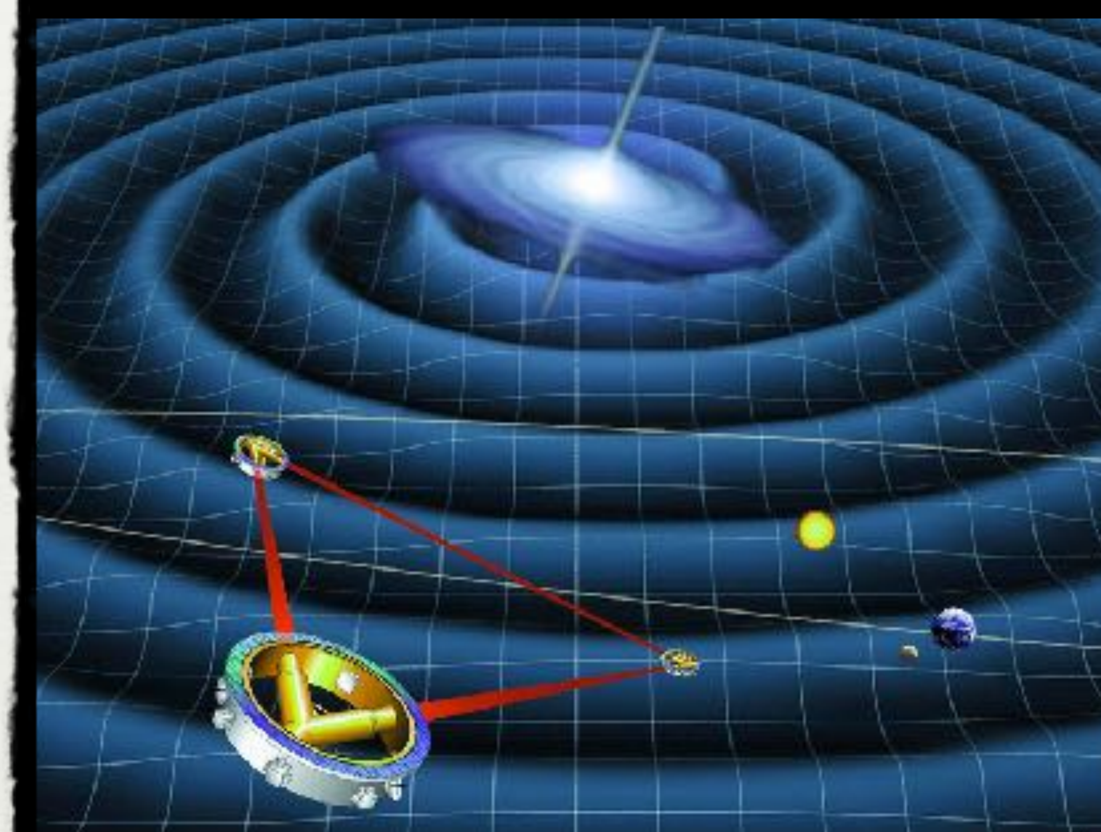


Current & Future GW Detectors

Noise curves of GW detectors

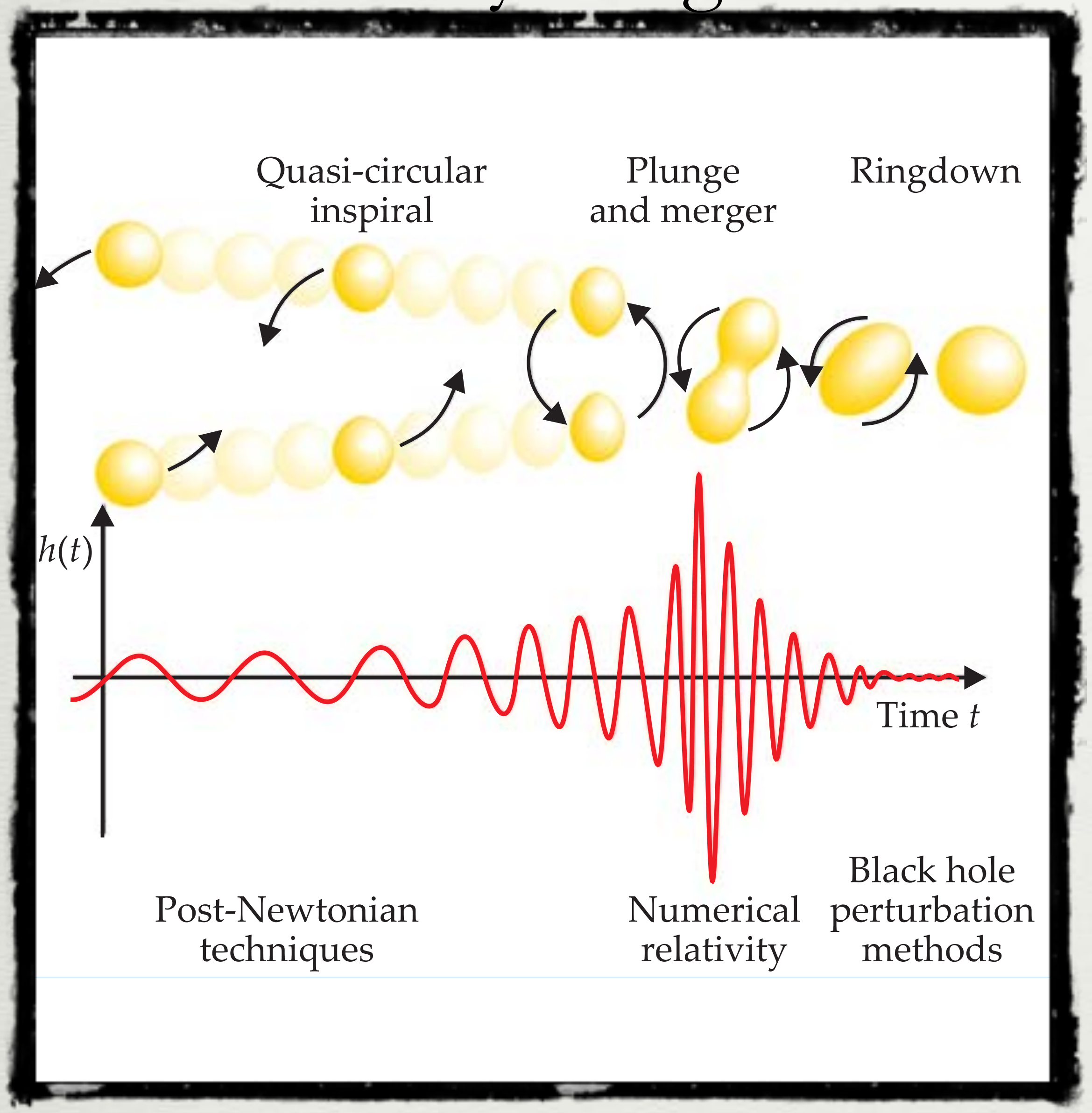


Conception of LISA

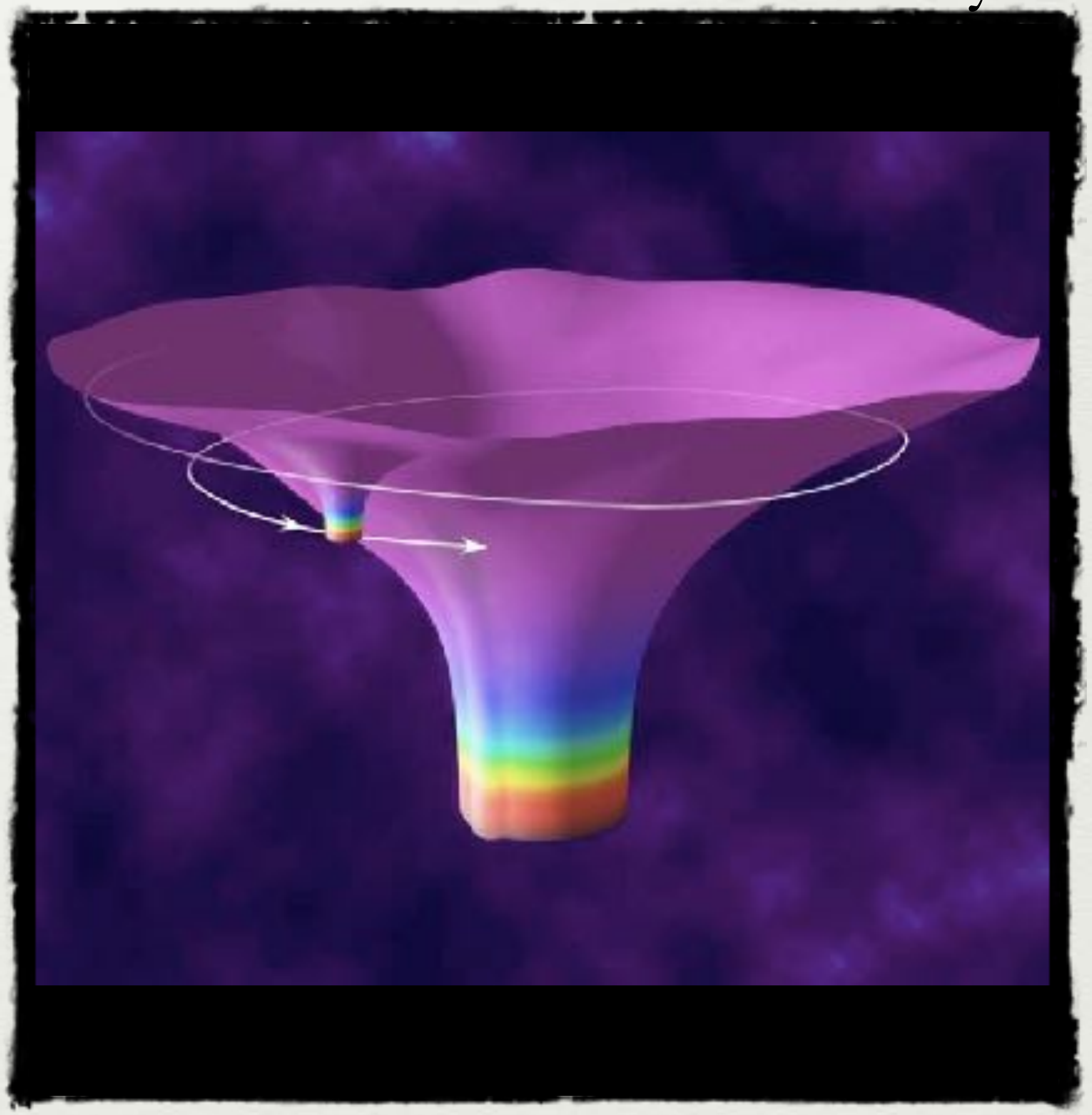


Binaries as GW Sources

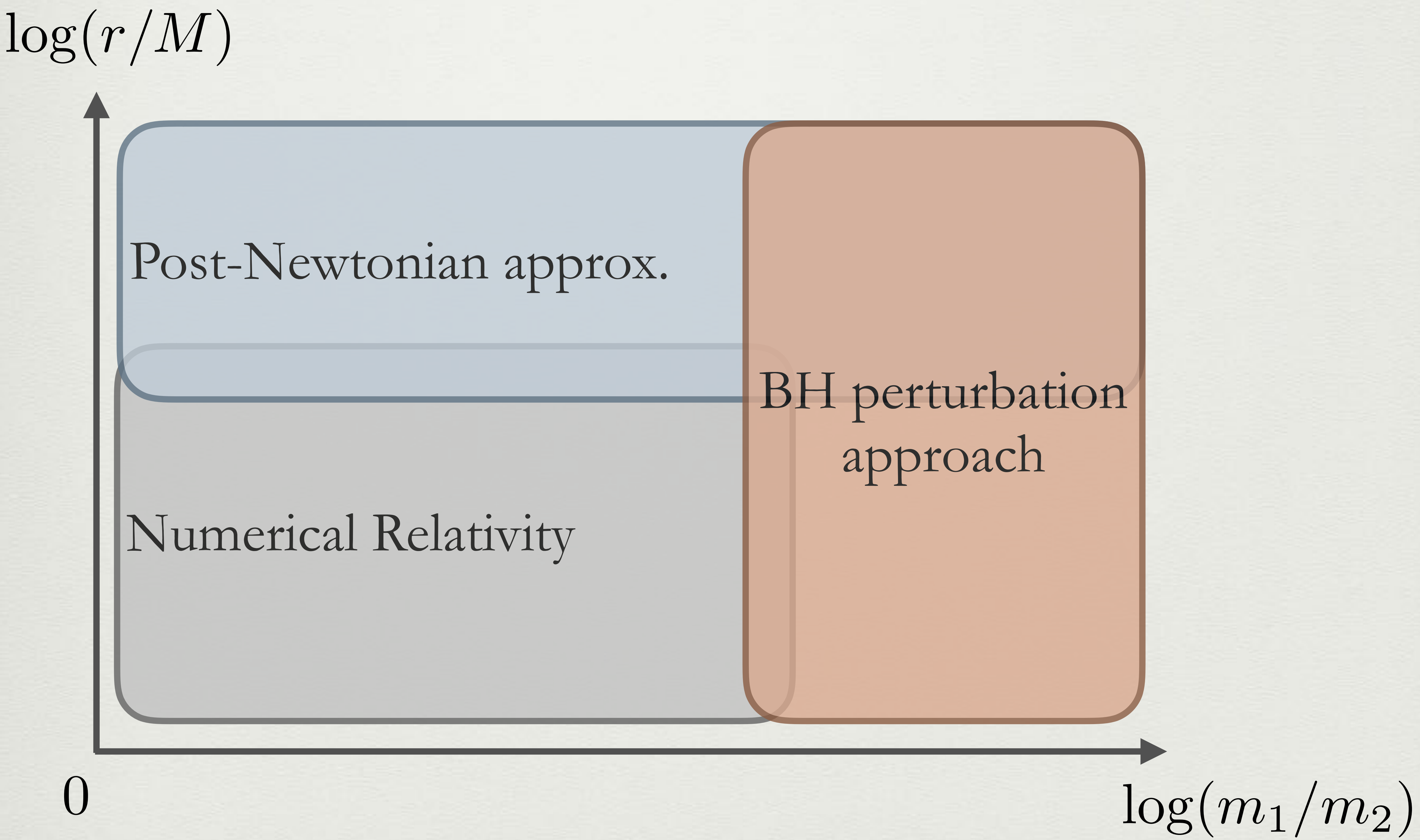
Binary merger



Extreme mass ratio binary



Methods for the 2-Body Problem



Black Hole Perturbation Approach

- Suppose the spacetime is perturbed from the background BH spacetime:

$$g_{\mu\nu} = g^{\mathrm{BG}}_{\mu\nu} + h_{\mu\nu}.$$

- Such perturbations were considered first to investigate the stability of the BH
  - Regge & Wheeler 1957, Zerilli 1970 for Schwarzschild BH.
  - Teukolsky 1973 for Kerr BH

Linearization of Einstein Equations

- The Einstein tensor is also expanded:

$$G_{\mu\nu}[g] = G_{\mu\nu}[g^{\mathrm{BG}}] + \delta G_{\mu\nu}[h] + \delta^2 G_{\mu\nu}[h,h] + \cdots ,$$

where (with $\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - g^{\mathrm{BG}}_{\mu\nu} h_\alpha{}^\alpha/2$,)

$$\delta G_{\mu\nu}[h] = -\frac{1}{2}\left[\Box \bar{h}_{\mu\nu} + 2R^\alpha{}_\mu{}^\beta{}_\nu \bar{h}_{\alpha\beta}\right],$$

$$\begin{aligned}\delta^2 G_{\mu\nu}[h] = &-\frac{1}{4}\bar{h}_{\alpha\beta;\mu}\bar{h}^{\alpha\beta}{}_{;\mu} - h^\alpha{}_\nu{}^{;\beta}\bar{h}_{\mu[\alpha;\beta]} - \frac{1}{2}\bar{h}_{\alpha(\mu;\nu)}\bar{h}_\beta{}^{\beta;\alpha} + \frac{1}{8}\bar{h}_\alpha{}^\alpha{}_{;\mu}\bar{h}_\beta{}^\beta{}_{;\nu} - \frac{1}{4}\bar{h}_{\mu\nu}\bar{h}_\alpha{}^\alpha{}_{;\beta}{}^{;\beta} \\ &+ \frac{1}{2}g^{\mathrm{BG}}_{\mu\nu}\left(\frac{1}{4}\bar{h}_\alpha{}^{\alpha;\gamma}\bar{h}_\beta{}^\beta{}_{;\gamma} + \frac{1}{4}\bar{h}_\alpha{}^\alpha\bar{h}_\beta{}^\beta{}_{;\gamma}{}^{;\gamma} - \frac{1}{2}\bar{h}^{\alpha\beta}\bar{h}_{\alpha\beta;\gamma}{}^{;\gamma} - R_{\alpha\gamma\beta\delta}\bar{h}^{\alpha\beta}\bar{h}^{\gamma\delta}\right)\end{aligned}$$

in the Lorenz gauge $\bar{h}_{\mu\alpha}{}^{;\alpha} = 0$.

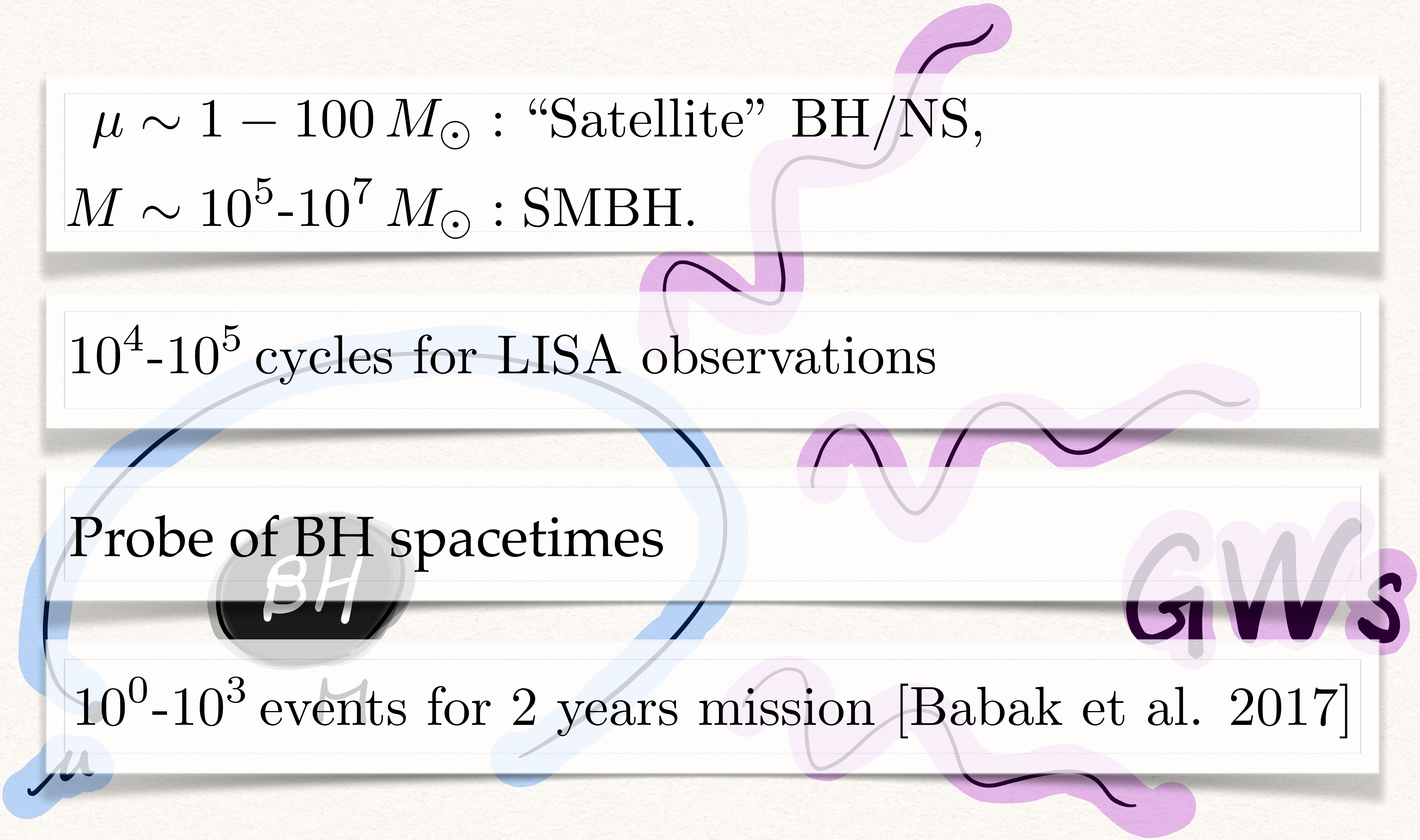
Extreme Mass Ratio Inspiral (EMRI)

$\mu \sim 1 - 100\, M_\odot$: "Satellite" BH/NS,
$M \sim 10^5$-$10^7\, M_\odot$: SMBH.

10^4-10^5 cycles for LISA observations

Probe of BH spacetimes

10^0-10^3 events for 2 years mission [Babak et al. 2017]



Gravitational Self-Force (GSF)

- Expand equations in the mass ratio:

$$\varepsilon \equiv \mu/M \ll 1,$$

$$g_{\mu\nu} = g^{\rm BG}_{\mu\nu} + \varepsilon h^{(1)}_{\mu\nu} + \varepsilon^2 h^{(2)}_{\mu\nu} + \cdots .$$

- Valid even if $v/c \sim 1$ ⟺ PN regime.

- EoM for the "satellite"

$$\ddot{z}^\mu = 0 + \varepsilon F^\mu_{(1)}(h) + \varepsilon^2 F^\mu_{(2)}(h) + \cdots .$$

- Formal expressions of GSF is known up to $\mathrm{O}(\varepsilon^2)$ [Pound, 2012].

What's Next?

- Calculate the waveform
  - Need (generic) orbits of the satellite in Kerr metric.

✓ Hamiltonian formulation for conservative GSF: Fujita et al. (2016).

✓ GSF up to first order: van de Meent (2017).

- Solve for second-order perturbations to obtain the dissipative GSF.

Why the Second Order?

- If neglect the second-order self-force $O(\varepsilon^2)$,
  ➡ error in acceleration is $\delta\ddot{z}^\mu \sim \varepsilon^2/M$.

  - Error in position is $\delta z^\mu \sim \varepsilon^2\tau^2/M$.

  - After inspiral time $\tau \sim M/\varepsilon$,
    error in position becomes $\delta z^\mu \sim M$.

- The second-order perturbation $h^{(2)}$ gives
  detectable effects on GW phase!

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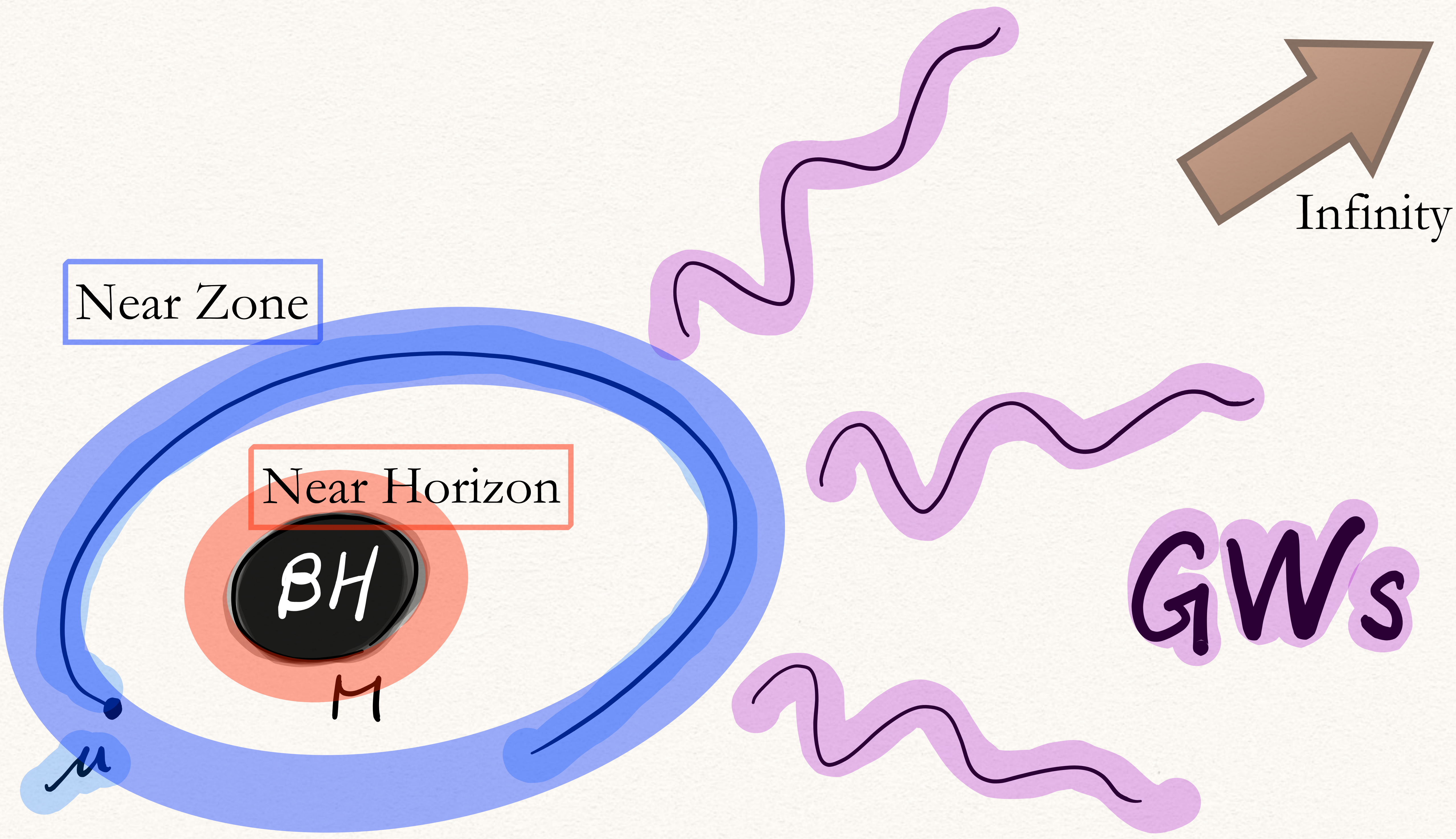
Extreme Mass Ratio Inspiral (EMRI)



Second-Order Vacuum Equations

- We expand equations in the mass ratio:

$$g_{\mu\nu} = g^{\mathrm{BG}}_{\mu\nu} + \varepsilon h^{(1)}_{\mu\nu} + \varepsilon^2 h^{(2)}_{\mu\nu} + \cdots .$$

- The field equations to the second-order are

$$\delta G^{\mu}{}_{\nu}[\varepsilon h^{(1)} + \varepsilon^2 h^{(2)}] = -\delta^2 G^{\mu}{}_{\nu}[\varepsilon h^{(1)}, \varepsilon h^{(1)}],$$

where $\delta G^{\mu}{}_{\nu}[h]$ & $\delta^2 G^{\mu}{}_{\nu}[h, h]$ are linear & quadratic in h .

IR Divergence Around Boundaries

- The field equations to the second-order are

$$\delta G^{\mu}{}_{\nu}[\varepsilon h^{(1)} + \varepsilon^2 h^{(2)}] = -\delta^2 G^{\mu}{}_{\nu}[\varepsilon h^{(1)}, \varepsilon h^{(1)}].$$

- The following integral diverges around boundaries:

$$h^{(2)}_{\omega\ell m} = \int_{r_{\mathrm{h}}}^{\infty} G_{\omega\ell m}(r, r')\delta^2 G_{\omega\ell m}(r')dr'.$$

✓ @infinity ➡ use the PN/PM results (cf. [Pound 2015]).

- We discuss the near-horizon expansion.

Physical Secular Growth

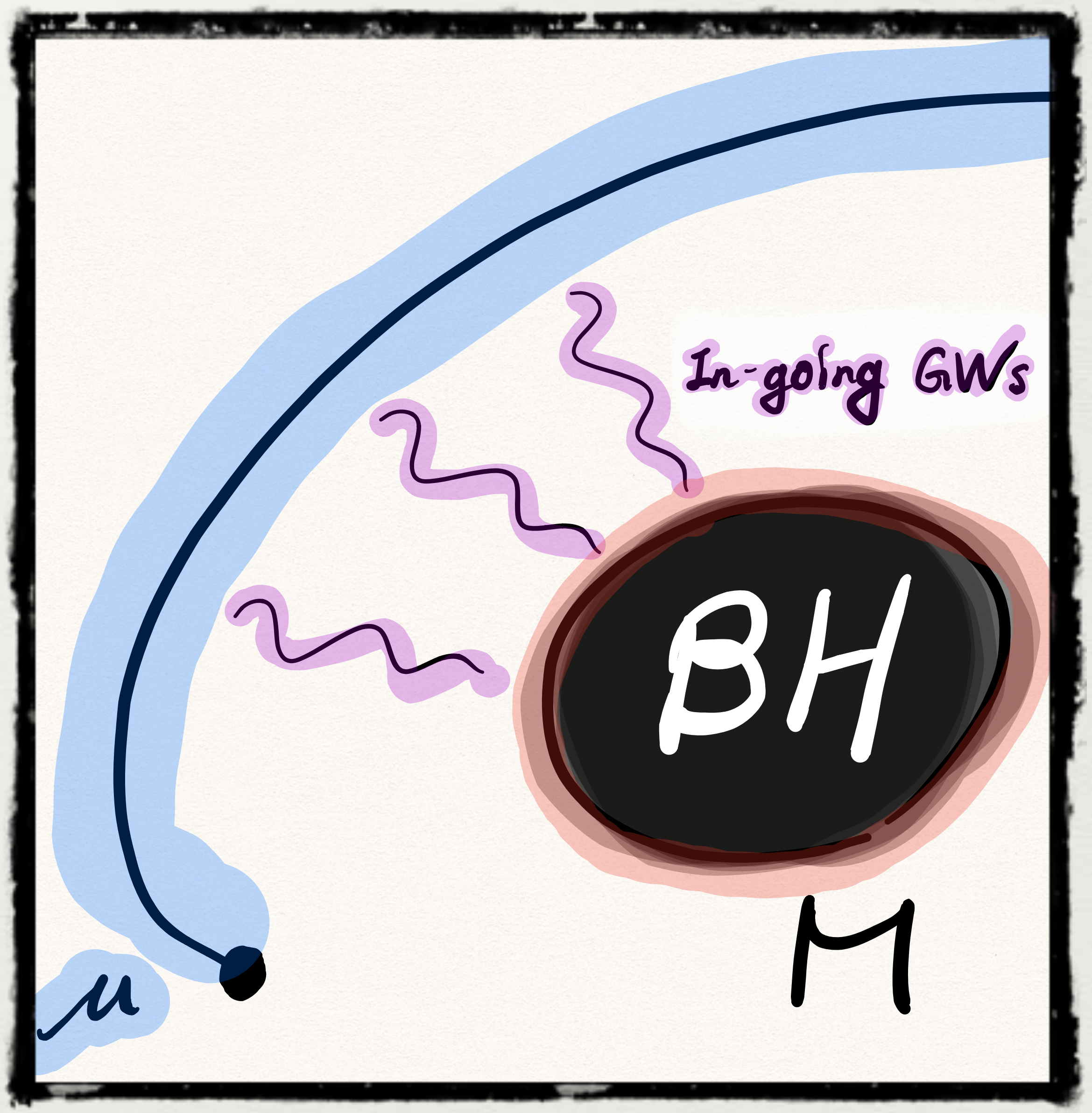
- $\{rv\}$ & $\{r\phi\}$ components of

$$\int \sqrt{-g}\,\delta G^{\mu}{}_{\nu}\,d\theta\,d\phi = \int \sqrt{-g}\left(-\delta^2 G^{\mu}{}_{\nu}\right)\,d\theta\,d\phi,$$

  determine the secular growth.

- The secular growth

  ➡the secular change of
  the BH's mass/spin
  $\delta M = \dot{E}\,\tilde{v}$ & $\delta a = \dot{L}\,\tilde{v}$.



Counter Term of Secular Growth

- We have found the secular growth δM & δa

$$h_{\mu\nu}^{(1)\delta M,\delta L} = \frac{\partial g_{\mu\nu}^{\mathrm{BG}}}{\partial M}\delta M + \frac{\partial g_{\mu\nu}^{\mathrm{BG}}}{\partial a}\delta a,$$

which reproduces the spurious divergence.

- Therefore, the effective source term,

$$S_{\mu\nu}^{\mathrm{eff}} = -\delta^2 G_{\mu\nu}[\varepsilon\, h^{(1)}, \varepsilon\, h^{(1)}] - \delta G_{\mu\nu}[\varepsilon\, h^{(1)\delta M,\delta L}]$$

is “regularized.”

Singular Asymptotic Sols.?

- We obtain inhomogeneous solutions

$$h^{(2)}_{\mathrm{BS1}} = \partial_{\tilde{t}} h^{(1)}_{\mathrm{BS2}} (1-f)\, r_* + \cdots\,,$$
$$h^{(2)}_{\mathrm{BS2}} = \partial_{\tilde{t}} h^{(1)}_{\mathrm{BS1}} r_* + \cdots\,,$$

  where the slow-time variable $\tilde{t} = \varepsilon t\,,\ \ f = 1 - 2M/r$

  & the tortoise coordinate $r_* = r + 2M \ln\left(\dfrac{r-2M}{2M}\right)$.

- The logarithmic divergences sourced
  by the secular growth appear near the horizon.

2 Types of Divergences Near Horizon

- Physical & spurious IR divergence
  - Secular changes of mass δM & spin δa of BG BH.
- Unphysical pure gauge degrees of freedom
  - Need to identify and remove
    for the boundary conditions.
- We focus on the quasi-stationary pert.

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The Eddington-Finkelstein Coordinates

- The Schwarzschild background metric is

$$ds^2 = -f dv^2 + 2dvdr + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

in the ingoing Eddington-Finkelstein coordinates, where $f = 1 - 2M/r$ & $v = t + r_*$.

✓ No singularity appears on the BH horizon.

✓ Ingoing GWs propagate along a null line, on which the "time coordinate" v is constant.

of Remaining Degrees of Freedom

- After regularizing the asymptotic sols., we still have remaining dof.
  - For $\ell = 0$ mode, 3 dof:
    2nd-order mass pert. (1) + gauge dof (2)
  - For $\ell = 1$ mode, 5 dof:
    2nd-order spin pert. (1) + gauge dof (4)
  - For $\ell \geq 2$ modes 6 dof:
    2nd-order GWs (2) + gauge dof (4)

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Summary

- Need the second-order metric perturbations
  for EMRI observations by LISA.
    - IR & gauge divergences appear near the BH horizon.
- We have found
    - singular behavior of asymptotic sols.
    - the appropriate gauge choice of near-horizon limit.
- Extend to the Kerr BH?

THANK YOU FOR YOUR ATTENTION