# ブラックホールまわりにおける磁気リコネクションの抵抗性一般相対論的MHD数値シミュレーション

# ーResistive GRMHD Simulations of Magnetic Reconnection around a Black Holeー


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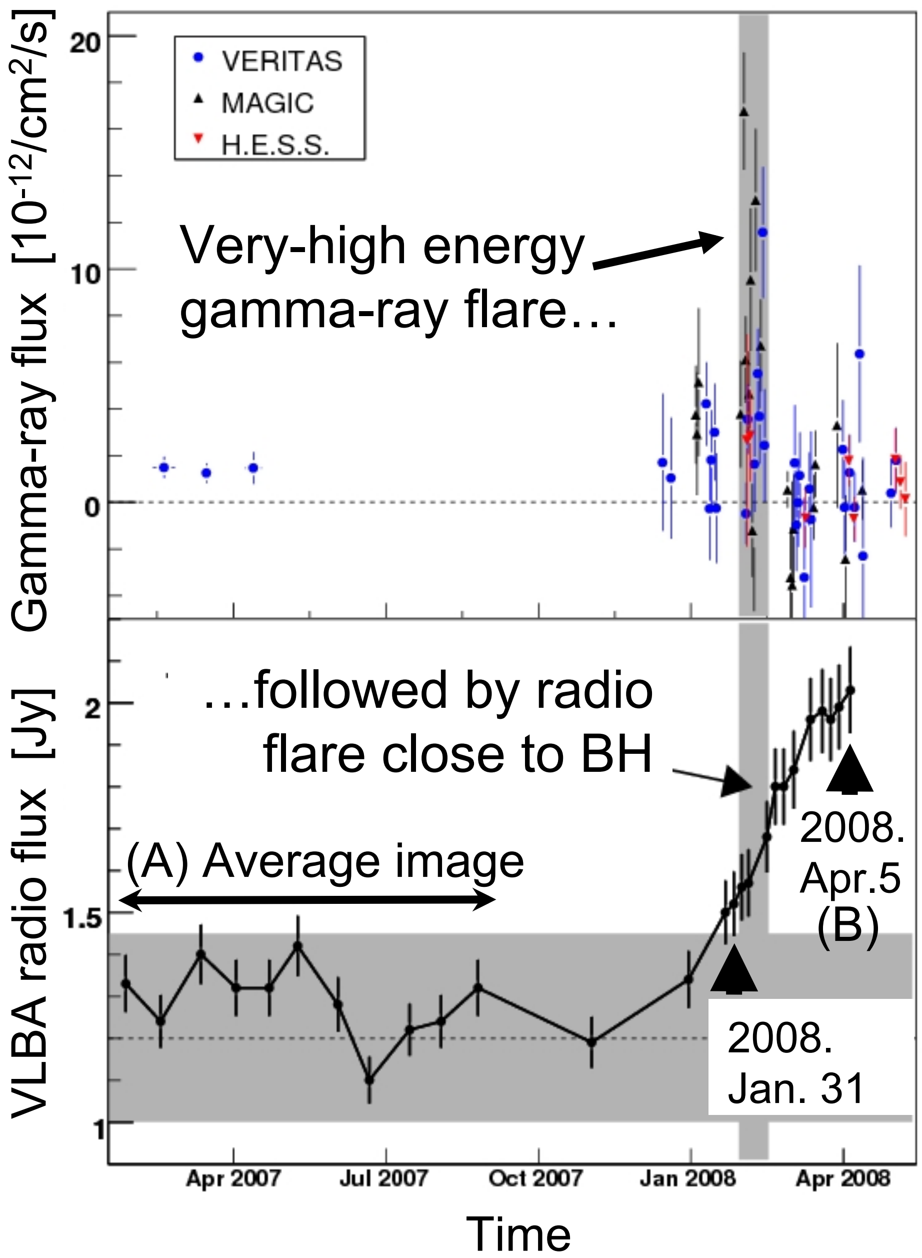
# Motivation from observation

# Flare observation in core of M87: TeV-γ & VLBI (43GHz)




TeV-gamma flare clearly synchronized with VLBA flare!

Acciari et al. Science 2009.
The VERITAS Collaboration,
the VLBA 43 GHz M87 Monitoring Team (P.E.Hardee),
the H.E.S.S. Collaboration,
the MAGIC Collaboration

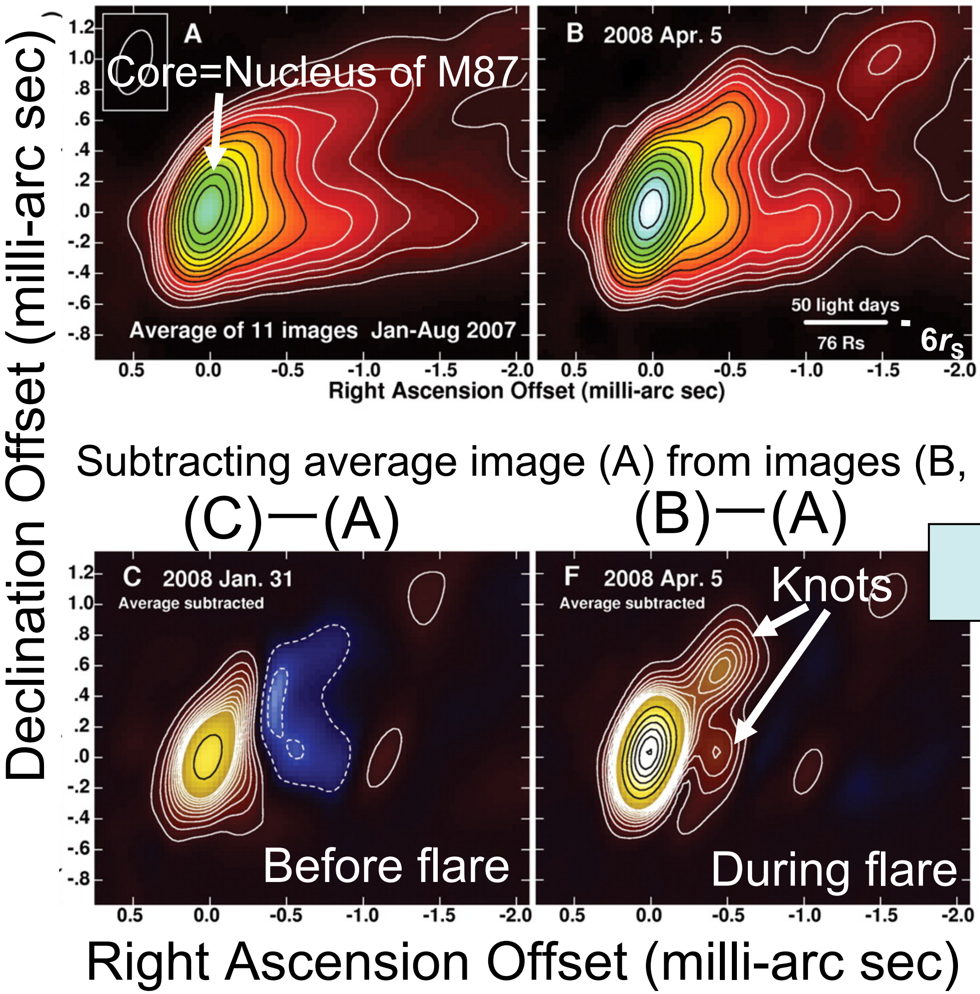
NRAO e-news (2009, July)

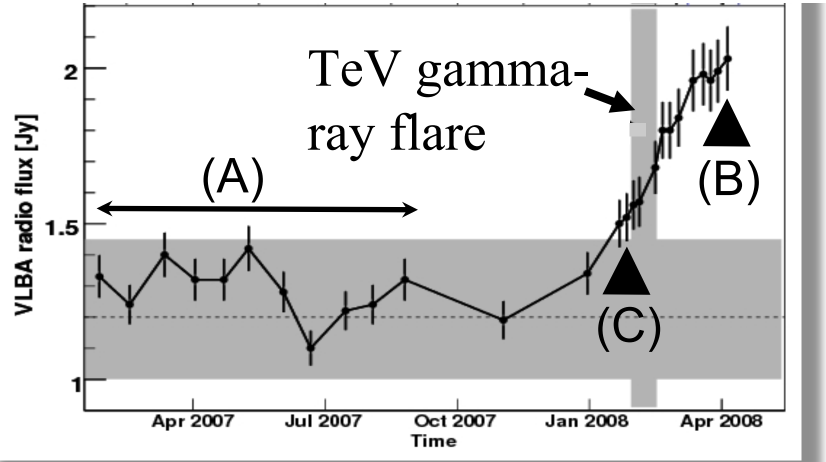
# Knot ejection synchronized with flare of M87

Acciari et al. Science 2009.

(A) Average

(B) Epoch image during flare



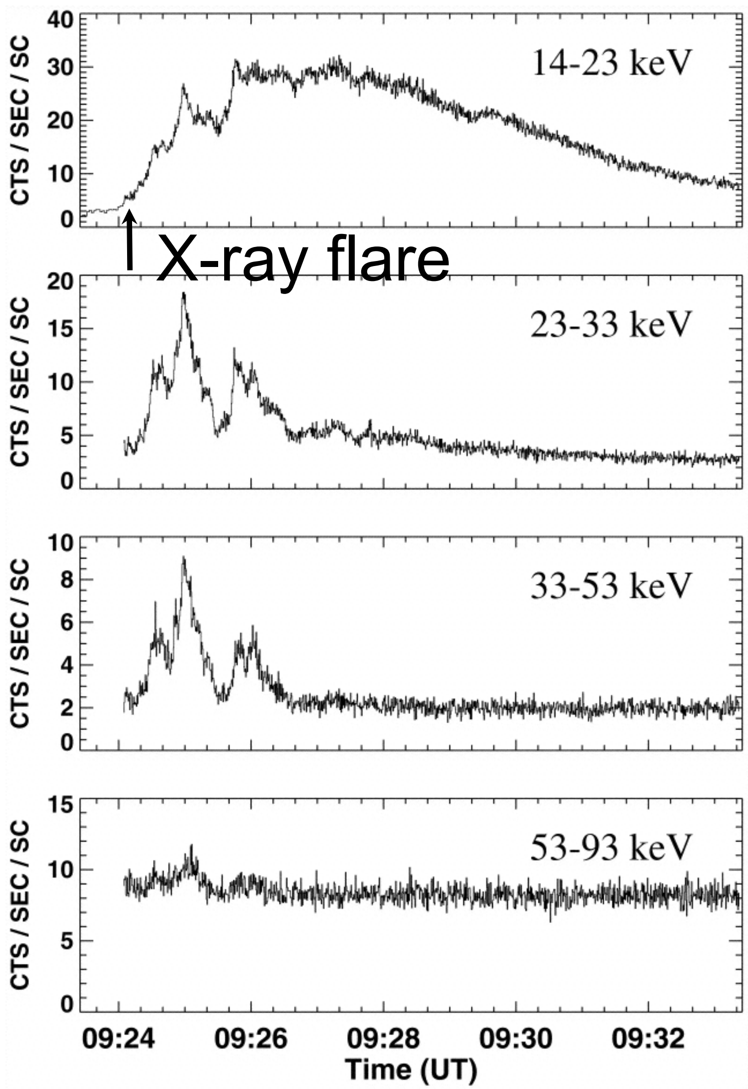



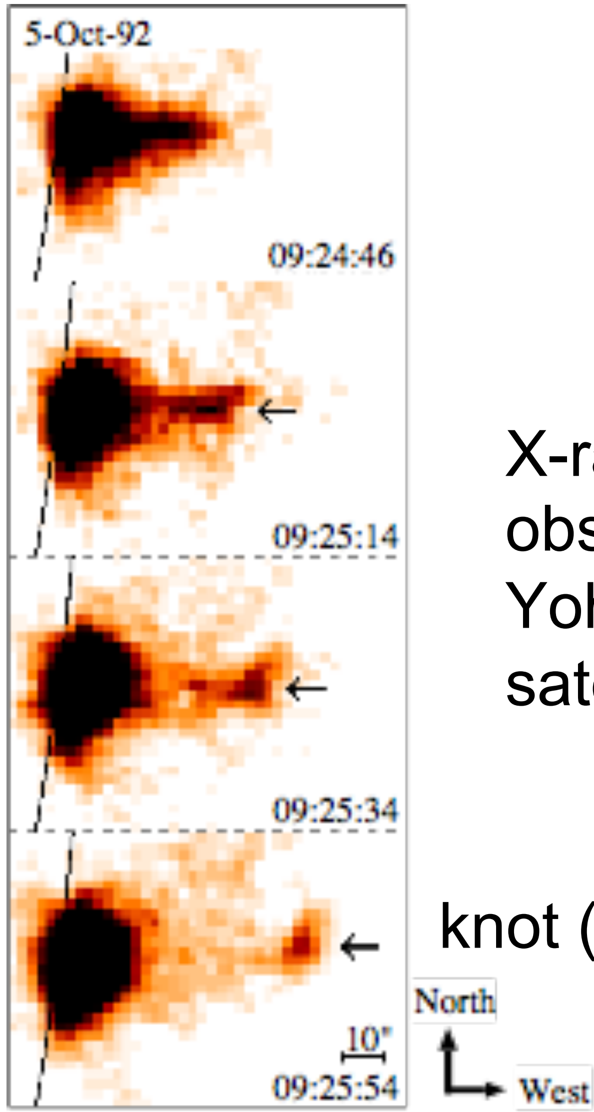

Knot ejection and flare at core correlation

Such correlation is also found in solar flares.

With respect to micro-QSOs, similar correlation was also found (Mirabel & Rodriguez 1990, Nature).

# “Knot” ejection and flare correlation in solar flare events: X-ray images of plasmoid ejection associated with a solar flare (Oct. 5, 1992).

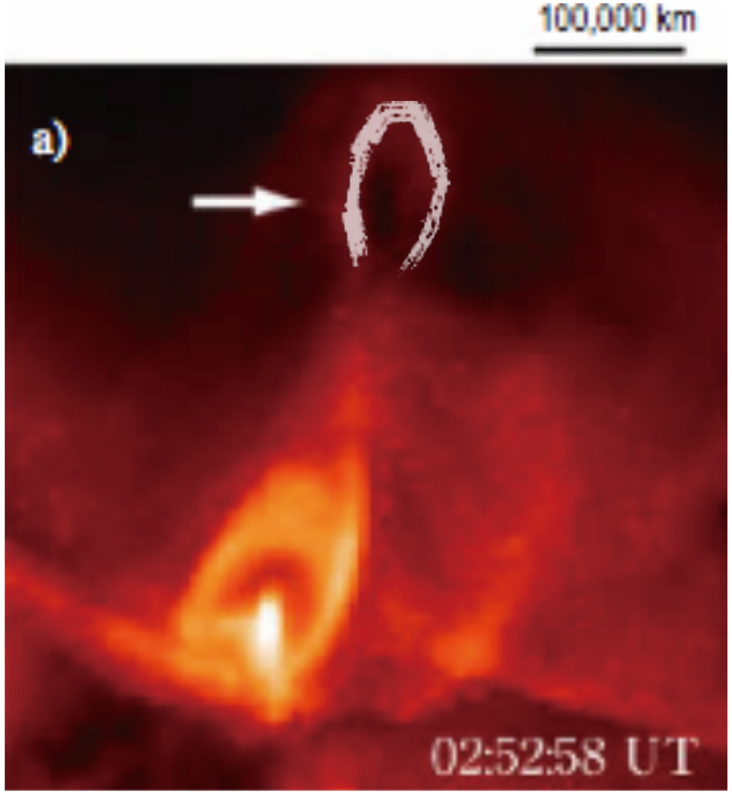



X-ray image observed by Yohkoh satellite

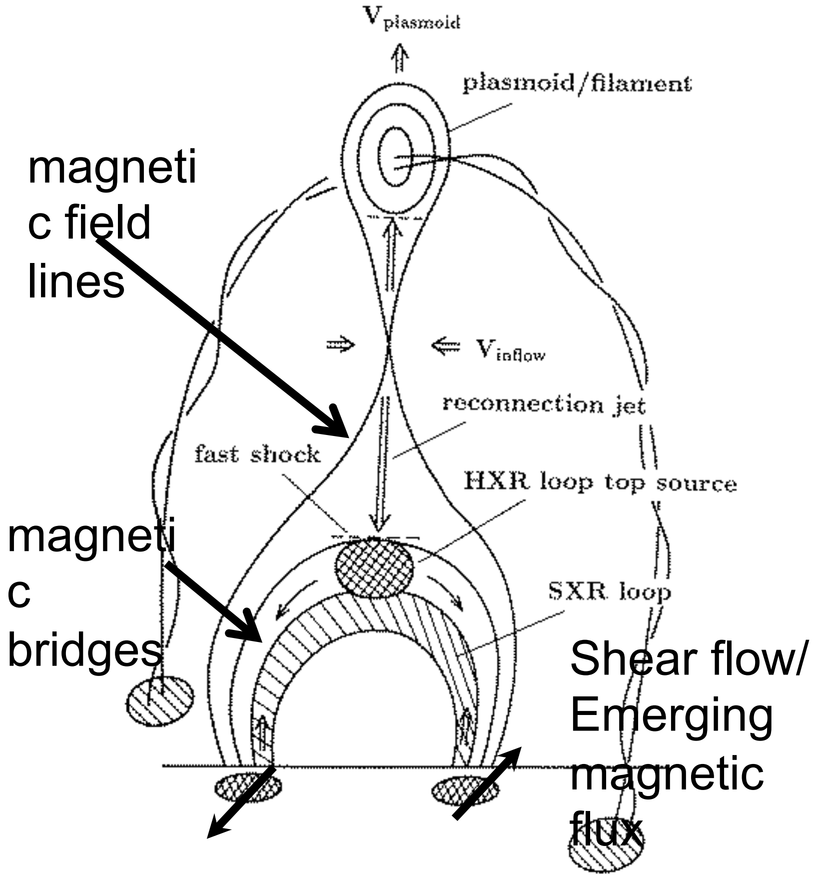
knot (plasmoid)

# Observation of plasmoid ejection after solar flare and magnetic reconnection




X-ray image of long-durational-event (LDE) flare observed by Yohkoh satellite.

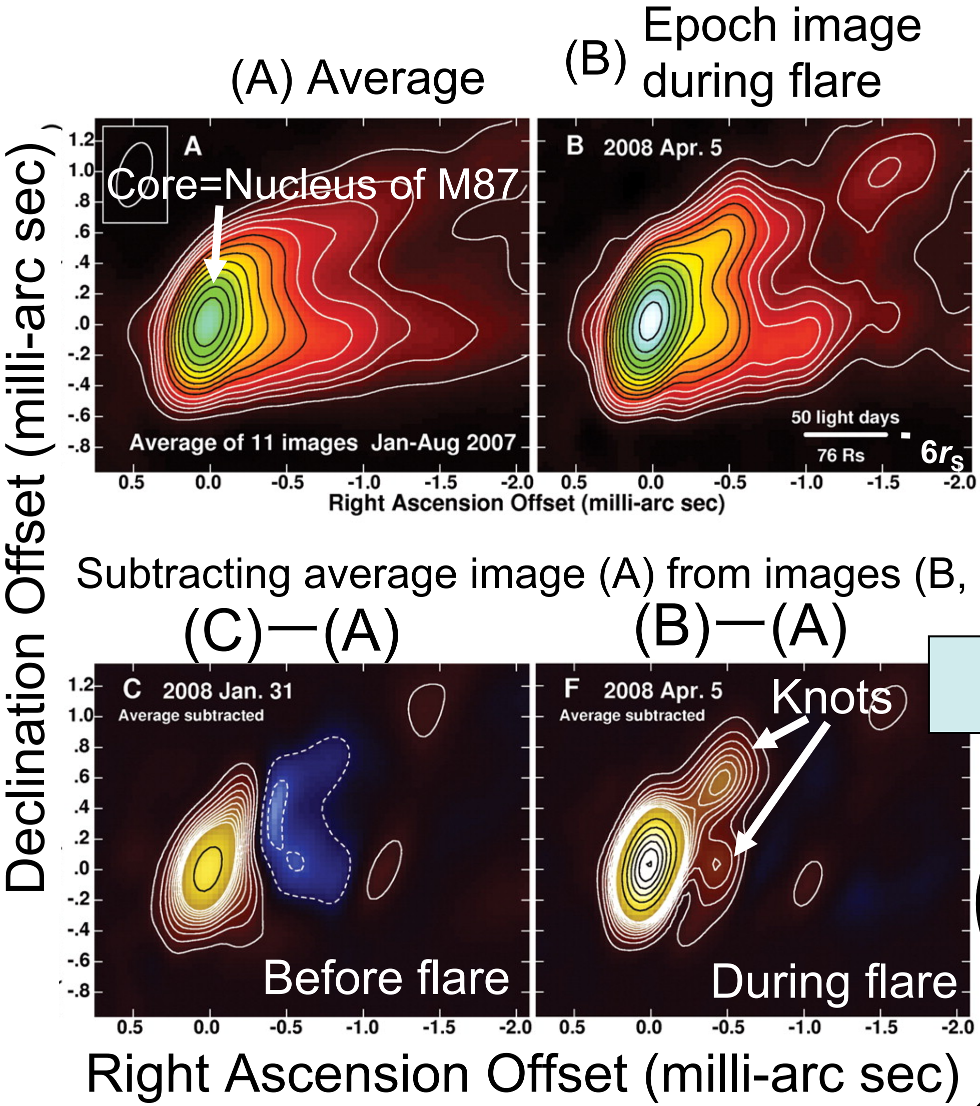
K. Shibata and T. Ohyama,
Living Review of Solar Physics 2011

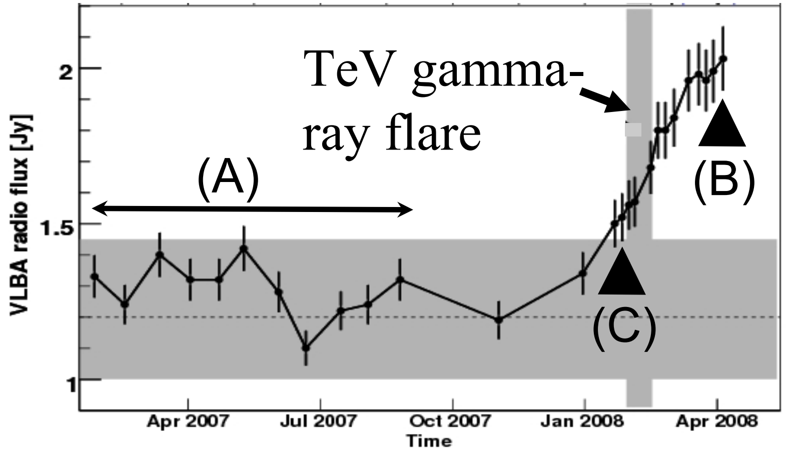



Magnetic bridges anchored at shear flow plasma cause magnetic reconnection to cause drastic phenomena like a flare with plasmoid ejection.

# Knot ejection synchronized with flare of M87

Acciari et al. Science 2009.




Knot ejection and flare at core correlation

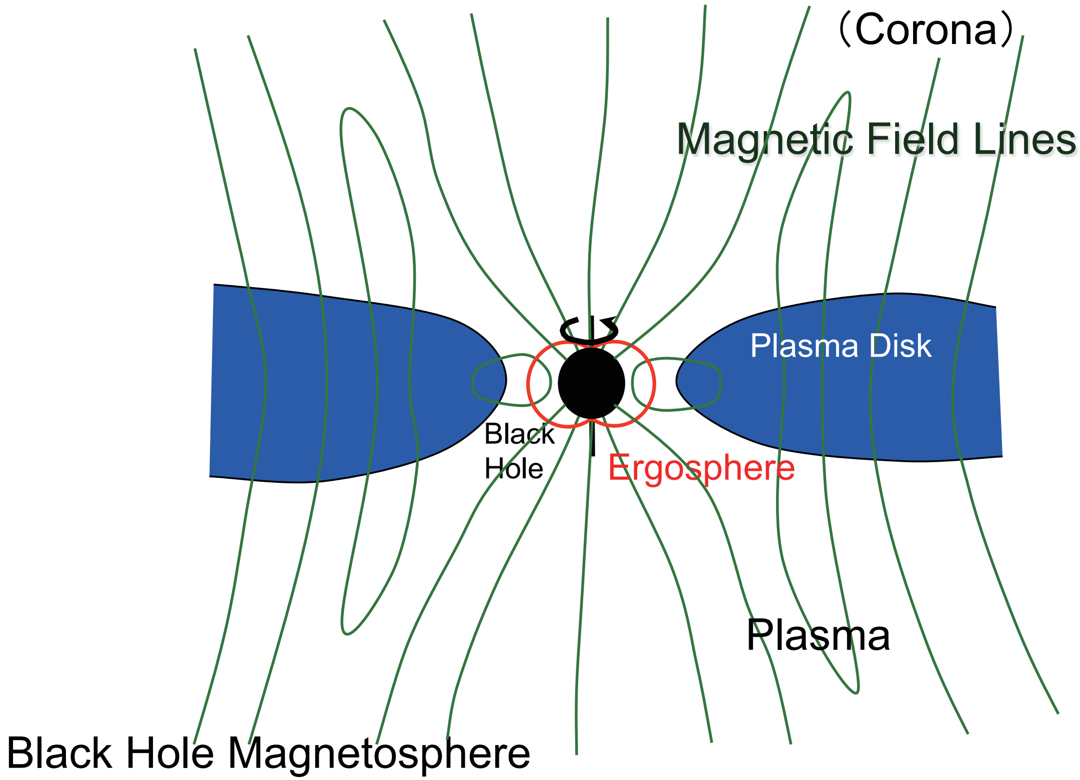
This flare was also caused by magnetic reconnection around black hole?

With respect to micro-QSOs, similar correlation was also found (Mirabel & Rodriguez 1990, Nature).

# General Relativistic Magnetohydrodynamics (GRMHD)

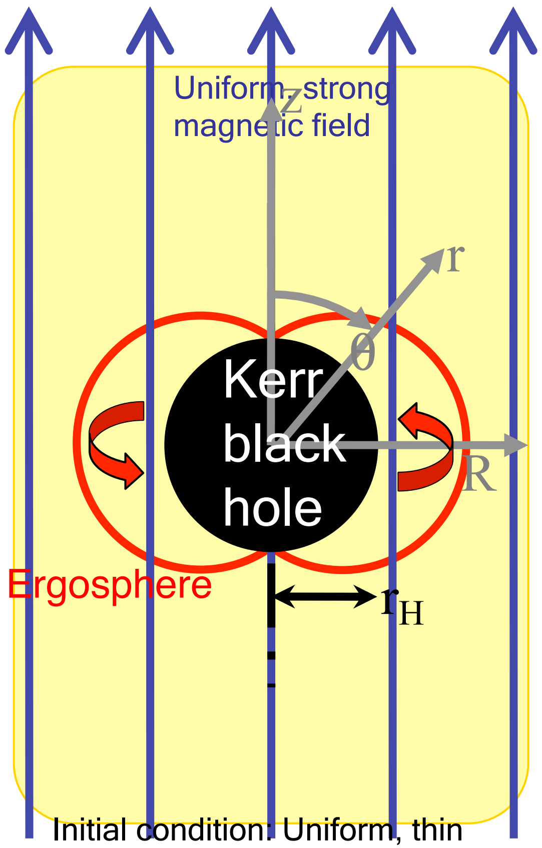
- Ideal GRMHD
- Resistive GRMHD

To simulate plasmas in the black hole magnetosphere

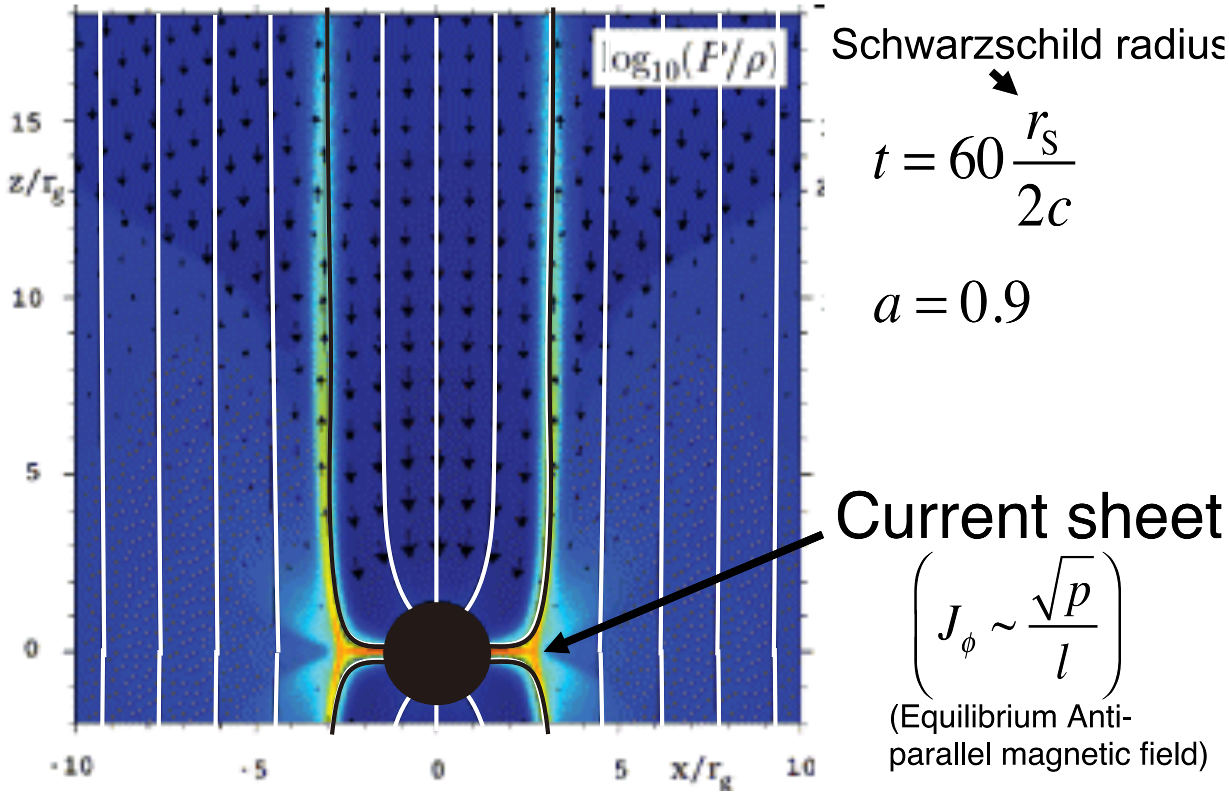


# Ideal GRMHD simulation of spontaneous formation of anti-parallel magnetic field around spinning black hole:

Even with uniform magnetic field as its initial condition around a rotating black hole, magnetic reconnection is expected to occur!




(Komissarov 2005)




To simulate magnetic reconnection around black hole, resistive GRMHD is required!

# Covariant form of standard resistive GRMHD equations

- General relativistic equations of conservation laws:

proper rest mass density

$$\nabla_\nu \left( \rho U^\nu \right) = 0 \quad \text{(particle number)}$$

4-velocity

$$\nabla_\nu T^{\mu\nu} = 0 \quad \text{(energy and momentum)}$$

Energy-momentum tensor

Maxwell equations:

Dual tensor of $F^{\mu\nu}$

Field strength tensor

4-current density

$$\nabla_\nu \, {}^*F^{\mu\nu} = 0 \qquad \nabla_\nu F^{\mu\nu} = -J^\mu$$

Ohm's law with resistivity:

$$F_{\mu\nu} U^\nu = \eta \left( J_\mu + \left( U_\nu J^\nu \right) U_\mu \right)$$

resistivity

Zero in the case of ideal GRMHD

Kerr Metric: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

$$g_{00} = -h_0^2; \quad g_{ii} = h_i^2; \quad g_{0i} = -h_i^2 \omega_i \quad (i = 1,2,3); \quad g_{ij} = 0 \quad (i \neq j)$$

Lapse function: $\alpha = \sqrt{h_0^2 + \sum_i \left( h_i \omega_i \right)^2}$

(gravitational time delay)

Shift vector: $\beta_i = h_i \omega_i / \alpha$

$$\boldsymbol{\beta} = \left( \beta_1, \beta_2, \beta_3 \right)$$

(velocity of dragged frame)

# 3+1 Formalism of Resistive GRMHD Equations

(conservative form)

Special relativistic mass density, γρ

$$\frac{\partial D}{\partial t} = -\nabla \cdot \left[ \alpha D \left( \hat{\boldsymbol{v}} + \boldsymbol{\beta} \right) \right]$$

general relativistic effect

(conservation of particle number)

Special relativistic total momentum density

$$\frac{\partial \hat{\boldsymbol{P}}}{\partial t} = -\nabla \cdot \left[ \alpha \left( \hat{\boldsymbol{T}} + \boldsymbol{\beta} \hat{\boldsymbol{P}} \right) \right] - \left( D + \varepsilon \right) \nabla \alpha + \alpha \boldsymbol{f}_{\mathrm{curv}} - \hat{\boldsymbol{P}} : \boldsymbol{\sigma}$$

special relativistic effect

(equation of motion)

$$\hat{\boldsymbol{P}} = h\gamma \hat{\boldsymbol{v}} + \hat{\boldsymbol{E}} \times \hat{\boldsymbol{B}} \qquad \hat{\boldsymbol{T}} = h\gamma^2 \hat{\boldsymbol{v}}\hat{\boldsymbol{v}} + \left( p + \frac{\hat{B}^2}{2} + \frac{\hat{E}^2}{2} \right) \boldsymbol{I} - \hat{\boldsymbol{B}}\hat{\boldsymbol{B}} - \hat{\boldsymbol{E}}\hat{\boldsymbol{E}}$$

Special relativistic total energy density

$$\frac{\partial \varepsilon}{\partial t} = -\nabla \cdot \left[ \alpha \left( \hat{\boldsymbol{P}} - D\hat{\boldsymbol{v}} + \varepsilon \boldsymbol{\beta} \right) \right] - \hat{\boldsymbol{P}} \cdot \nabla \alpha - \hat{\boldsymbol{T}} : \boldsymbol{\sigma}$$

(equation of energy)

$$\frac{\partial \hat{\boldsymbol{B}}}{\partial t} = -\nabla \times \left[ \alpha \left( \hat{\boldsymbol{E}} - \boldsymbol{\beta} \times \hat{\boldsymbol{B}} \right) \right] \qquad \alpha \left( \hat{\boldsymbol{J}} + \rho_{\mathrm{e}} \boldsymbol{\beta} \right) + \frac{\partial \hat{\boldsymbol{E}}}{\partial t} = \nabla \times \left[ \alpha \left( \hat{\boldsymbol{B}} + \boldsymbol{\beta} \times \hat{\boldsymbol{E}} \right) \right]$$

$$\nabla \cdot \hat{\boldsymbol{B}} = 0 \qquad \rho_{\mathrm{e}} = \alpha \nabla \cdot \hat{\boldsymbol{E}}$$

(Maxwell equations)

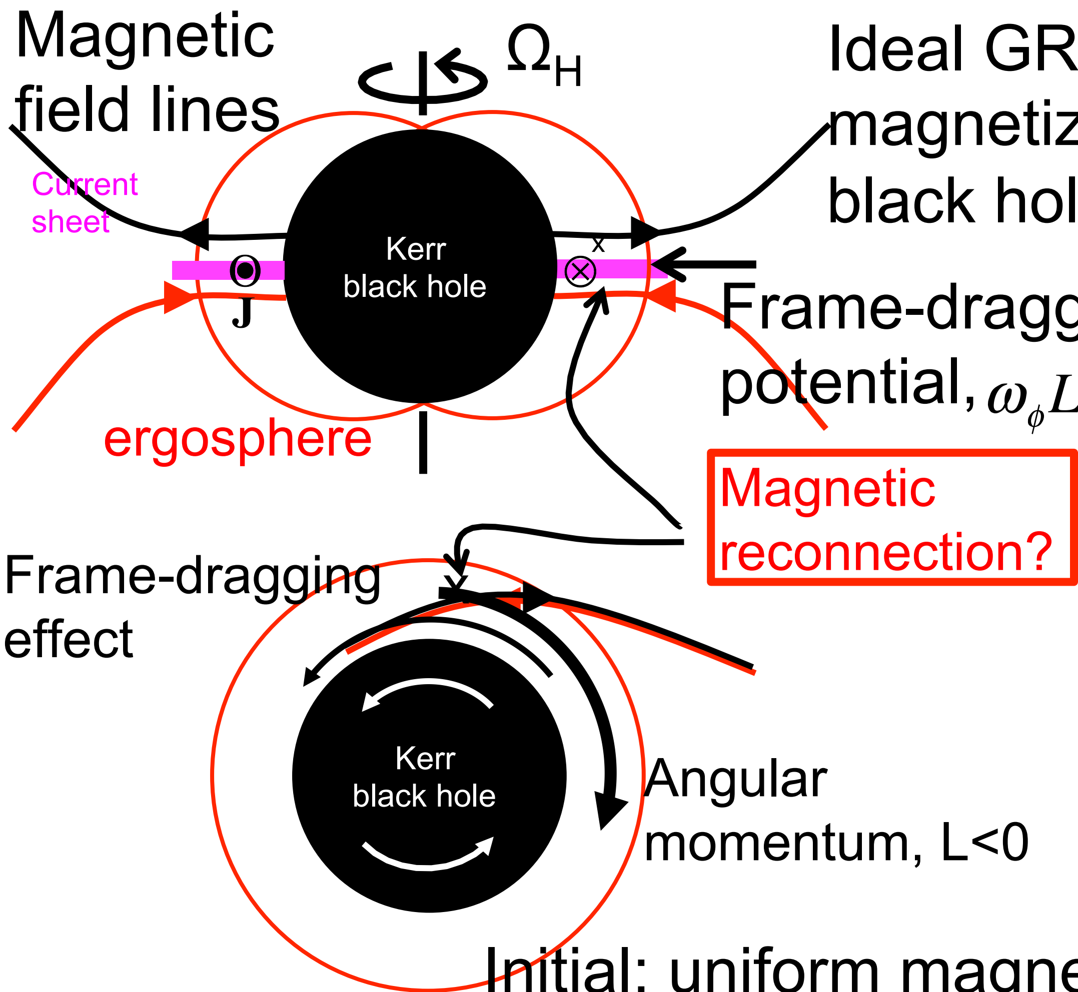
$$\hat{\boldsymbol{E}} + \hat{\boldsymbol{v}} \times \hat{\boldsymbol{B}} = \frac{\eta}{\gamma} \left[ \hat{\boldsymbol{J}} - \gamma^2 \left( \rho_{\mathrm{e}} - \left( \hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{J}} \right) \right) \hat{\boldsymbol{v}} \right]$$

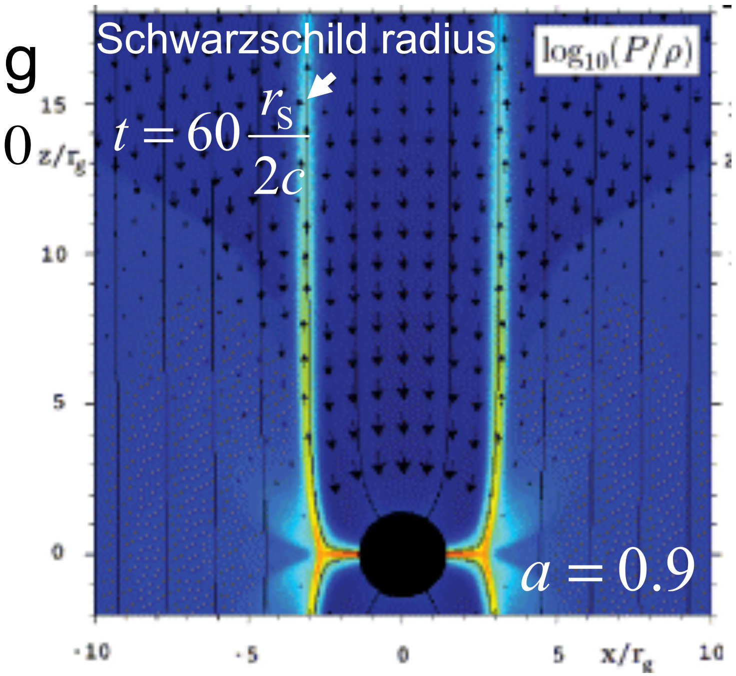
(Ohm’s law with finite resistivity)

We treat Ampere’s law as an equation of time evolution of electric field (Watanabe & Yokoyama 2006).

# Longer term ideal GRMHD simulation of uniformly magnetized plasmas around Kerr BH

Ideal GRMHD simulation of uniformly magnetized plasma around rotating black hole (Komissarov 2005)



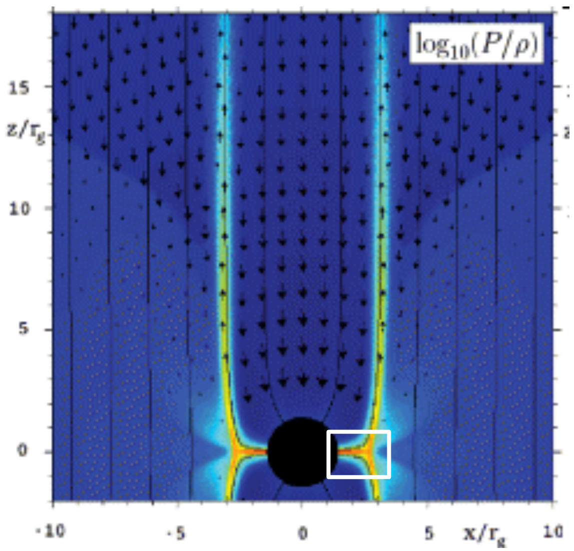

Initial: uniform magnetic field (Wald solution)

⇒ sprit monopole-like magnetic field in ergosphere

⇒ magnetic reconnection in ergosphere

# Initial condition of a simple case with split-monopole field

Long-term simulation of Ideal GRMHD




Komissarov (2005)

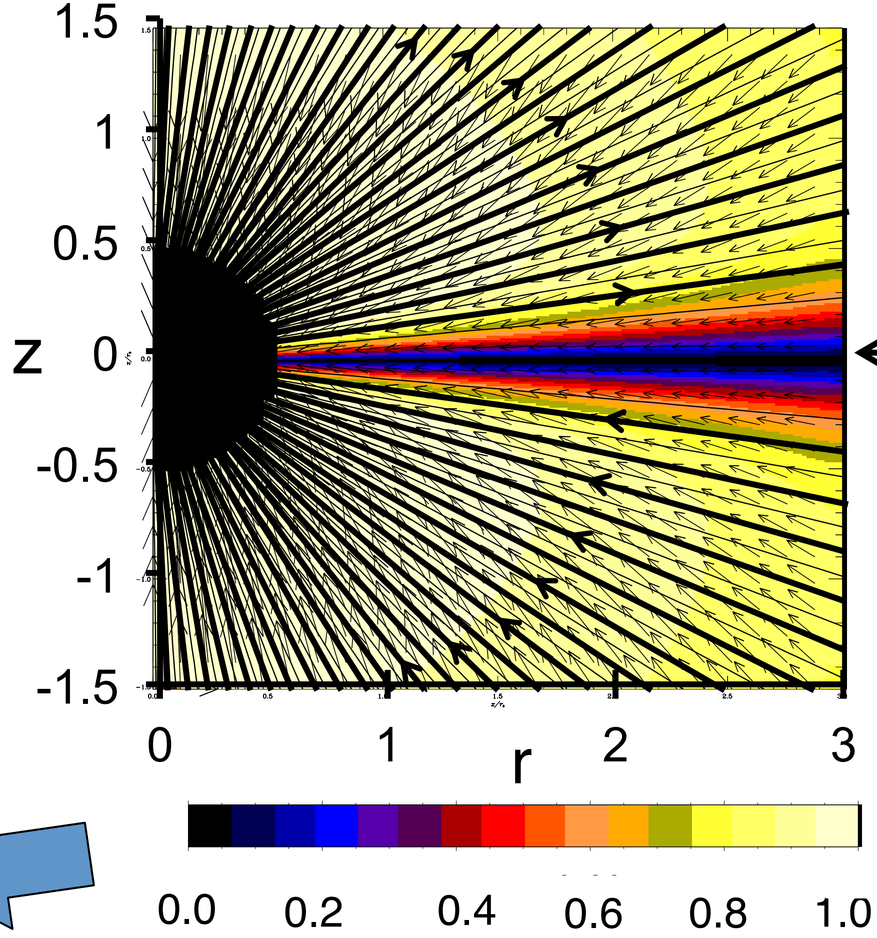
Relativistic magnetic reconnection:

Resistivity: uniform

$v_A \sim c$

$a = 0.99995$

$t = 0$




thin current sheet where pressure is high.

Relativistic Alfven velocity

$$v_A = \frac{B}{\sqrt{\rho c^2 + \Gamma p/(\Gamma - 1) + B^2}} c$$

# Results:

## Simulations of magnetic reconnection around Schwarzschild black hole by resistive GRMHD

# Initial condition: Split monopole magnetic field around Schwarzschild black hole

- Black Hole: $a \equiv \dfrac{J}{J_{\max}} = 0$ (black hole spin parameter)
- Magnetic Field： Split monopole magnetic field

$$B_r = B_0 \frac{1}{r^2} \tanh\left(\frac{\theta - \pi/2}{\Delta\theta}\right),\ B_\theta = B_\phi = 0,$$

$$B_0 = B\left(R = 0,\ \ z = r_{\mathrm{S}}\right)$$

Vertical equilibrium around current sheet due to Harris magnetic field

- Plasma：

$$\rho = \frac{\rho_0}{\sqrt{2Mr^3}}, \quad p = \frac{B_0^2}{2r^4} \frac{1}{\cosh^2\left(\left(\theta - \pi/2\right)/\Delta\theta\right)} + p_{\mathrm{b}}$$

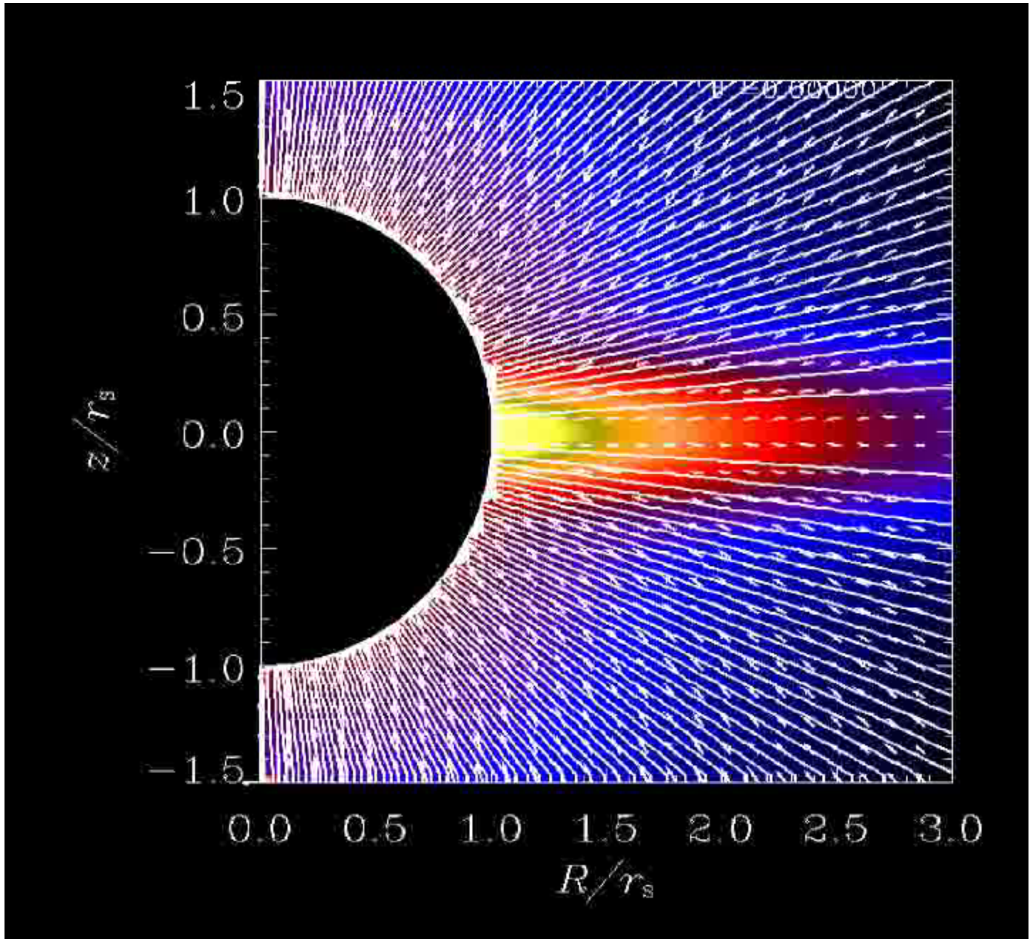
$$\hat{v}^r = -0.8\sqrt{\frac{2M}{r}}, \quad \hat{v}^\varphi = \hat{v}^\theta = 0,$$

$$p_{\mathrm{b}} = \frac{\beta_{\mathrm{p}} B_0^2}{8\pi} \rho^\Gamma$$

$$\beta_{\mathrm{p}} = 0.025$$

# Magnetic reconnection with uniform resistivity in split-monopole magnetic field around black holes

- Schwarzschild black hole case




$$a = 0$$

$$\rho_0 = 1$$

$$B_0 = 10, \quad \Delta\theta = 0.1$$

$$\eta = 0.001 r_S \text{ (uniform)}$$

$$(S = 100)$$

Magnetic Reynolds number:

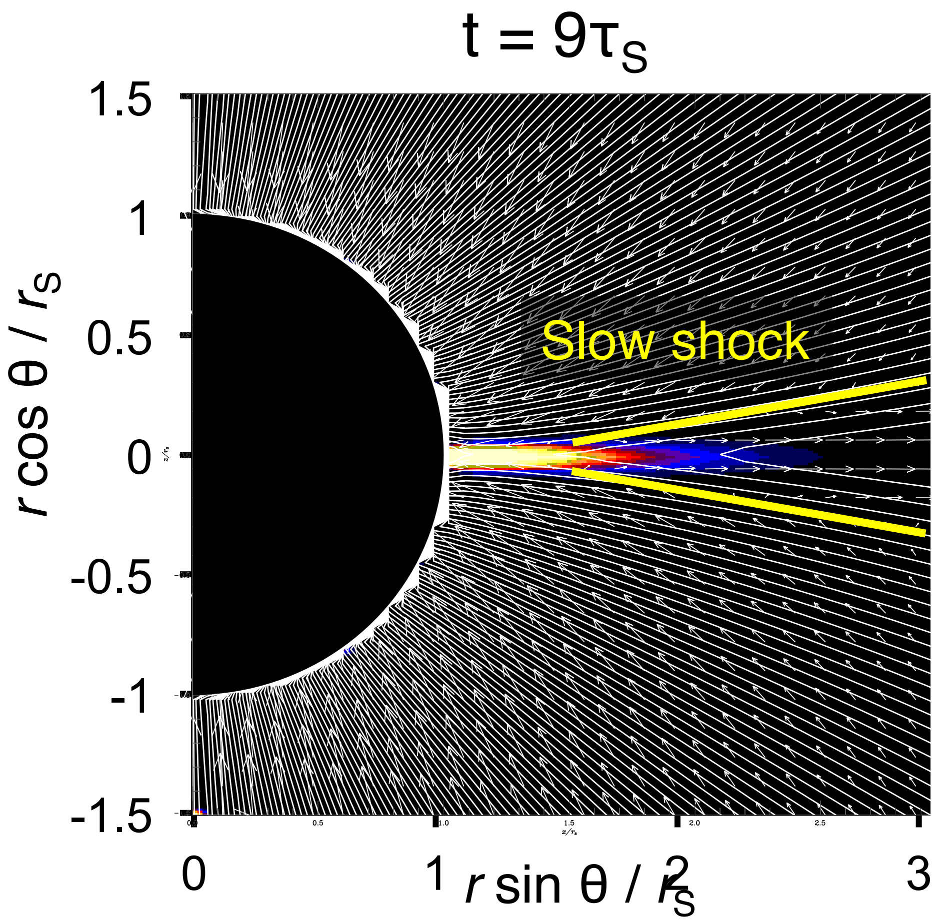
$$S = \frac{\delta_c v_A}{\eta}$$

Color: pressure
Lines: magnetic field
Arrows: velocity

# Magnetic reconnection with uniform resistivity in split-monopole magnetic field around black holes

- Schwarzschild black hole case:

$$\eta = 0.001 r_S \ (S = 100)$$




Petschek type fast magnetic reconnection is found even with uniform resistivity!

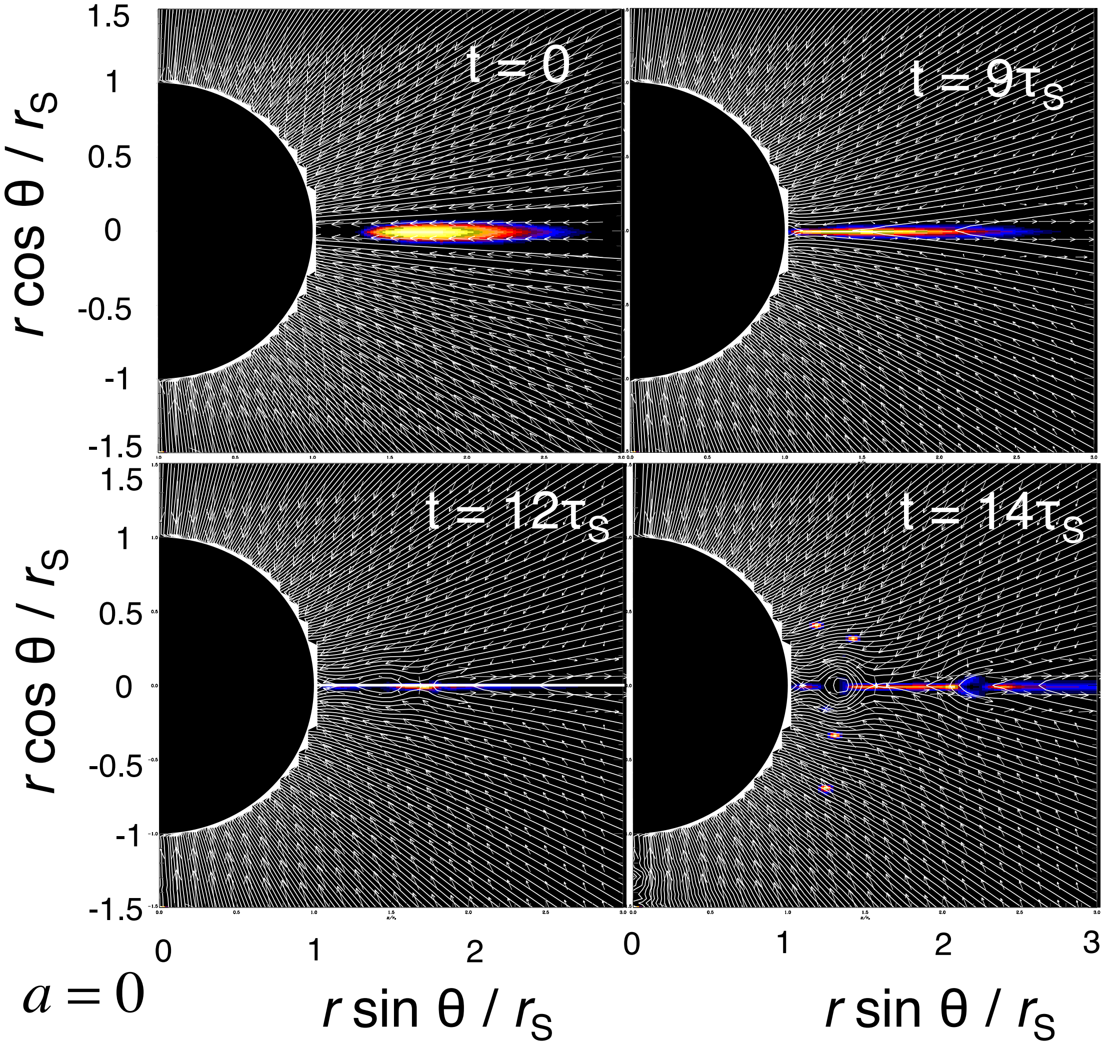
$$\rho_0 = 1 \quad a = 0$$

$$B_0 = 10, \quad \Delta\theta = 0.1$$

Color: pressure
Lines: magnetic field
Arrows: velocity

# Magnetic reconnection with uniform resistivity in split-monopole magnetic field around black holes

"Diffusive slip-through rate of magnetic field lines across plasma"




Lapse function

$$R_{\mathrm{ms}} = \frac{\alpha E'_{\phi}}{v_{\mathrm{A}} B_0} = \frac{\alpha \eta J_{\phi}}{v_{\mathrm{A}} B_0}$$

$$B_0 = 10 \quad \rho_0 = 1$$

Electric field measured by plasma rest frame

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \left( \alpha \left( \boldsymbol{v} \times \boldsymbol{B} + \boldsymbol{E}' \right) \right)$$

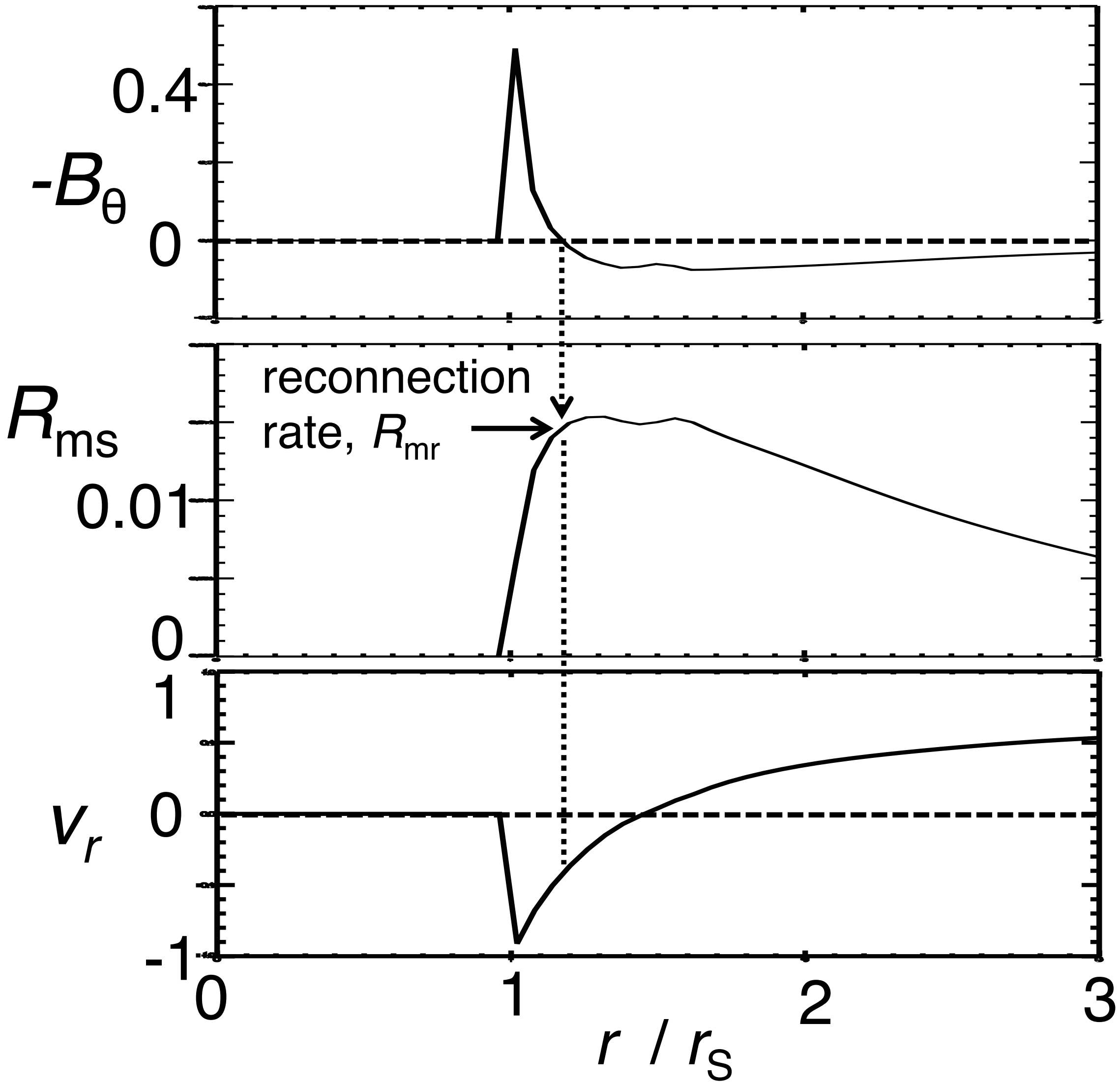
$$\eta = 0.001 r_{\mathrm{S}} \ \left( S = 100 \right)$$

Color: $R_{\mathrm{ms}}$

Lines: magnetic field

Arrows: velocity

# Reconnection rate of magnetic reconnection: Definition




t = 9$\tau_S$

$$R_{mc} = \frac{\alpha E'_\phi}{v_A B_0} = \frac{\alpha \eta J_\phi}{v_A B_0}$$

$$a = 0$$

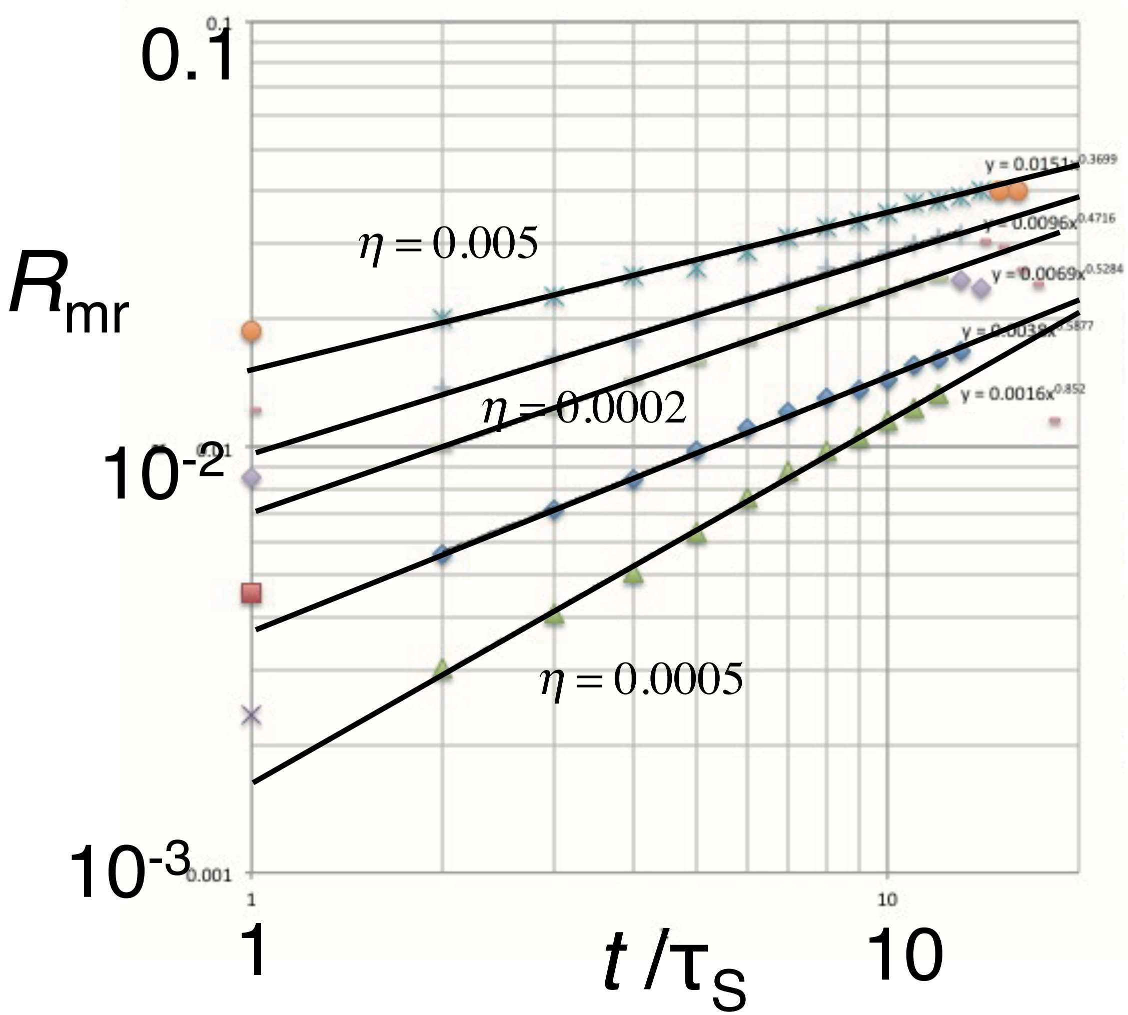
$$\rho_0 = 1$$

$$B_0 = 10$$

$$\Delta\theta = 0.1$$

$$\eta = 0.001 r_S$$

# Time evolution of magnetic reconnection rate: Dependence on resistivity




$$R_{\mathrm{mc}} \propto t^{b}$$

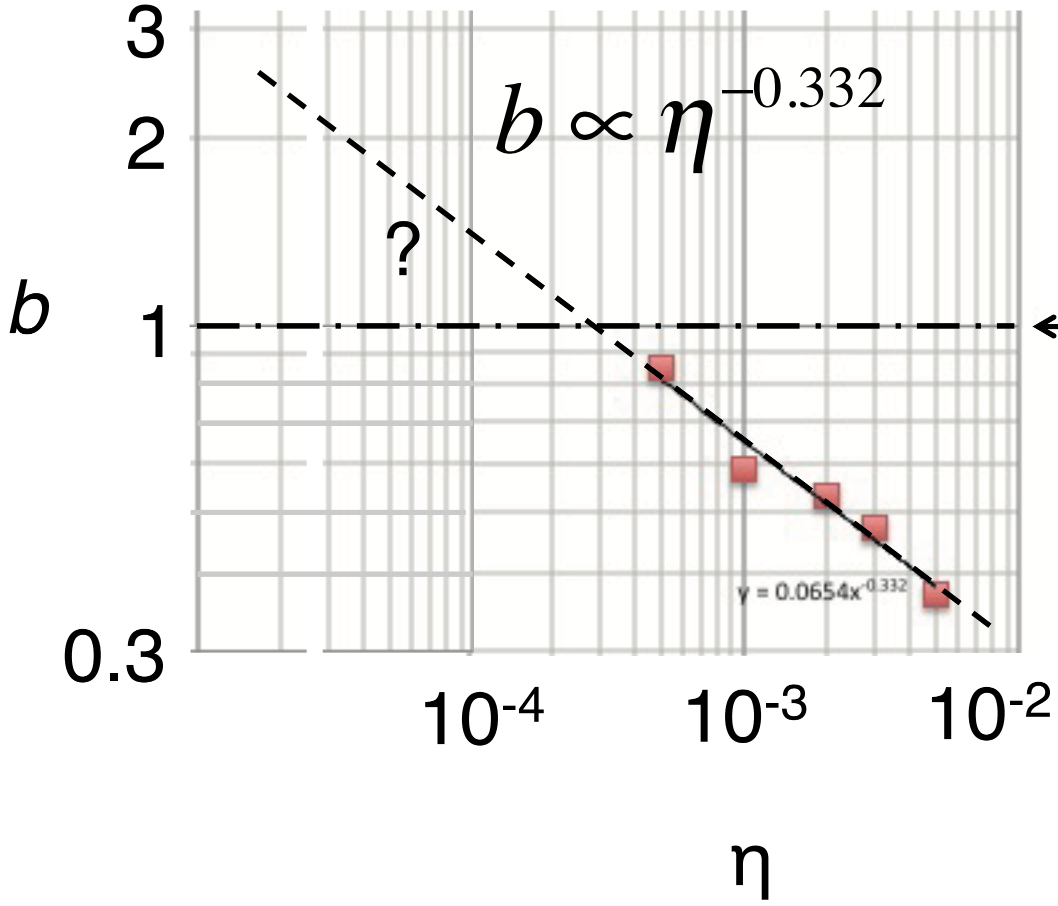
$$a = 0$$

$$\rho_0 = 1$$

$$B_0 = 10$$

$$\Delta\theta = 0.1$$

# Dependence of power index of reconnection rate on resistivity




$$R_{\mathrm{mc}} \propto t^{b}$$

Rutherford regime of tearing mode

$$b = 1 \ \ (S >> 1)$$

$$a = 0$$

$$\rho_0 = 1$$

$$B_0 = 10$$

$$\Delta\theta = 0.1$$

# Models of magnetic reconnection

- Linear growth
  – Tearing instability

$$R_{\mathrm{mr}} \propto e^{\gamma t}, \quad \gamma \propto \frac{1}{\sqrt{S}},$$

- Non-linear growth
  – Rutherford regime of tearing instability

$$R_{\mathrm{mr}} \propto t \quad (S >> 1)$$

- Stationary model $R_{\mathrm{mr}} = \mathrm{const.}$
  – Sweet-Parker model
  – Petschek model

$$R_{\mathrm{mr}} = 1/\sqrt{S}$$

$$R_{\mathrm{mr}} = \pi / 8 \ln S \quad (S >> 1)$$

($S$>>1 except for reconnection region)

# Magnetic reconnection rate

$$\eta = 0.001 r_{\mathrm{S}}, \quad \delta_{\mathrm{CS}} = 0.1, \quad v_{\mathrm{A}} = 1, \quad B_0 = 1 \sim 10$$

$\delta_{\mathrm{CS}}$: Thickness of current sheet

$$\tau_{\mathrm{A}} = 0.1 \tau_S, \quad S = 100$$

$\tau_A = \dfrac{\delta_{\mathrm{CS}}}{v_A}$ (Alfven transit time) $\quad S = \dfrac{\delta_{\mathrm{CS}} v_{\mathrm{A}}}{\eta}$ (Magnetic Reynolds number)

- Sweet-Parker model: possible even in uniform resistivity

$$R_{\mathrm{mr}} = \frac{v_{\mathrm{in}}}{v_{\mathrm{A}}} = \frac{\delta}{L} \cong \frac{1}{S^{1/2}} = 0.1$$

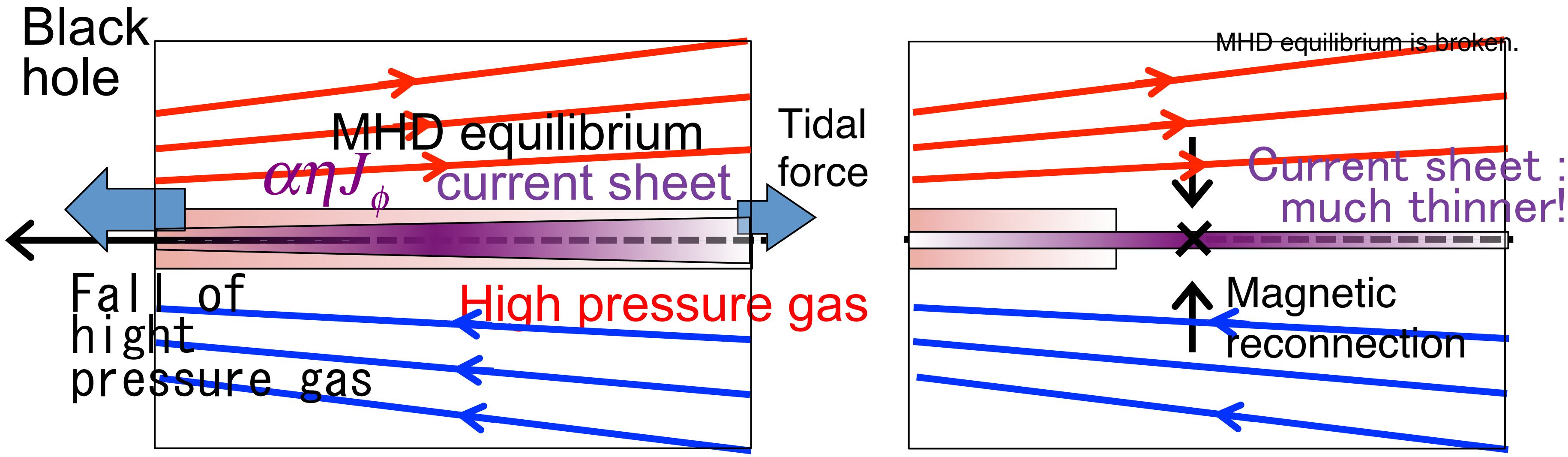
- Petschek model: anomalous resistivity is required

$$R_{\mathrm{mr}} = \frac{v_{\mathrm{in}}}{v_{\mathrm{A}}} \cong \frac{\pi}{8 \ln S} = 0.085$$

Simulations with much lower resistivity ($\eta \leqq 10^{-4}$) are required.

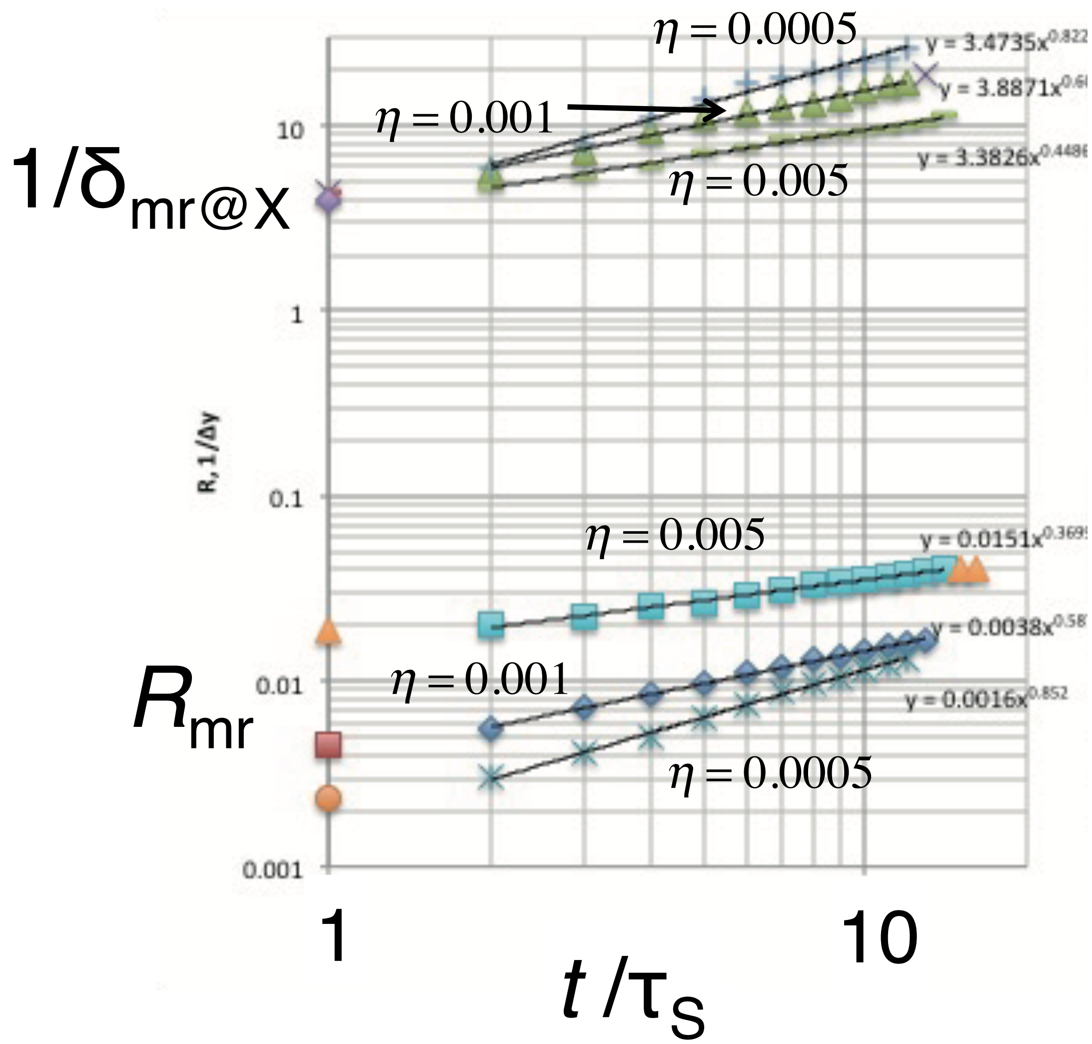
# A “Toy” model of the fast magnetic reconnection in ergoshpere: Mechanism of “Daruma-Otoshi”

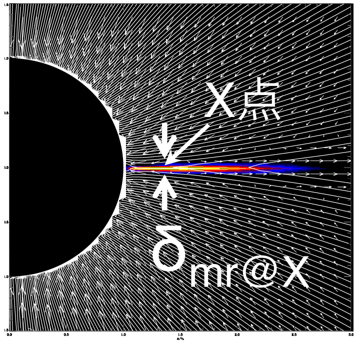
Model: Simple magnetic reconnection caused by localized current density $\alpha J_\phi$ in thin current sheet




Daruma-otoshi: Japanese traditional toy. When one piece is hit to go out, the pieces upper and lower collide.

# Correlation between reconnection rate and thickness of current sheet




thickness of current sheet around the X-point

$$a = 0$$

$$\rho_0 = 1$$

$$B_0 = 10$$

$$\Delta\theta = 0.1$$

# Summary

- We performed resistive GRMHD simulations of magnetic reconnection in split monopole field around Schwarzschild black hole (the simplest situation).
- We found magnetic reconnection like Petchek model reconnection which has point-like reconnection region. The time evolution of the reconnection rate shows power law. The power index depends on the resistivity. We cannot identify the reconnection around the black hole as known reconnection models.
- A simple toy model: thinner current sheet is induced by the reconnection outflows and the localized current density in the current sheet induces the reconnection..

## Future plan

Simulations with much lower resistivity ($\eta \leqq 10^{-4}$) are required.