

# BH-NS

# MAGNETOSPHERE

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# GW ASTRONOMY

Binary Black Hole → GW150916, .. 5 detection

Binary Neutron Star → **GW170817**

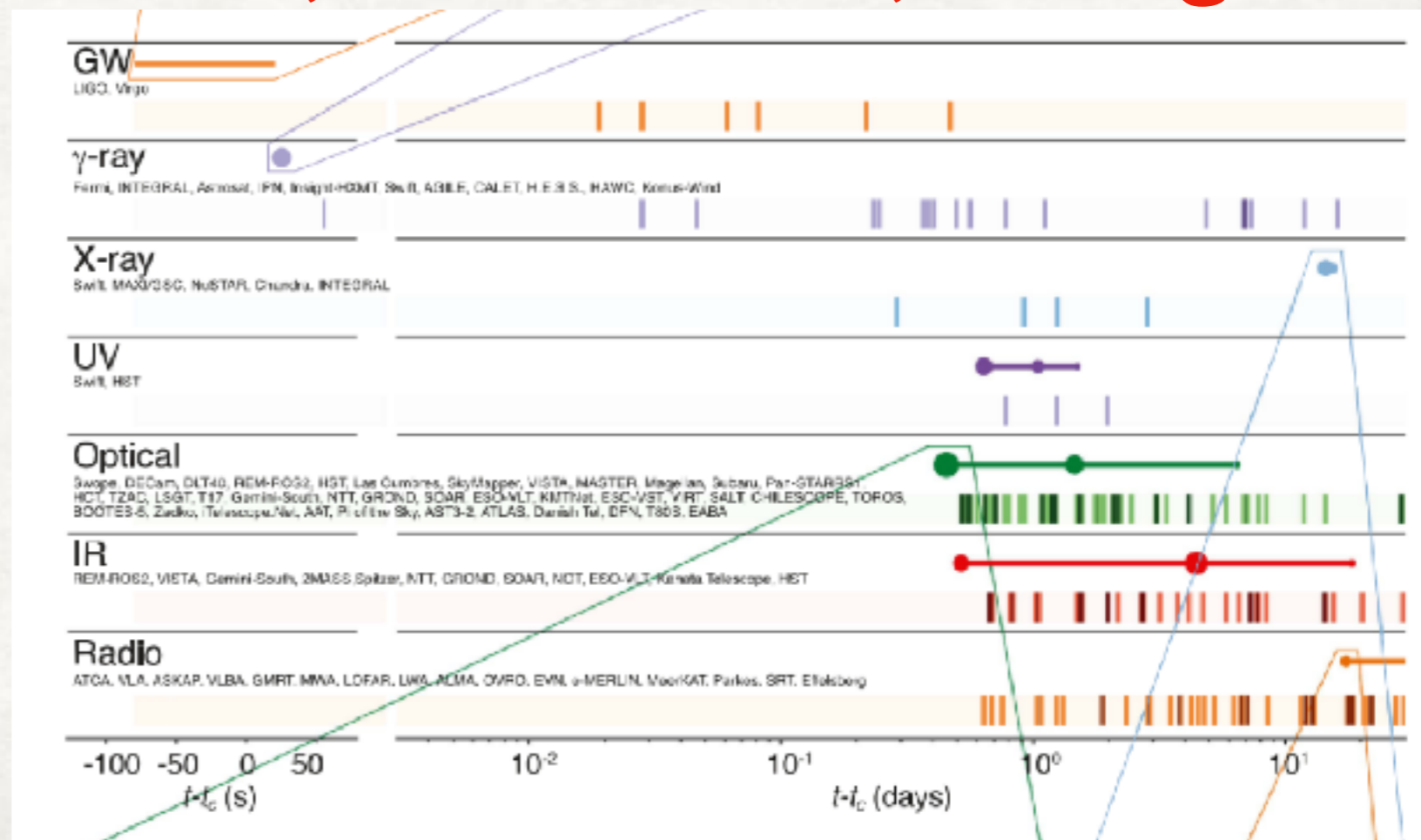
& **GRB170817A, macronova, afterglow**

$\gamma$ -ray  $\sim$  Radio

Electromagnetic

Counterpart

was detected !!!



# GW ASTRONOMY

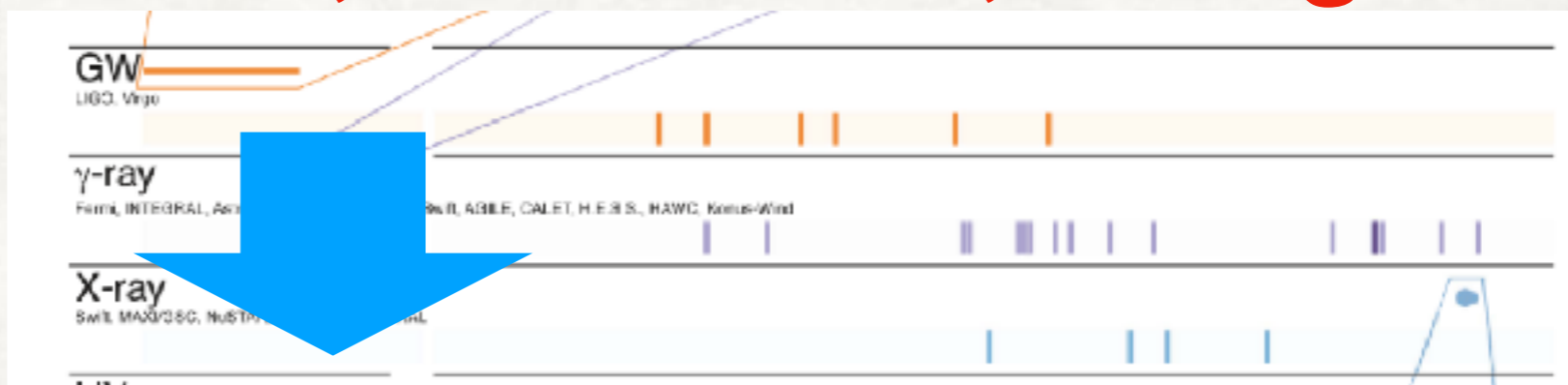
Binary Black Hole → GW150916, ...5.5 detection

Binary Neutron Star → **GW170817**

& **GRB170817A, macronova, afterglow**

$\gamma$ -ray  $\sim$  Radio

**Electromagnetic**



**What can we detect with GW  
from BH-NS merger ???**



# COUNTERPART WITH BH-NS MERGER

What happens with BH-NS merger ?

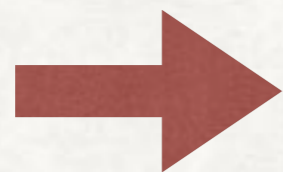
Typically

- **light BH** ( $M_{\text{BH}}/M_{\text{NS}} \lesssim 5, a \gtrsim 0.5$ )

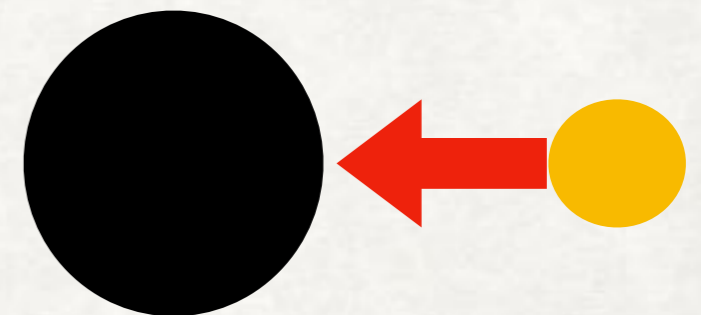
→ tidal disruption ... destroyed NS will light

- **heavy BH** (Else)

→ NO tidal disruption



Nothing will light ??



# COUNTERPART WITH BH-NS MERGER

What happens with BH-NS merger ?

Typically

- **light BH** ( $M_{\text{BH}}/M_{\text{NS}} \lesssim 5$ ,  $a \gtrsim 0.5$ )

→ tidal disruption ... destroyed NS will light

- **heavy BH** (Else)

→ NO tidal disruption



Nothing will light ??

**NS ... STRONG MAGNETIC FIELD**

**→ LIGHT EMISSION ?**



# PULSAR MAGNETOSPHERE

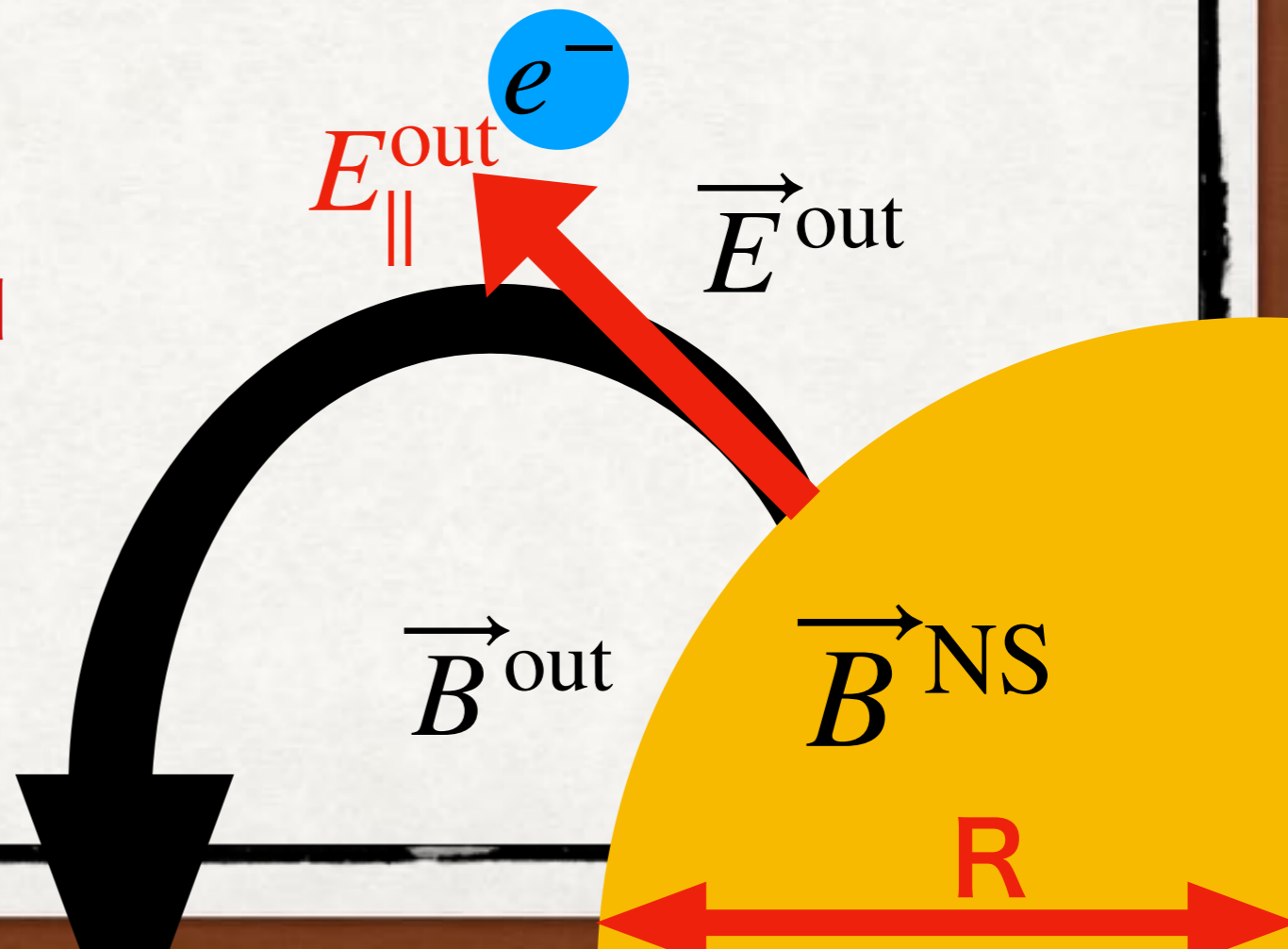
- assume pulsar is dipole
- NS is conductor  
 $\rightarrow \vec{E} = \vec{0}$  in coronation frame
- Poisson equation

$\rightarrow \vec{B}^{\text{NS}} \quad \vec{B}^{\text{out}}$

$\rightarrow \vec{E}^{\text{NS}} = -\frac{(\vec{\Omega} \times \vec{r})}{c} \times \vec{B}^{\text{NS}}$

$\rightarrow E^{\text{out}} \sim \frac{\Omega B^{\text{NS}} R^5}{cr^4}$

- Electric field parallel to  
Magnetic field  $E_{\parallel}^{\text{out}} \sim \frac{\Omega R}{c} B$  will  
accelerate charged particles



LIGHT EMISSION



# PULSAR MAGNETOSPHERE

- assume pulsar is dipole

$$\vec{B}^{\text{NS}} \quad \vec{B}^{\text{out}}$$

- NS is conductor

$$\vec{E}^{\text{NS}} = -\frac{(\vec{\Omega} \times \vec{r})}{c} \times \vec{B}^{\text{NS}}$$

→  $\vec{E} = \vec{0}$  in coronation frame

- Poisson equation

$$E^{\text{out}} \sim \frac{\Omega B^{\text{NS}} R^5}{4}$$

**IF THIS HAPPENS IN BH-NS BINARY,  
CHARGED PARTICLE WILL EMIT LIGHT  
→ THIS CAN BE COUNTERPART  
OF GW FROM BH-NS MERGER**

**LIGHT EMISSION**

$$\vec{B}^{\text{out}}$$

$$\vec{B}^{\text{NS}}$$

**R**



# PROBLEM

Pulsar ...  $E_{\parallel}^{\text{out}}$  accelerates particles

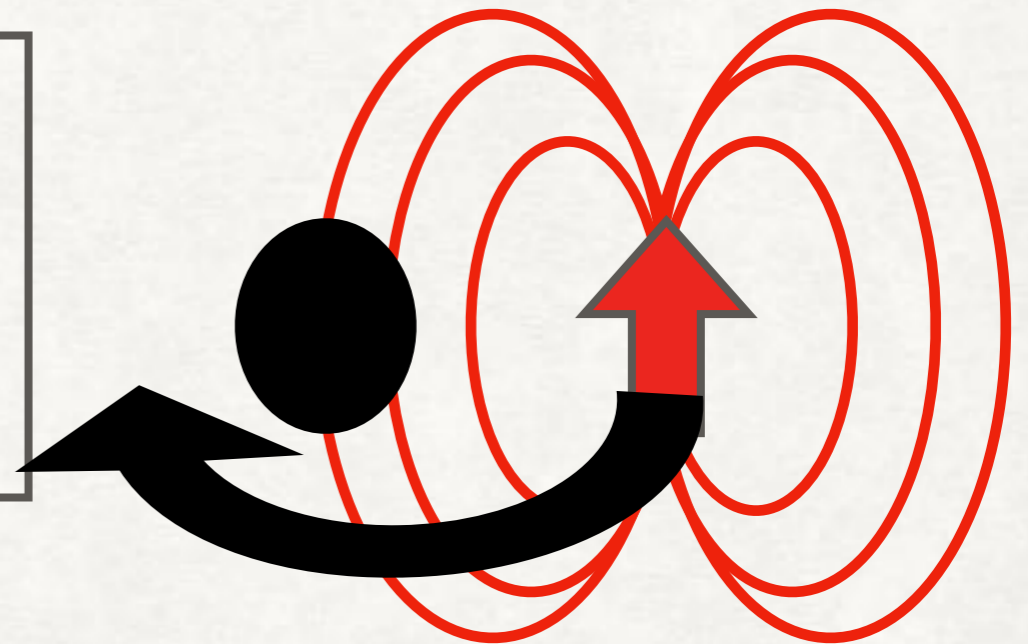
→ Does  $E_{\parallel}^{\text{out}}$  exist in binary ??

## Setting

Consider magnetic dipole  
rotating BH

Calculate induced electric field

Is there  $E_{\parallel}^{\text{out}}$  which accelerate particles from NS ??





# AIM

## Pulsar magnetosphere

Assume NS is dipole which is spinning

→ Calculate induced electric field

→  $E_{\parallel}^{\text{out}}$  will accelerate particles



## Binary magnetosphere

Assume NS is dipole which is **rotating BH**

→ Calculate induced electric field

→ is there  $E_{\parallel}^{\text{out}}$  ?

# BASIC EQUATIONS

Maxwell equations ( in flat spacetime )

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \partial_t \vec{B} + \vec{\nabla} \times \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \quad \partial_t \vec{E} - \vec{\nabla} \times \vec{B} = -4\pi\vec{J}$$

Current is rotating dipole

$$J^\mu(x) = -\nabla_\lambda \int d\tau m^{\lambda\mu} \frac{\delta(x^\mu - x_{NS}^\mu(\tau))}{\sqrt{-g}}$$

$m^{\lambda\mu}$  : magnetic moment  
 $x_{NS}^\mu(\tau)$  : orbit of NS

# VECTOR HARMONICS EXPANSION

Expand vector fields using vector harmonics

$$\mathbf{B}_i = \sum_{l,m} \left[ \mathbf{B}_1^{lm} \left( Y_{lm}(\theta, \phi), 0, 0 \right) + \mathbf{B}_2^{lm} \left( 0, Y_{lm,\theta}(\theta, \phi), Y_{lm,\phi}(\theta, \phi) \right) + \mathbf{B}_3^{lm} \left( 0, -\frac{Y_{lm,\phi}(\theta, \phi)}{\sin \theta}, \sin \theta Y_{lm,\phi}(\theta, \phi) \right) \right]$$

Maxwell equations



Time evolution equation

$B_1^{lm}$  : radial direction

$B_2^{lm}$  : angular direction  
party even

$B_3^{lm}$  : angular direction  
party odd

$$\partial_t^2 \tilde{B}_1^{lm}(t, r) = \partial_r \left( \partial_r \tilde{B}_1^{lm}(t, r) \right) - \frac{l(l+1)}{r^2} \tilde{B}_1^{lm} - 4\pi l(l+1) J_3^{lm}(r)$$

$$\tilde{B}_1^{lm}(t, r) = r^2 B_1^{lm}(t, r)$$

$$\partial_r \tilde{B}_1^{lm}(t, r) = l(l+1) B_2^{lm}(t, r)$$

$$\partial_t^2 \tilde{B}_3^{lm}(t, r) = \partial_r \left( \partial_r \tilde{B}_3^{lm}(t, r) \right) - \frac{l(l+1)}{r^2} \tilde{B}_3^{lm} - 4\pi \left[ J_1^{lm}(r) - \partial_r \left( J_2^{lm}(r) \right) \right]$$

# GREEN FUNCTION METHOD

- Fourier transformation

$$\frac{d^2}{dr^2} \tilde{B}_1^{lm}(\omega, r) + \left( \omega^2 - \frac{l(l+1)}{r^2} \right) \tilde{B}_1^{lm}(\omega, r) = 4\pi l(l+1) J_3^{lm}(\omega, r)$$
$$\frac{d^2}{dr^2} B_3^{lm}(\omega, r) + \left( \omega^2 - \frac{l(l+1)}{r^2} \right) B_3^{lm}(\omega, r) = 4\pi (J_1^{lm}(\omega, r) - \partial_r J_2^{lm}(\omega, r))$$

- give boundary condition  Construct Green function & Solve equation

Regular at  $r = 0$

$$\omega r j_l(\omega r)$$

outgoing wave at infinity

$$\omega r h_l^{(1)}(\omega r)$$



# MAGNETIC FIELD

$$B_1^{lm}(t, r) = -i \frac{4\pi m^z}{|\dot{t}|} \frac{1}{r^2} Y_{lm, \theta}^* \left( \frac{\pi}{2}, 0 \right) e^{-im\Omega t} \left\{ \theta(r - R) \left[ \frac{1}{r'} \partial_{r'} (r' j_l(m\Omega r')) \right]_{r'=R} m\Omega r h_l^{(1)}(m\Omega r) + \theta(R - r) \left[ \frac{1}{r'} \partial_{r'} (r' h_l^{(1)}(m\Omega r')) \right]_{r'=R} m\Omega r j_l(m\Omega r) \right\} \quad (47)$$

$$B_2^{lm}(t, r) = -i \frac{4\pi m^z}{|\dot{t}|} \frac{1}{l(l+1)} Y_{lm, \theta}^* \left( \frac{\pi}{2}, 0 \right) e^{-im\Omega t} \left\{ \theta(r - R) \left[ \frac{1}{r'} \partial_{r'} (r' j_l(m\Omega r')) \right]_{r'=R} \frac{d}{dr} [m\Omega r h_l^{(1)}(m\Omega r)] + \theta(R - r) \left[ \frac{1}{r'} \partial_{r'} (r' h_l^{(1)}(m\Omega r')) \right]_{r'=R} \frac{d}{dr} [m\Omega r j_l(m\Omega r)] - \frac{i}{R} \delta(r - R) \right\} \quad (48)$$

$$B_3^{lm}(t, r) = \frac{4\pi m^z}{|\dot{t}|} \frac{m}{l(l+1)} Y_{lm}^* \left( \frac{\pi}{2}, 0 \right) e^{-im\Omega t} \left\{ \theta(r - R) (m\Omega)^2 j_l(m\Omega R) m\Omega r h_l^{(1)}(m\Omega r) + \theta(R - r) (m\Omega)^2 h_l^{(1)}(m\Omega R) m\Omega r j_l(m\Omega r) - \frac{i}{R} \delta(r - R) \right\} \quad (49)$$

times vector harmonics  
& sum up l, m

# ELECTRIC FIELD

$$E_1^{lm}(t, r) = \frac{4\pi m^z}{|\dot{t}|} \frac{im}{r^2} Y_{lm}^* \left( \frac{\pi}{2}, 0 \right) e^{-im\Omega t} \left\{ \theta(r - R) m\Omega j_l(m\Omega R) m\Omega r h_l^{(1)}(m\Omega r) \right. \\ \left. + \theta(R - r) m\Omega h_l^{(1)}(m\Omega R) m\Omega r j_l(m\Omega r) \right\}$$

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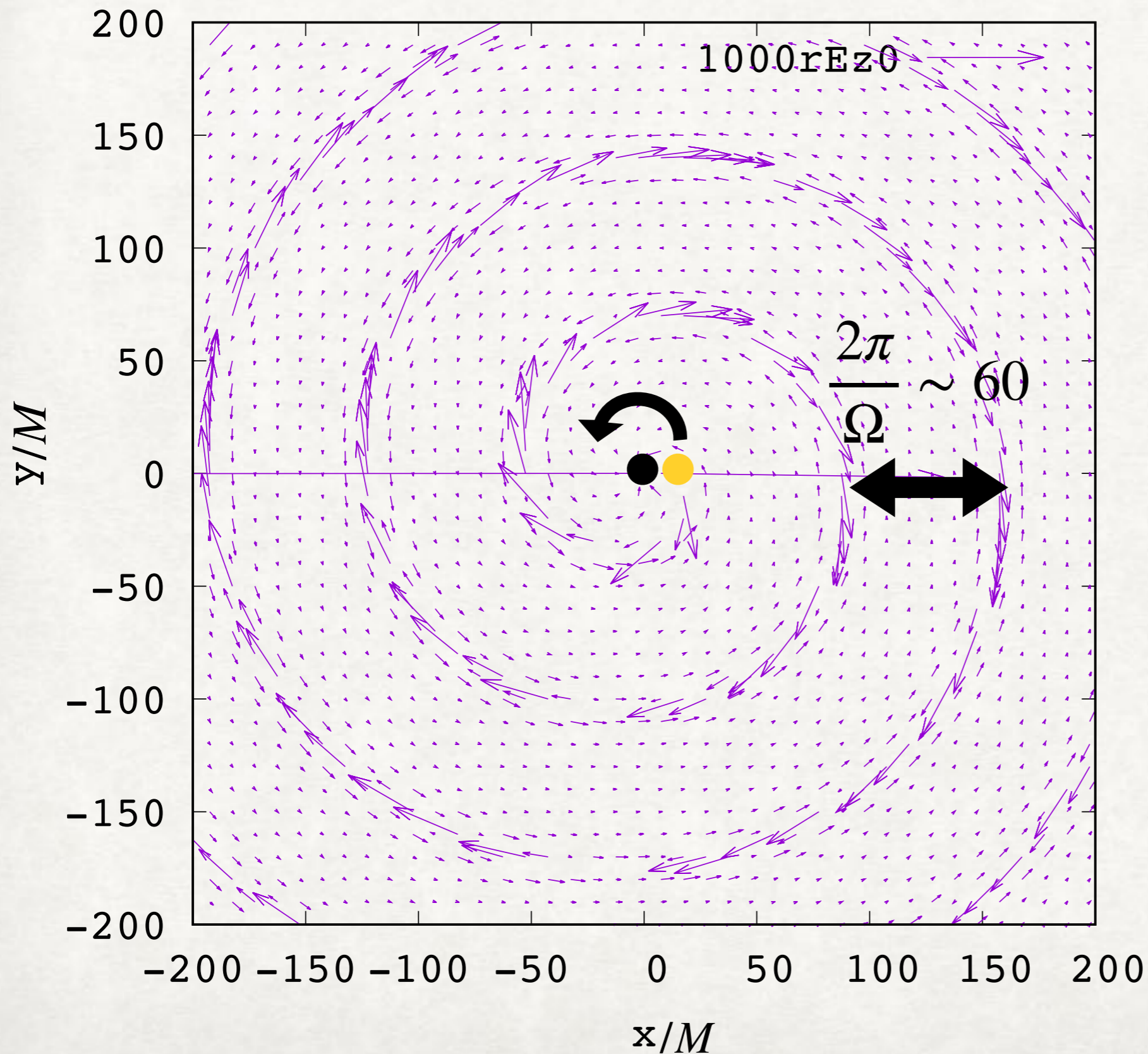
times vector harmonics  
sum up l, m

# ELECTRIC FIELD

$$E_1^{lm}(t, r) = \frac{4\pi m^z}{|\dot{t}|} \frac{im}{r^2} Y_{lm}^* \left( \frac{\pi}{2}, 0 \right) e^{-im\Omega t} \left\{ \theta(r - R) m\Omega j_l(m\Omega R) m\Omega r h_l^{(1)}(m\Omega r) \right. \\ \left. + \theta(R - r) m\Omega h_l^{(1)}(m\Omega R) m\Omega r j_l(m\Omega r) \right\}$$

WHAT IS  
HAPPENING ????

# NUMERICAL ELECTRIC (Z=0)



$r \cdot E$   
@ constant time  
surface

separation  
 $R = 5$

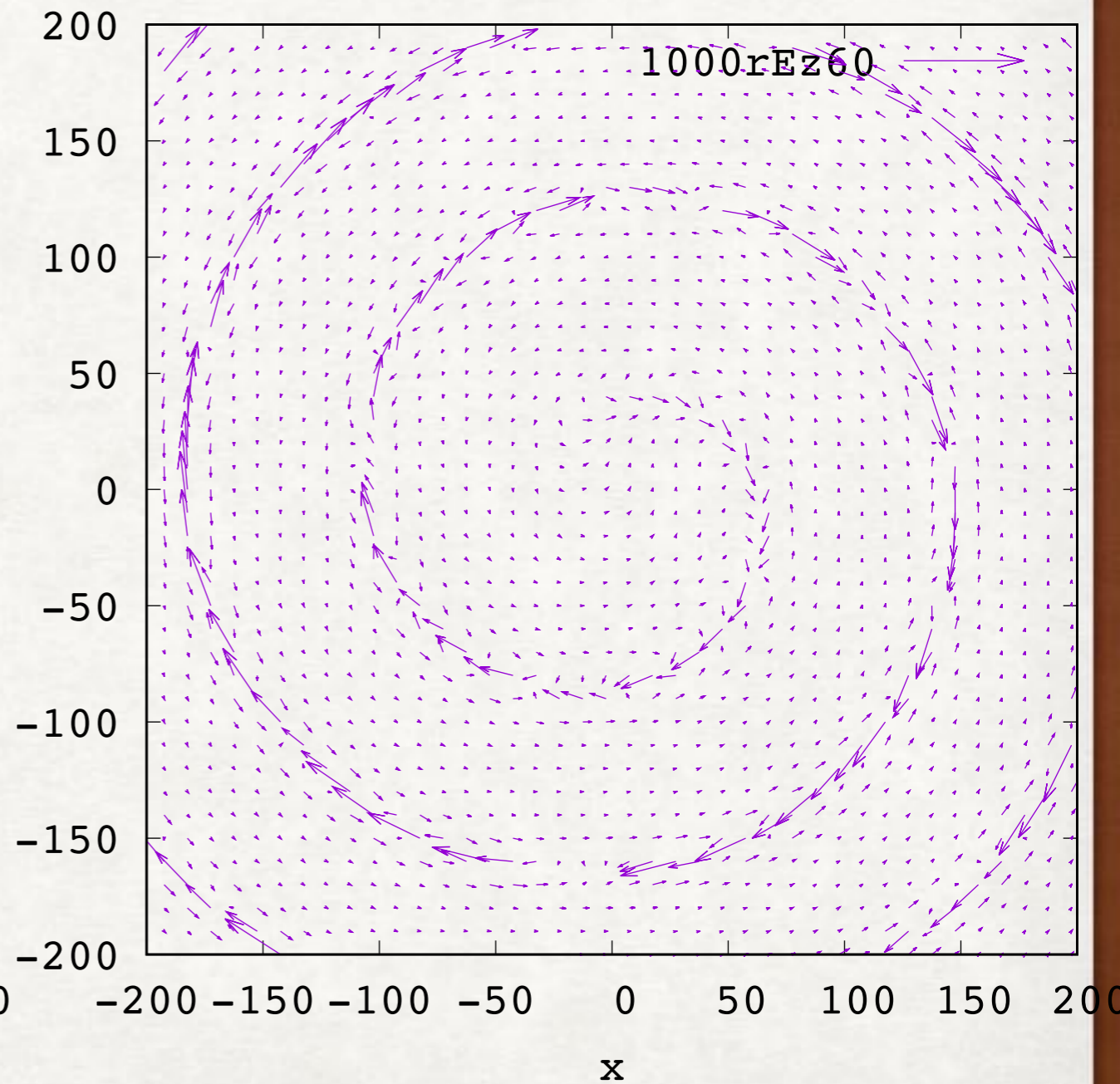
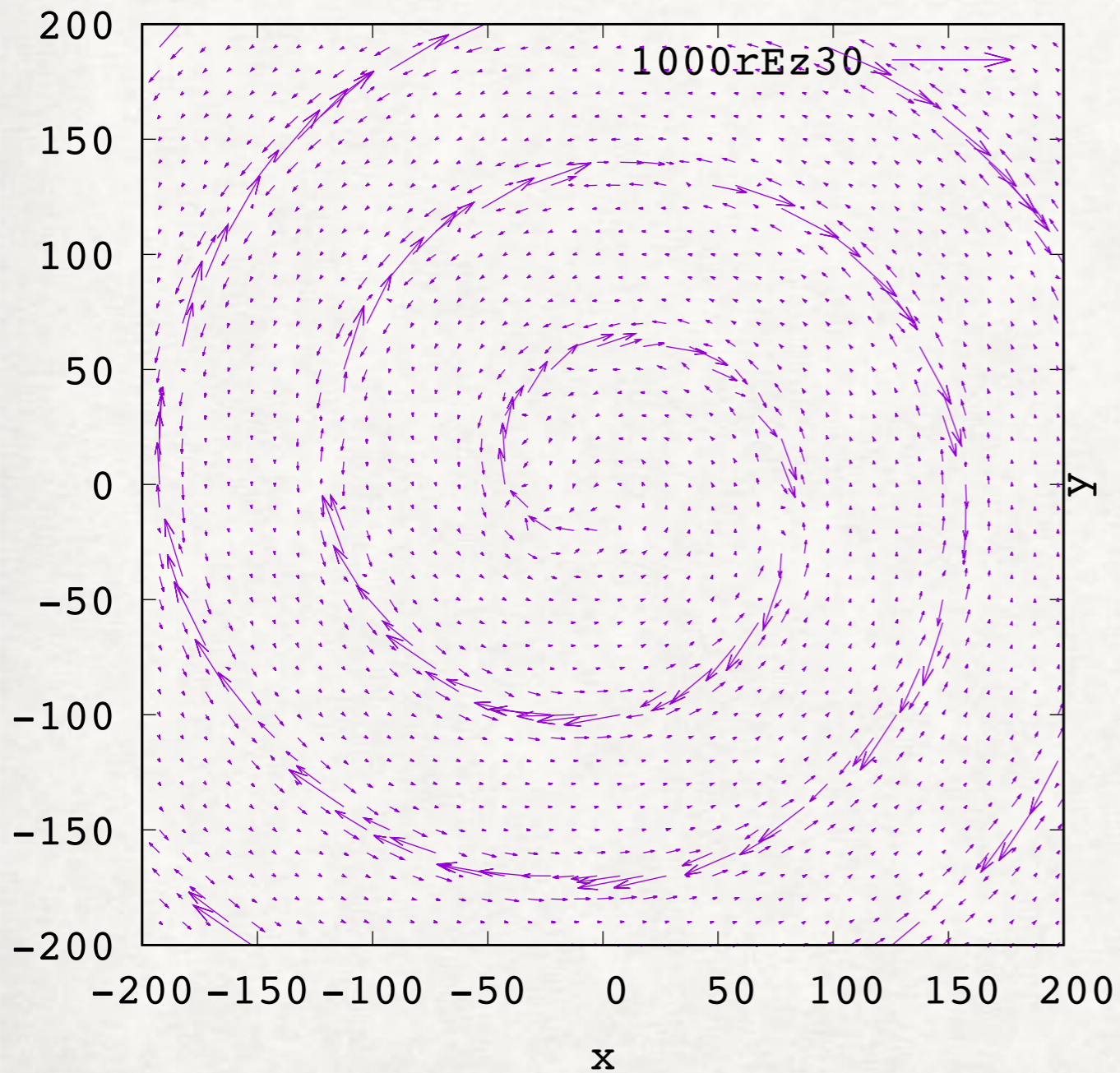
total mass  
 $M_{\text{tot}} = 10M_{\odot}$



# NUMERICAL ELECTRIC ( $rE$ )

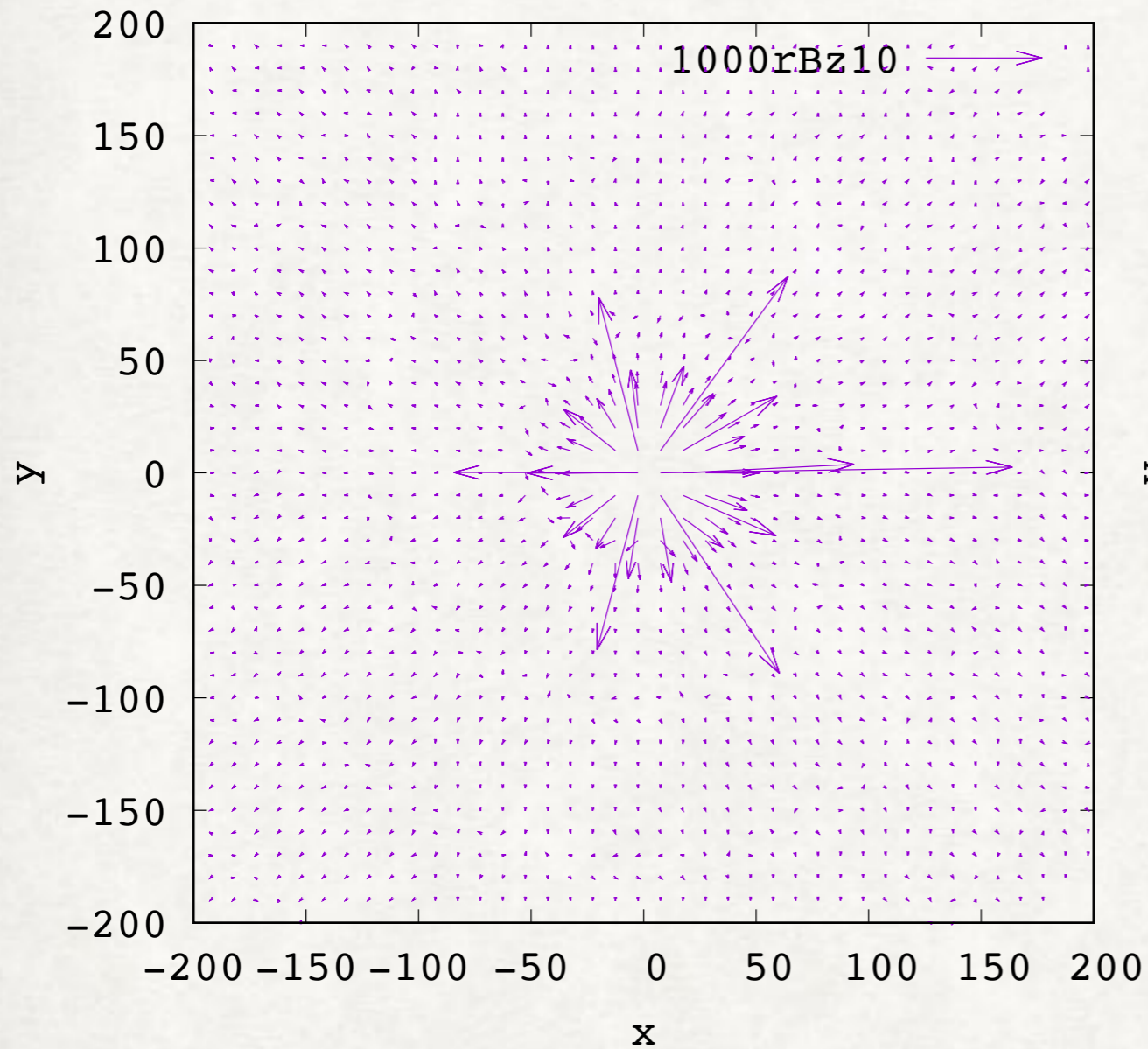
$z=30$

$z=90$



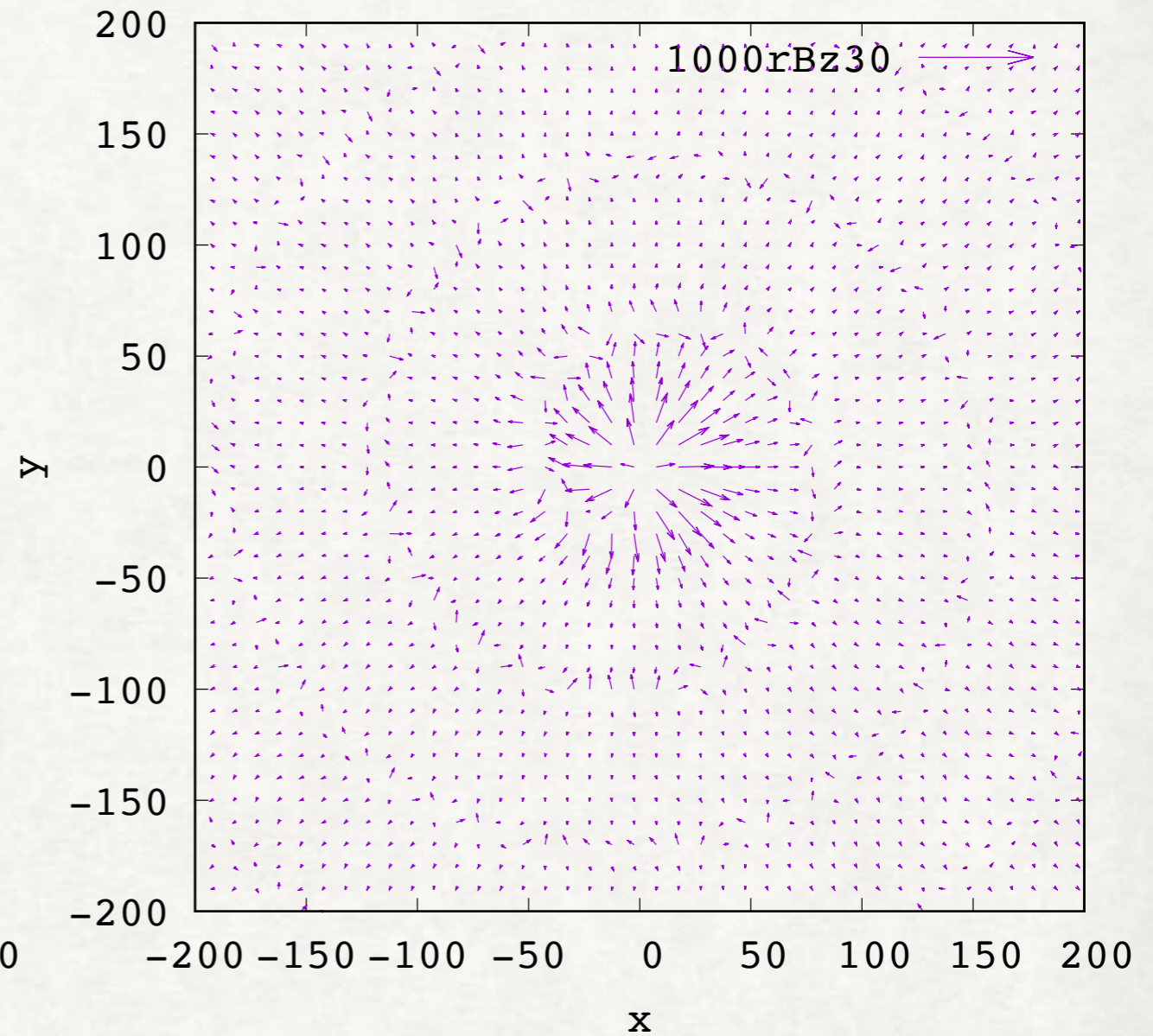
# NUMERICAL MAGNETIC ( $r \cdot B$ )

$z=10$



almost dipole

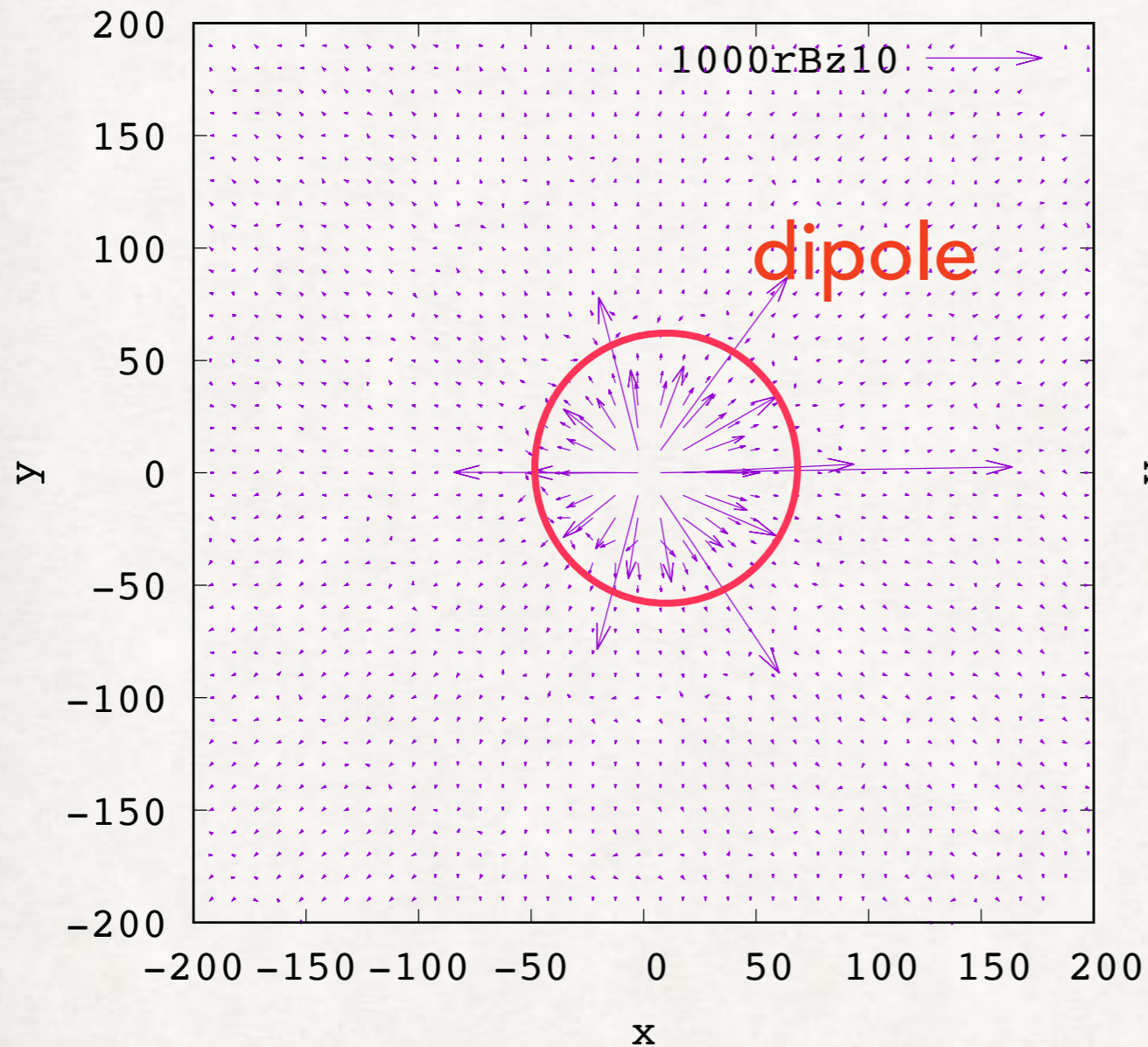
$z=30$



dipole + spiral arm (radiation)

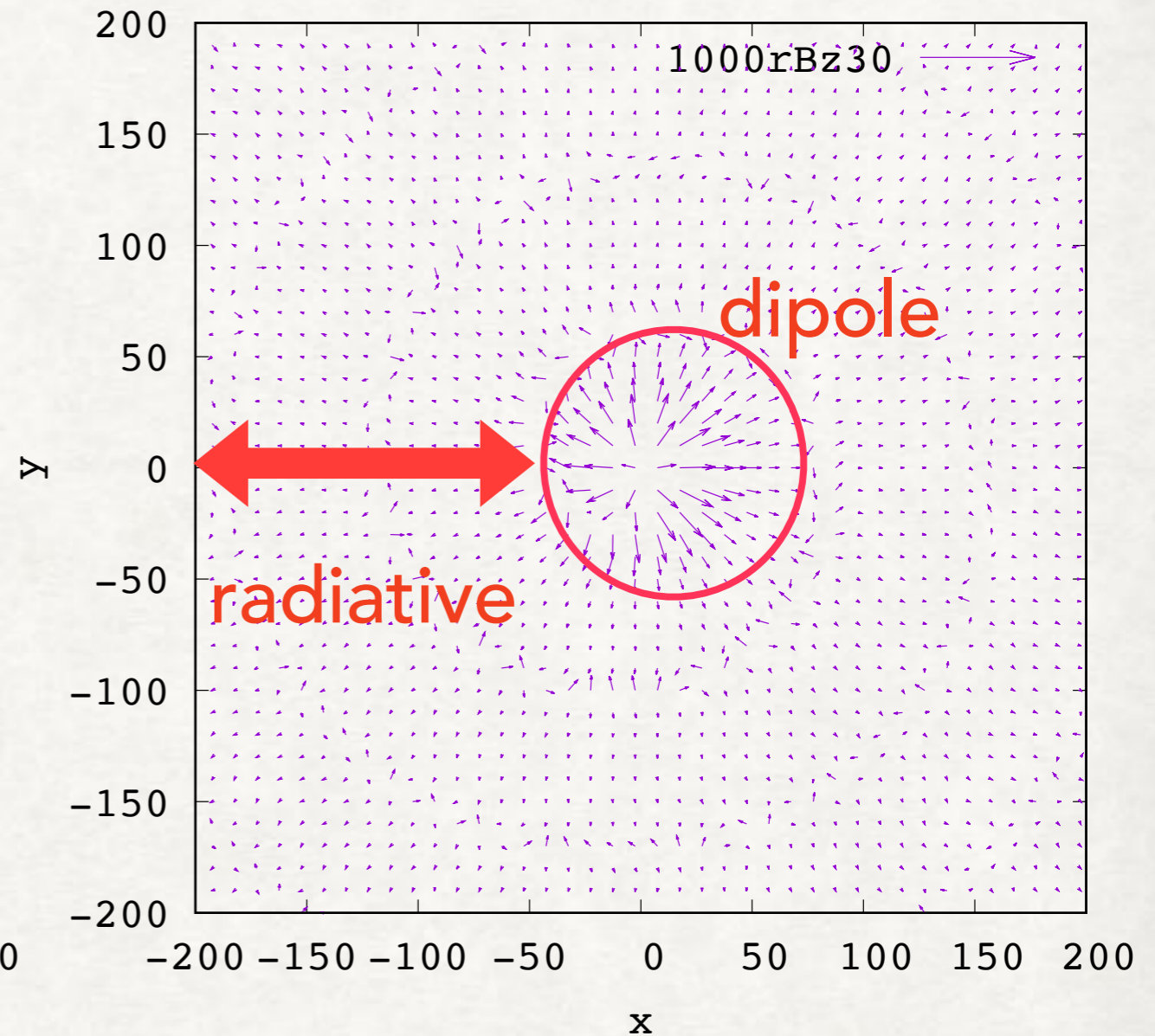
# NUMERICAL MAGNETIC ( $r \cdot B$ )

$z=10$



almost dipole

$z=30$

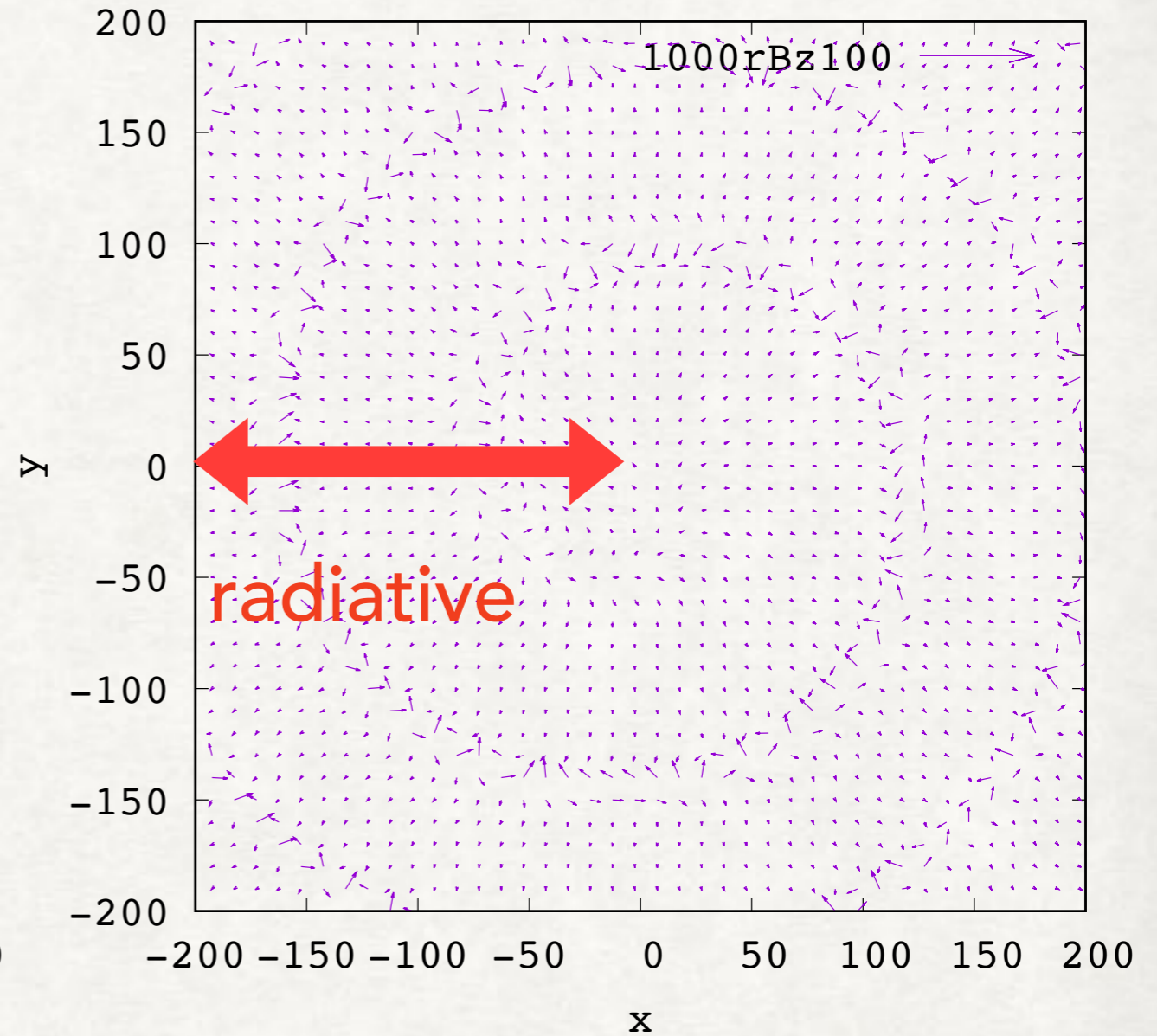
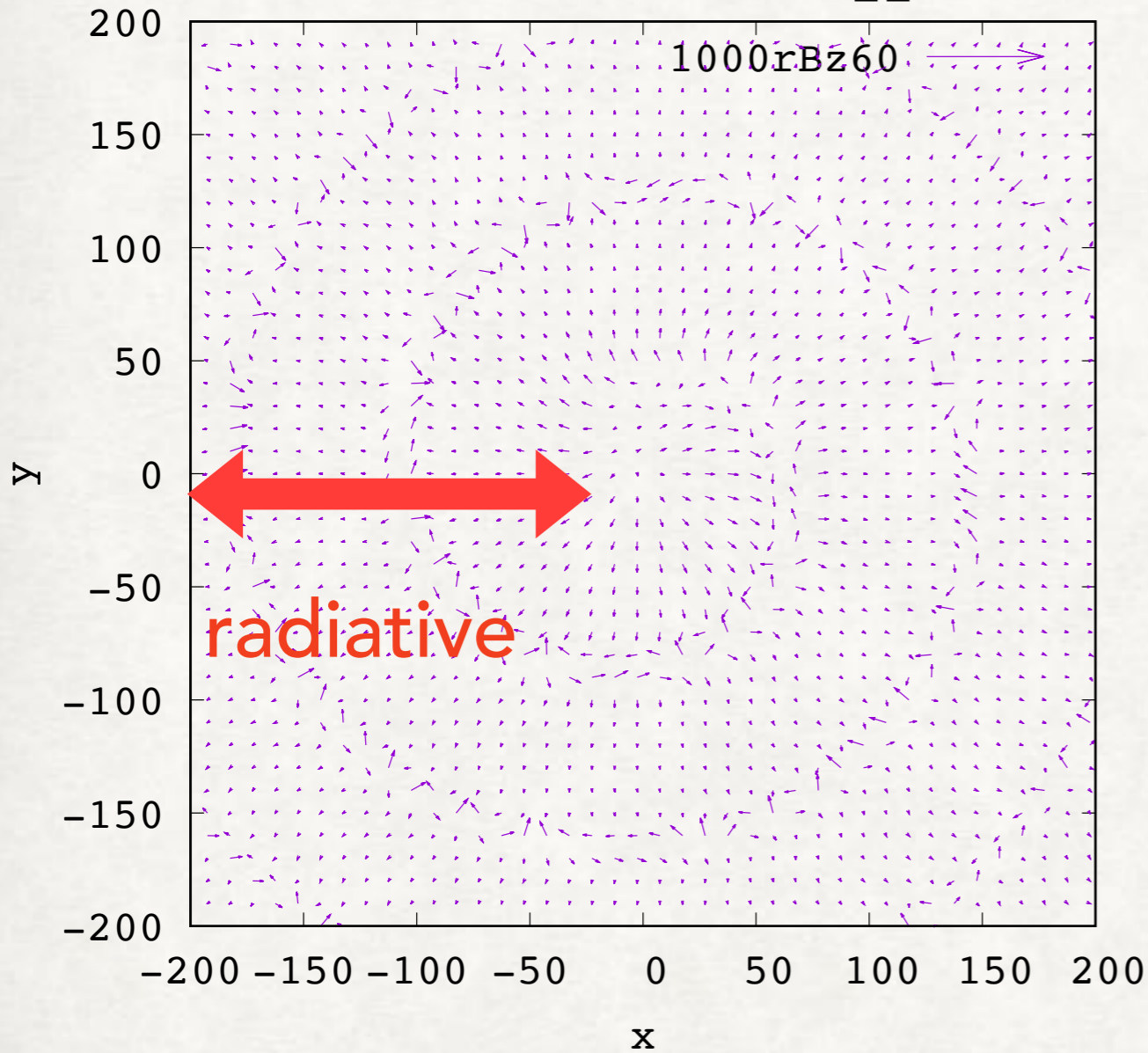


dipole + spiral arm (radiation)

# NUMERICAL MAGNETIC

$$z=60 \sim \frac{2\pi}{\Omega}$$

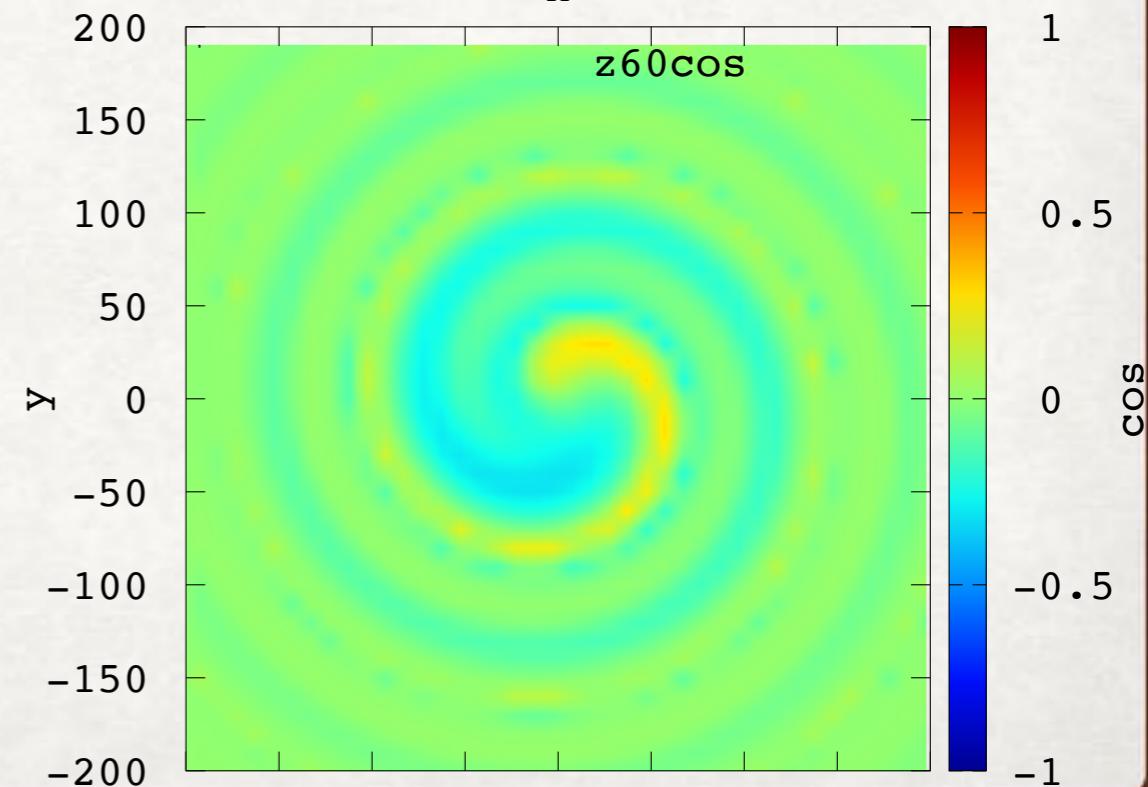
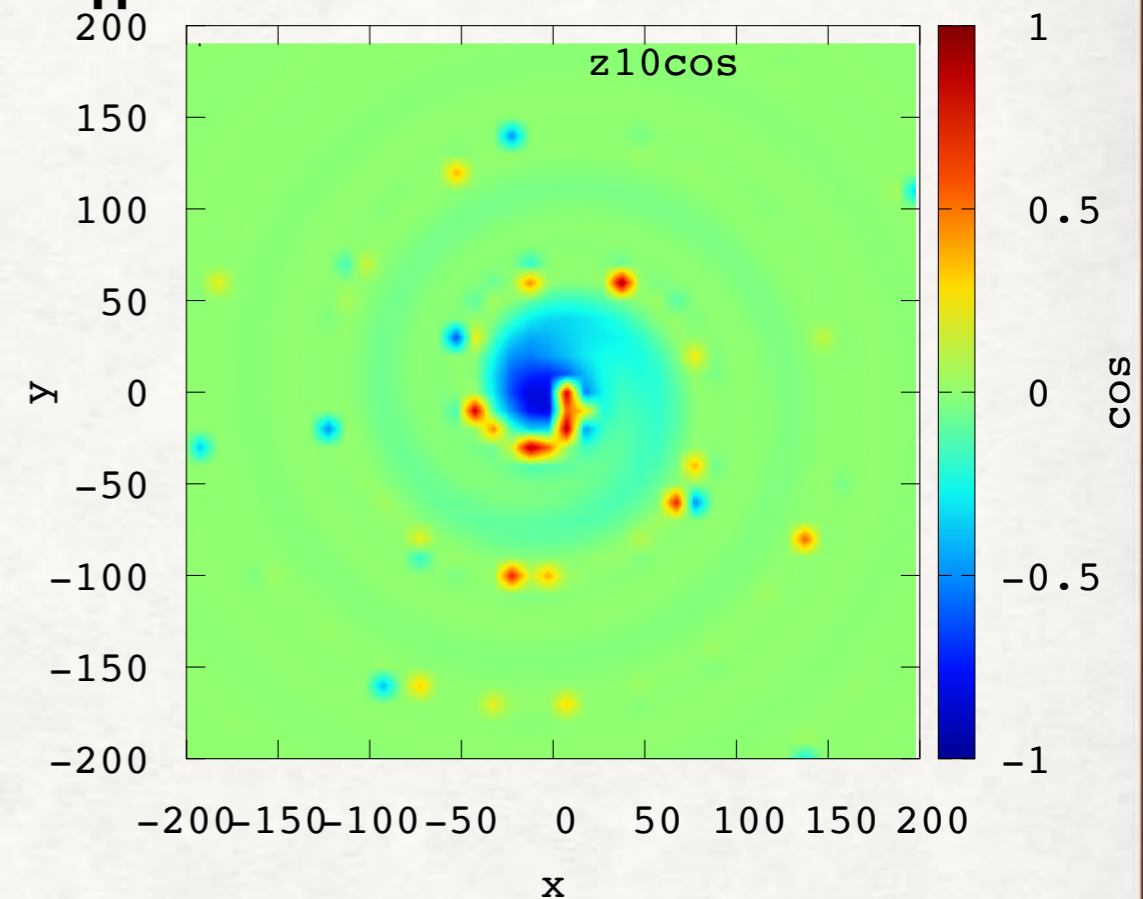
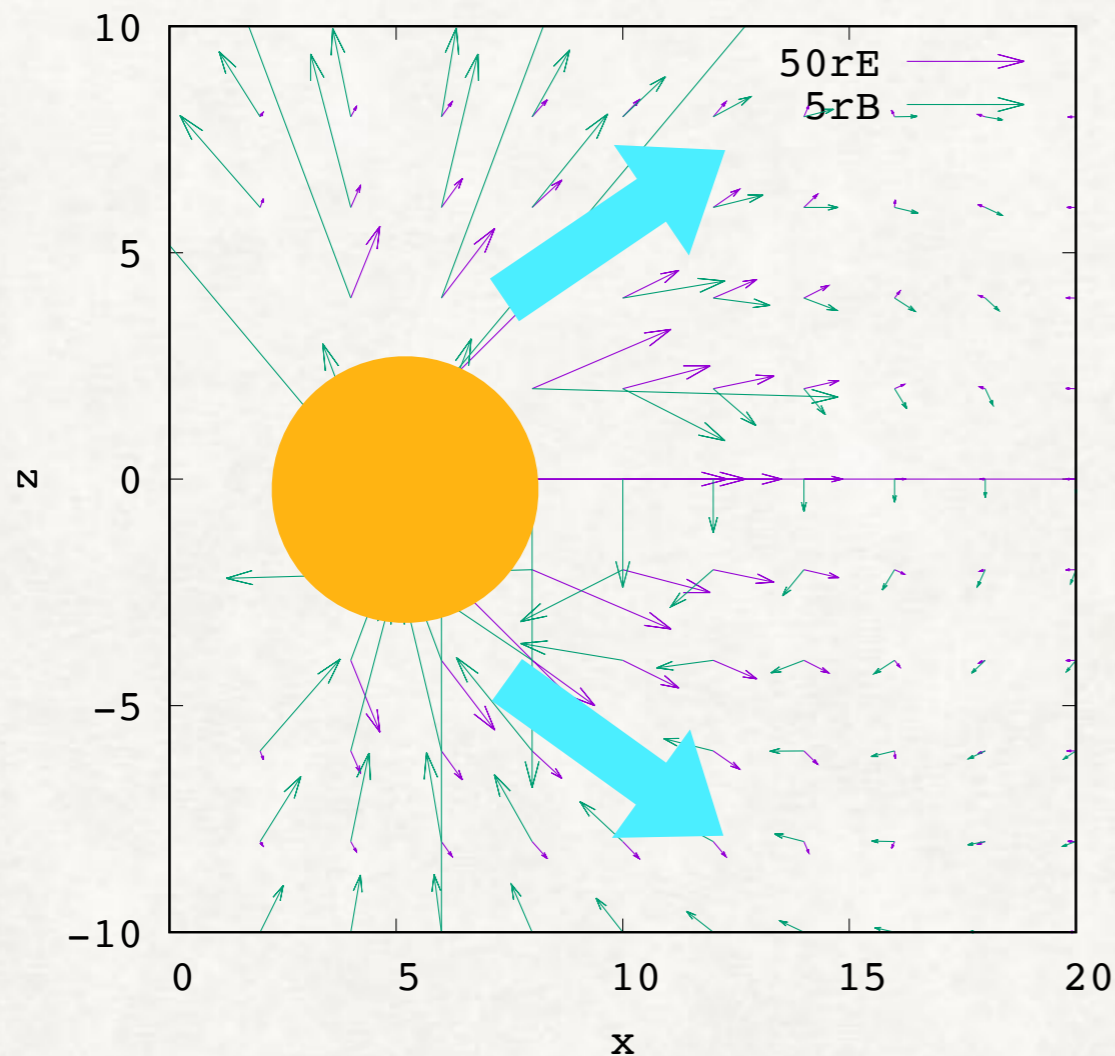
$z=100$



almost spiral arm (radiation)

spiral arm (radiation)

# IS THERE $E_{\parallel}^{\text{out}}$ ??



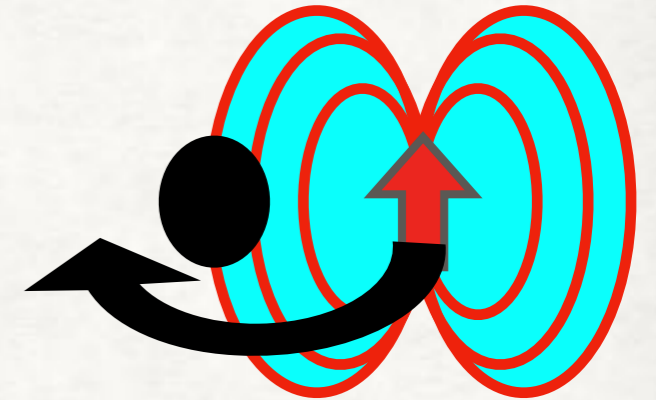
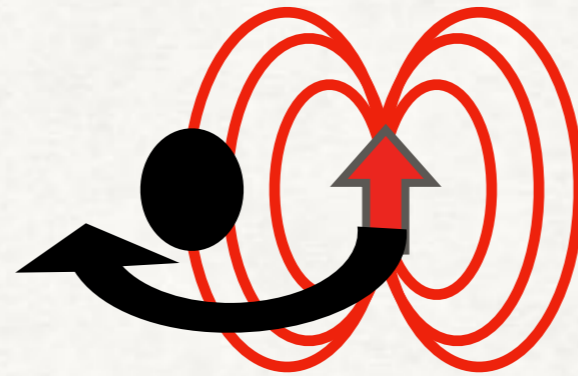
## PARTICLE ACCELERATION

There is  $E_{\parallel}^{\text{out}}$  and  
Particle acceleration  
will happen

# BINARY MAGNETOSPHERE

There is  $E_{\parallel}^{\text{out}}$

→ magnetosphere can't be vacuum  
and may filled with plasma..



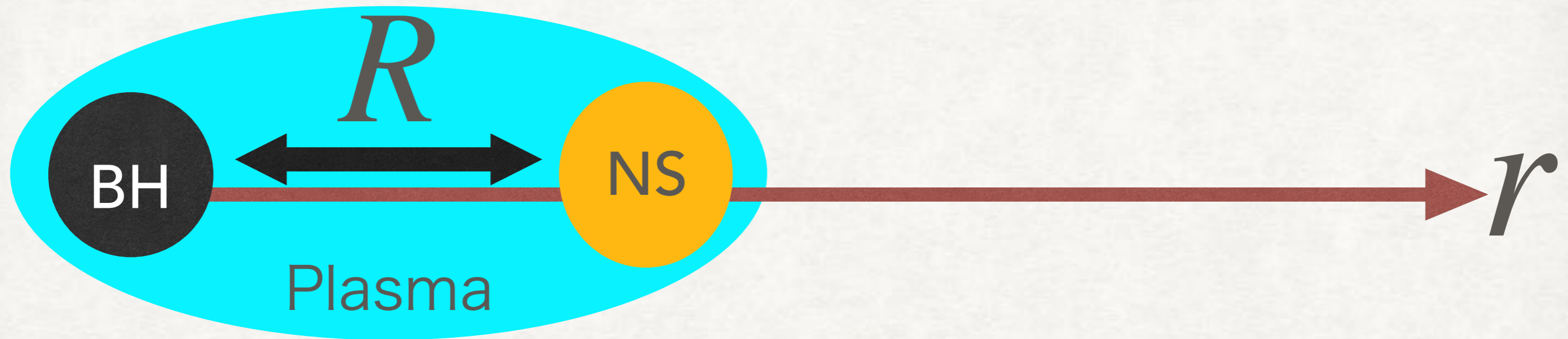
There are gaps in co-rotating magnetosphere  
→ particle acceleration  
& charged particle emits light



In binary, electromagnetic field changes dynamically  
→ there must be many gaps

# PARTICLE ACCELERATION

Order estimate of particle acceleration



Considering only  $l = 1$  mode

$$B_1^{lm}(t, r) = -i \frac{4\pi m^2}{|\dot{l}|} \frac{1}{r^2} Y_{lm, \theta}^* \left( \frac{\pi}{2}, 0 \right) e^{-im\Omega t} \left\{ \theta(r-R) \left[ \frac{1}{r'} \partial_{r'} (r' j_l(m\Omega r')) \right]_{r'=R} m\Omega r h_l^{(1)}(m\Omega r) + \theta(R-r) \left[ \frac{1}{r'} \partial_{r'} (r' h_l^{(1)}(m\Omega r')) \right]_{r'=R} m\Omega r j_l(m\Omega r) \right\} \quad (47)$$

$$B_2^{lm}(t, r) = -i \frac{4\pi m^2}{|\dot{l}|} \frac{1}{l(l+1)} Y_{lm, \theta}^* \left( \frac{\pi}{2}, 0 \right) e^{-im\Omega t} \left\{ \theta(r-R) \left[ \frac{1}{r'} \partial_{r'} (r' j_l(m\Omega r')) \right]_{r'=R} \frac{d}{dr} [m\Omega r h_l^{(1)}(m\Omega r)] + \theta(R-r) \left[ \frac{1}{r'} \partial_{r'} (r' h_l^{(1)}(m\Omega r')) \right]_{r'=R} \frac{d}{dr} [m\Omega r j_l(m\Omega r)] - \frac{i}{R} \delta(r-R) \right\} \quad (48)$$

$$B_3^{lm}(t, r) = \frac{4\pi m^2}{|\dot{l}|} \frac{m}{l(l+1)} Y_{lm, \theta}^* \left( \frac{\pi}{2}, 0 \right) e^{-im\Omega t} \left\{ \theta(r-R) (m\Omega)^2 j_l(m\Omega R) m\Omega r h_l^{(1)}(m\Omega r) + \theta(R-r) (m\Omega)^2 h_l^{(1)}(m\Omega R) m\Omega r j_l(m\Omega r) - \frac{i}{R} \delta(r-R) \right\} \quad (49)$$

$$E_1^{lm}(t, r) = \frac{4\pi m^2}{|\dot{l}|} \frac{im}{r^2} Y_{lm, \theta}^* \left( \frac{\pi}{2}, 0 \right) e^{-im\Omega t} \left\{ \theta(r-R) m\Omega j_l(m\Omega R) m\Omega r h_l^{(1)}(m\Omega r) + \theta(R-r) m\Omega h_l^{(1)}(m\Omega R) m\Omega r j_l(m\Omega r) \right\}$$

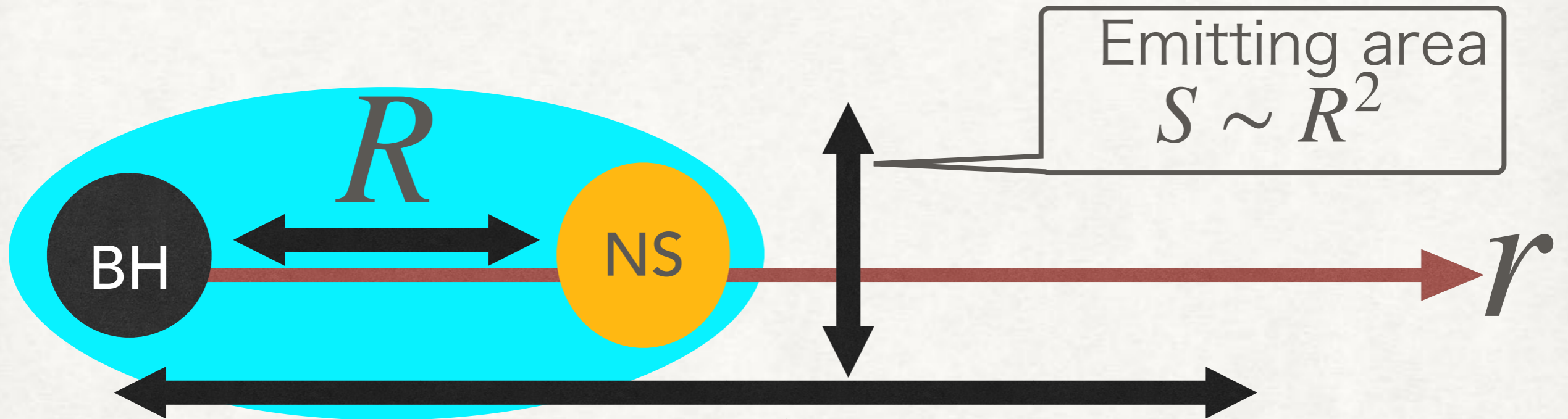
$$E_2^{lm}(t, r) = -\frac{4\pi m^2}{|\dot{l}|} \frac{im}{l(l+1)} Y_{lm, \theta}^* \left( \frac{\pi}{2}, 0 \right) e^{-im\Omega t} \left\{ \theta(r-R) m\Omega j_l(m\Omega R) \frac{d}{dr} (m\Omega r h_l^{(1)}(m\Omega r)) + \theta(R-r) m\Omega h_l^{(1)}(m\Omega R) \frac{d}{dr} (m\Omega r j_l(m\Omega r)) \right\}$$

$$E_3^{lm}(t, r) = -\frac{4\pi m^2}{|\dot{l}|} \frac{m\Omega}{l(l+1)} Y_{lm, \theta}^* \left( \frac{\pi}{2}, 0 \right) e^{-im\Omega t} \left\{ \theta(r-R) \left[ \frac{1}{r'} \partial_{r'} (r' j_l(m\Omega r')) \right]_{r'=R} m\Omega r h_l^{(1)}(m\Omega r) + \theta(R-r) \left[ \frac{1}{r'} \partial_{r'} (r' h_l^{(1)}(m\Omega r')) \right]_{r'=R} m\Omega r j_l(m\Omega r) \right\}$$

$$\rho \sim \frac{E}{R} \sim v \frac{B(R)}{R} \sim \frac{m^2 \Omega}{R^3}$$

$$E_r \sim \frac{m^2 \Omega^2 R}{r^2}$$

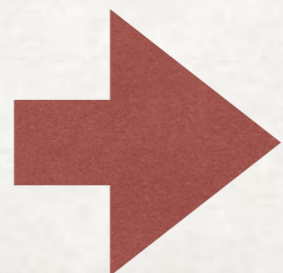
# PARTICLE ACCELERATION



$$\rho \sim \frac{E}{R} \sim v \frac{B(R)}{R} \sim \frac{m^z \Omega}{R^3}$$

$$E_r(r) \sim \frac{m^z \Omega^2 R}{r^2}$$

Accelerating region  
 $R_{\text{acc}} \sim \Omega^{-1}$

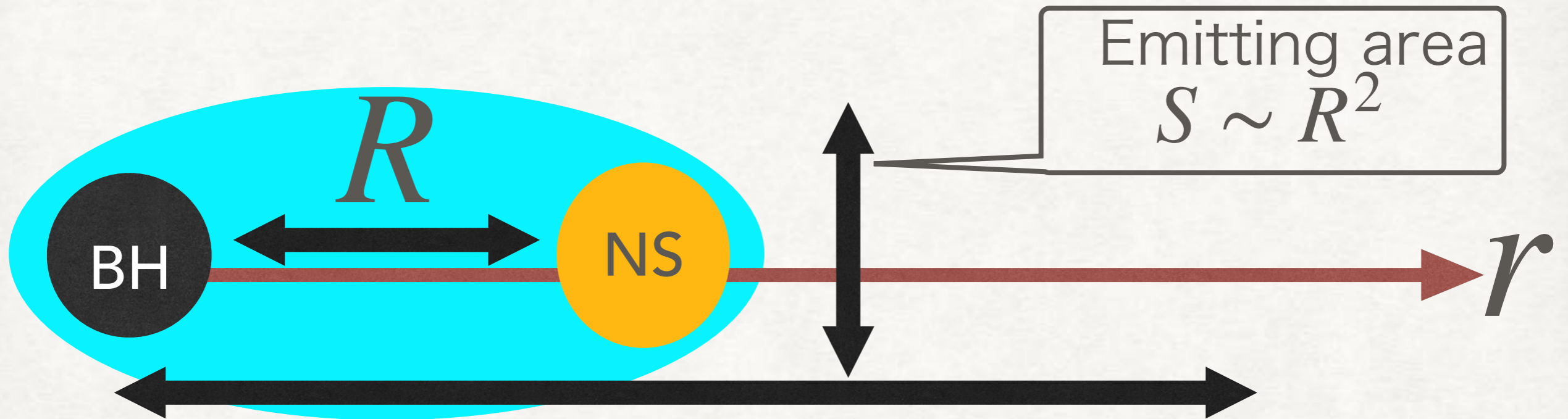


$$L \sim IV \sim (\rho c R^2) \cdot (E_r(R_{\text{acc}}) R_{\text{acc}})$$

$$\sim (m^z)^2 \Omega^3 R_{\text{acc}}^{-1} \propto R^{-6}$$



# PARTICLE ACCELERATION



Accelerating region  
 $R_{\text{acc}} \sim \Omega^{-1}$

$$L \sim IV \sim (\rho c R^2) \cdot (E_r(R_{\text{acc}}) R_{\text{acc}})$$

$$\sim (m^z)^2 \Omega^3 R_{\text{acc}}^{-1} \propto R^{-6}$$

cf. rotating dipole

$$L \sim \propto R^{-7}$$

$$L \sim (m^z)^2 \Omega^3 R_{\text{acc}}^{-1}$$

$$\sim 10^{42} \text{ erg/s}$$

# SUMMARY

- BH-NS binary ... there is  $E_{\parallel}^{\text{out}}$   
especially near binary
  - **magnetosphere must filled with plasma**
- charged particle may be **accelerated**
  - luminosity  $L \sim 10^{42}$  erg/s
  - it can be electromagnetic counterpart  
as precursor

**FUTURE WORK**

- frequency
- evaluate acceleration
- effect of plasma
- effect of GR