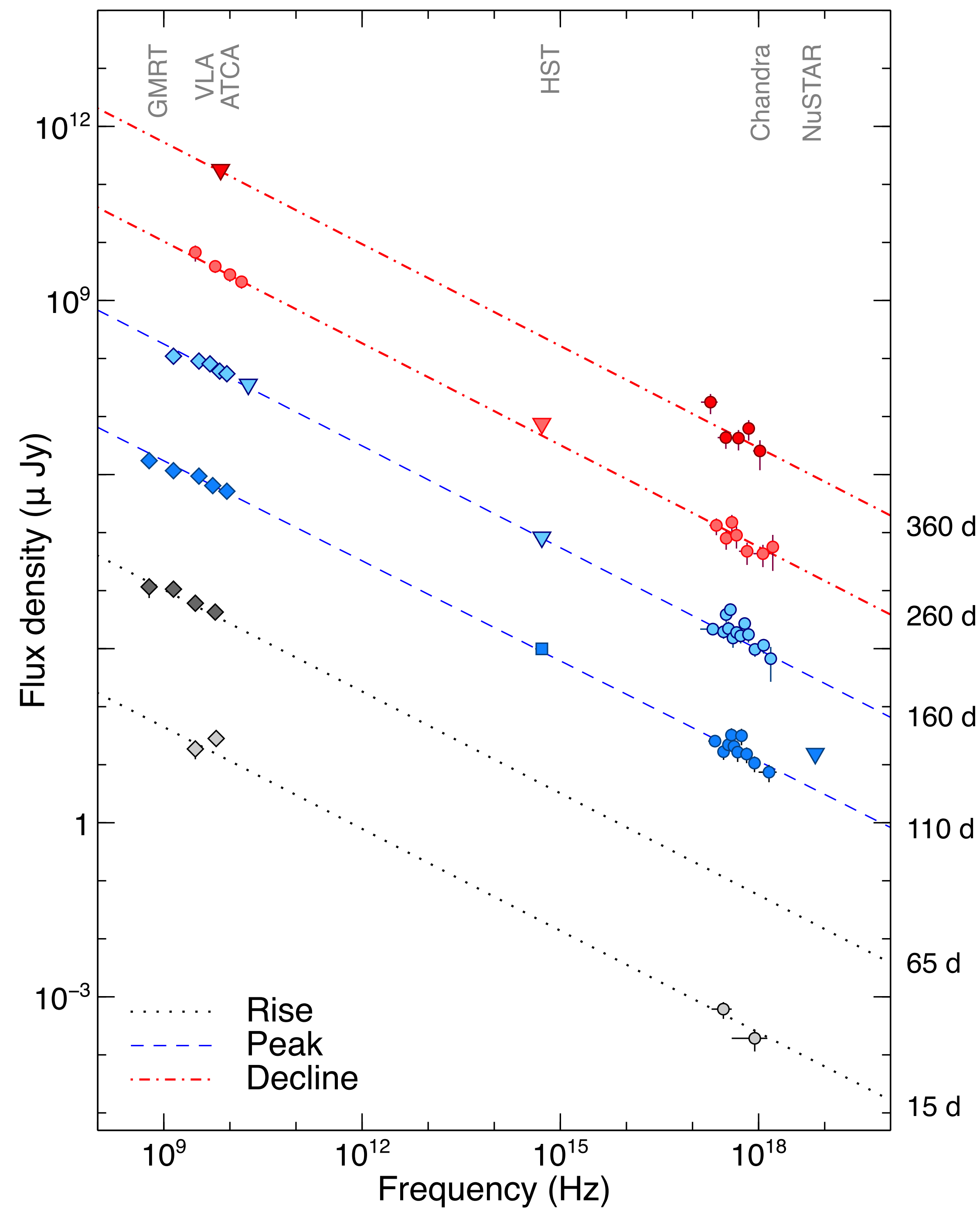
Nonthermal afterglow of GW170817:
a more natural electron energy distribution leads to a new solution with radio flux in the low frequency synchrotron tail

LIN, HAOXIANG 林 浩翔 (UTokyo)
COLLABORATORS: **TOTANI, TOMONORI** (UTokyo), **KIUCHI, KENTA** (KyotoU)

Nonthermal afterglow of GW170817

One-year afterglow of GW170817:

- Nonthermal spectrum $F_\nu \sim \nu^{-0.6}$: synchrotron radiation from shock-accelerated electrons (electron spectral index $p \approx 2.2$)
- Slow rising ($F_\nu \sim t^{0.8}$), peak ~ 160d, fast decay ($F_\nu \sim t^{-2}$). Simple pictures are challenged by slow rising pattern:
  - *Uniform jet* viewed *off-axis* (F_ν ~ t^3)
  - *Single-velocity spherical outflow* (F_ν ~ t^3)
  - Slow rising → Gradual input of energy → more complicated outflow structure



Afterglow SED of GW170817 (Troja+18)

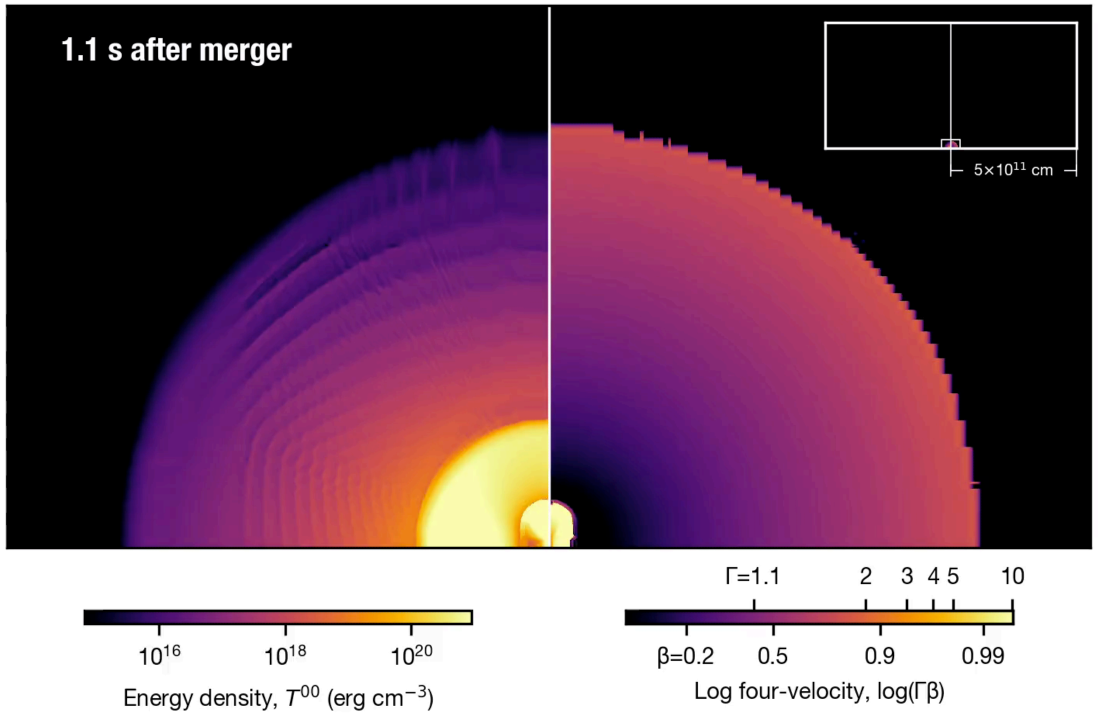
Nonthermal afterglow of GW170817

Structured jet viewed off-axis (***Jet***, e.g. Lazzati+17, Margutti+18):

- Ultra-relativistic core + slower wings
- Initially beamed-away core emission gradually enters line of sight to off-axis observer

Radially-stratified (quasi-)spherical outflow (***Sph***, e.g. Mooley+18):

- "Cocoon" created by a successful/chocked jet
- Radially-stratified layers: fast layers traveling ahead gradually caught up by slow layers and energized



"Cocoon" (Kasliwal+17)

	Jet	**Sph**
Decay rate	$F_\nu \sim t^{-2}$	$F_\nu \sim t^{-1.2}$
Cooling passage	~ 10^4 d	~ 10^2 d
Radio imaging	Large axial ratio, flux centroid large offset	Spherical, small offset
Linear polarization	10% – 60%	< 10%

Obs (Alexander+, Corsi+, Mooley+, Troja+18):

- $F_\nu \sim t^{-2}$ after ~ 160d
- No cooling passage till ~ 360d
- Not resolved by VLBI, but superluminal motion
- $\Pi < 12\%$ (2.8 GHz) at 244d

jet-dominated late-time emission

Our research

Motivation

*f injection efficiency, i.e. number fraction of accelerated electrons

Previous studies commonly apply a conventional GRB afterglow theory (e.g. **Sari+, 1998, ApJ, 497, L17**), which

- assumed ***all electrons in the shock are accelerated (*$f^* = 1$*)***, ignoring possible majority of ***thermal electrons*** (e.g. normally observed in SNR)
- formulated in ***ultra-relativistic*** limit, but BNS merger ejecta is ***mildly or non-relativistic***
- Degeneracy in f (Eichler & Waxman, 2005) and paucity of broadband GRB afterglow data

We re-examine afterglow modeling of GW170817 with a more natural electron spectrum:

- f is allowed to vary freely
- In conventional model, minimum electron energy γ_m is controlled by other model parameters. We determine γ_m independently by 1 extra d.o.f. (electron-proton coupling level)
- Applicable to trans-relativistic regime

We perform Markov-Chain Monte-Carlo (MCMC) analysis to estimate parameters of GW170817

Modeling: electron spectrum

Given shock Lorentz factor Γ_s

(i) power-law energy spectrum with minimum injection energy $\frac{dN_{\rm NT}}{d\gamma_e} \propto \gamma_e^{-p}, \quad \gamma_e \geq \gamma_m$

Conventional $f = 1$ model

(ii) energy as fraction ϵ_e of shock energy

$$f m_e \langle \gamma_e \rangle = \epsilon_e m_p \Gamma_s$$

average

(iii) all electrons are accelerated

$$f = 1$$

$$\gamma_m = \epsilon_e \frac{p-2}{p-1} \frac{m_p}{m_e} \Gamma_s$$

γ_m controlled by the spectrum energy ϵ_e and index p

Our free f model

(ii) energy as fraction ϵ_e (rest mass excluded)

$$f m_e (\langle \gamma_e \rangle - 1) = \epsilon_e m_p (\Gamma_s - 1)$$

(iii) degree of electron-proton coupling

$$m_e(\gamma_m - 1) = \eta_e m_p (\Gamma_s - 1)$$

$$\gamma_m - 1 = \eta_e \frac{m_p}{m_e} (\Gamma_s - 1)$$

γ_m independently parametrized by η_e (as 1 new d.o.f)

Modeling: electron spectrum

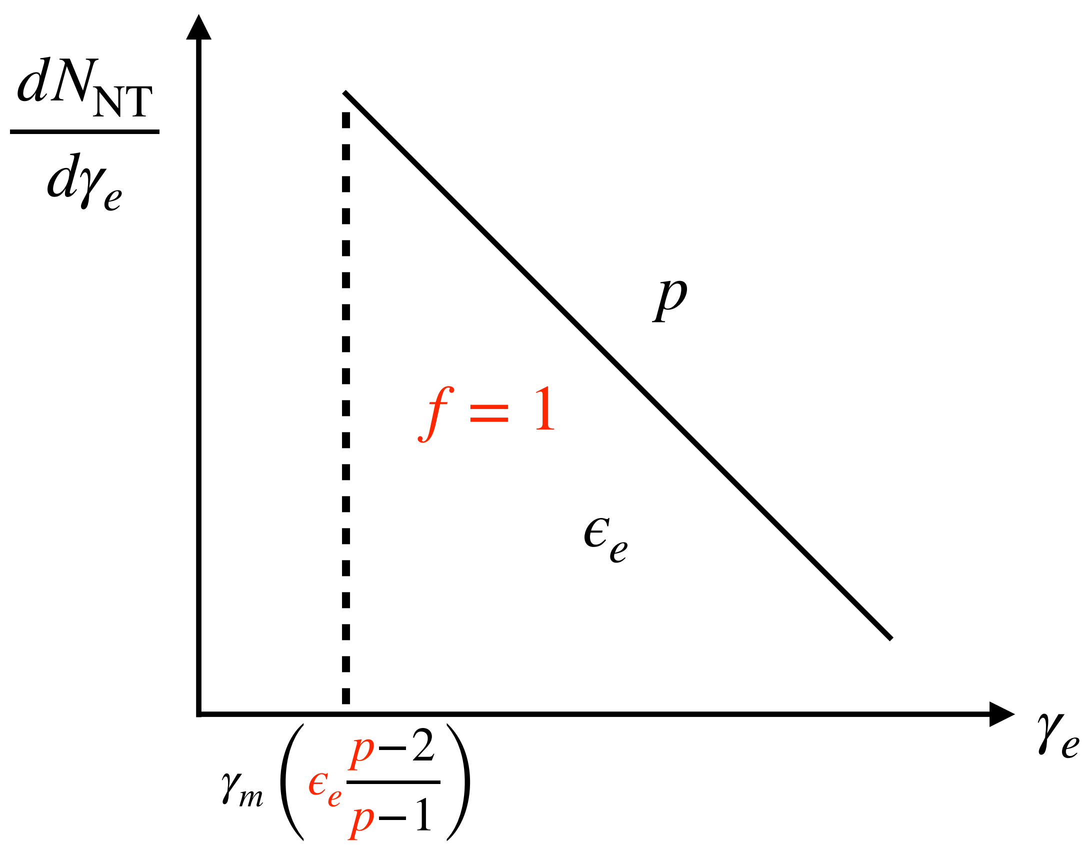
Spectral index p

Injection efficiency f

Total energy ϵ_e (as fractional shock energy)

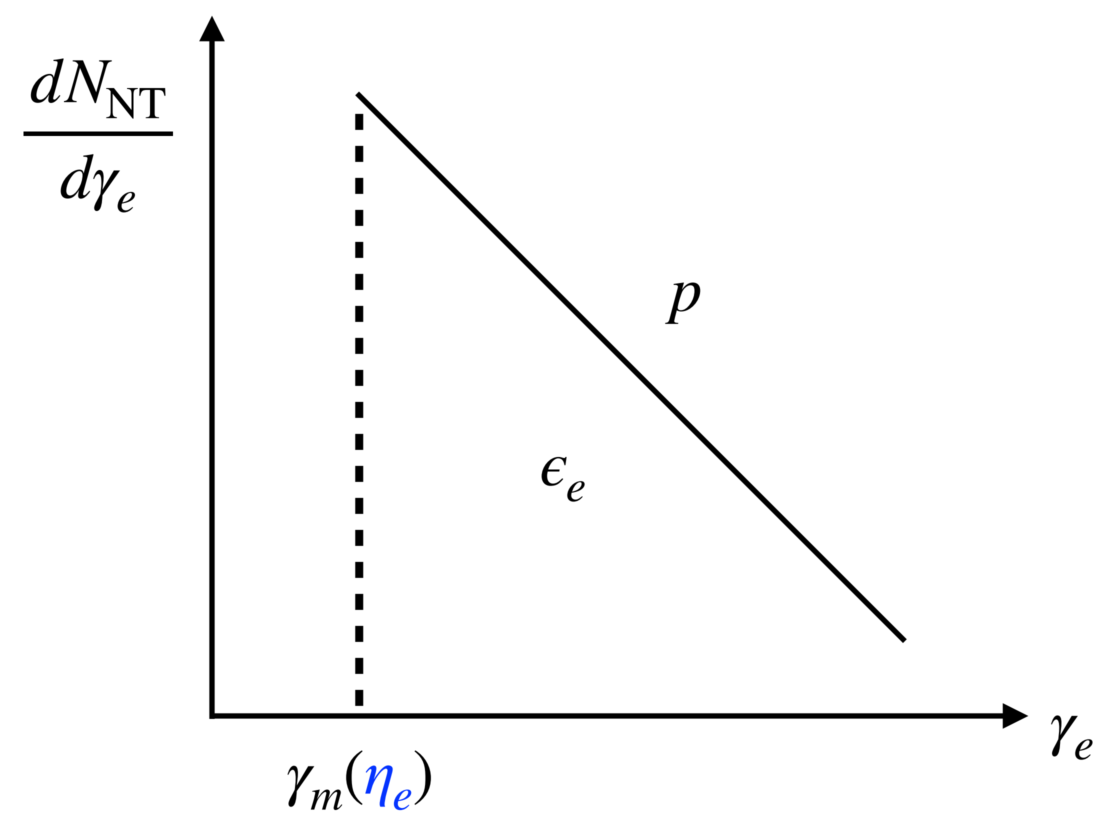
Degree of electron-proton coupling η_e
(1 extra d.o.f.)

Conventional $f = 1$ model



γ_m NOT independent parameter

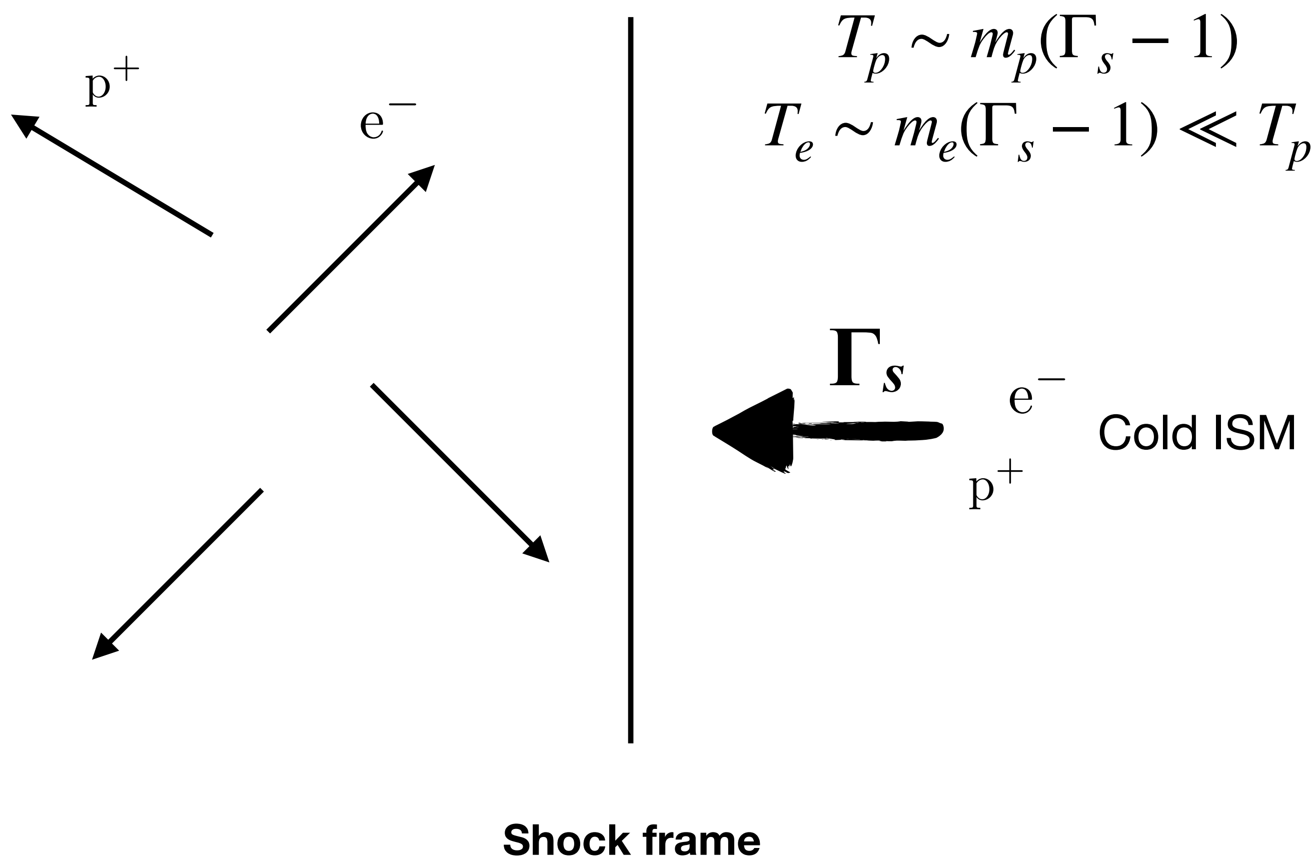
Our free f model



$$f = \frac{\epsilon_e}{\eta_e} \frac{p-2}{p-1} \quad (\gamma_m \gg 1)$$

How to parametrize γ_m?

Modeling: electron spectrum



$$T_p \sim m_p(\Gamma_s - 1)$$
$$T_e \sim m_e(\Gamma_s - 1) \ll T_p$$

Electrons further heated by protons up to a higher temperature up to equilibrium

Natural assumption: shock acceleration starts somewhere between T_e and T_p

$$T_e < m_e(\gamma_m - 1) < T_p$$
$$m_e/m_p < \eta_e < 1$$

$$\eta_e \equiv \frac{m_e(\gamma_m - 1)}{m_p(\Gamma_s - 1)}$$

Degree of electron-proton coupling: η_e

$\eta_e \sim 1$: equilibrium (heated $T_e \sim T_p$)

$\eta_e \sim m_e/m_p$: no energy transfer from protons to electrons

$m_e/m_p < \eta_e < 1$: inefficiently-heated electrons

Modeling: shock dynamics

Jet model

Gaussian angular profile for energy

$$E_{k,\mathrm{iso}}(\theta) = E_{c,\mathrm{iso}} \exp\left(-\frac{\theta^2}{2\theta_c^2}\right)$$

$$\Gamma_0(\theta) - 1 = (\Gamma_c - 1)\exp\left(-\frac{\theta^2}{2\theta_c^2}\right)$$

Single injection (no radial profile)

Spherical model

Power-law radial profile in proper velocity u

$$E(>u) \propto u^{-k} \quad (u_{\min} < u < u_{\max})$$

Continuous injection

$$E_{\mathrm{inj}} = E(>u_{\mathrm{col}}) \qquad u_{\mathrm{col}} = \frac{R}{\sqrt{c^2t^2 - R^2}}$$

$$(\beta_{\mathrm{col}} = R/ct)$$

Shock dynamics

Energy equation $E_{\mathrm{inj}} = M(R)\,(cu_s)^2$

Swept-up ISM mass $M(R) = \frac{4}{3}\pi R^3 n m_p$

Shock velocity $u_s = \frac{\dot{R}}{c}\left[1 - \left(\frac{\dot{R}}{c}\right)^2\right]^{-\frac{1}{2}}$

Modeling: synchrotron flux

Comoving magnetic field strength (parametrized by ϵ_B)

$$B' = (32\pi\epsilon_B n m_p)^{1/2}\Gamma_s c \longrightarrow B' = \left[8\pi\epsilon_B \frac{\hat\gamma\Gamma_s + 1}{\hat\gamma - 1} n m_p c^2 (\Gamma_s - 1)\right]^{1/2} \qquad \hat\gamma = \frac{4}{3} + \frac{1}{3\Gamma_s}$$

Relativistic limit (standard GRB theory) — Trans-relativistic — adiabatic index (mono-energetic gas approx.)

Comoving synchrotron spectral power

$$P'_m = \frac{8\pi}{9}\frac{e^3 B'}{m_e c^2}\frac{M(R)}{m_p} f \qquad P'_{\nu'}/P'_m = \begin{cases} (\nu'_c/\nu'_m)^{-(p-1)/2} (\nu'/\nu'_c)^{-p/2} & (\nu'_c < \nu') \\ (\nu'/\nu'_m)^{-(p-1)/2} & (\nu'_m < \nu' < \nu'_c) \\ (\nu'/\nu'_m)^{1/3} & (\nu' < \nu'_m) \end{cases}$$

peak frequency $\nu'_m = \frac{1}{2\pi}\frac{eB'}{m_e c}\gamma_m^2$ absorption frequency ν'_a cooling frequency $\nu'_c = \frac{1}{2\pi}\frac{eB'}{m_e c}\left(\frac{6\pi m_e c}{\sigma_T B'^2 \Gamma_s t}\right)^2$

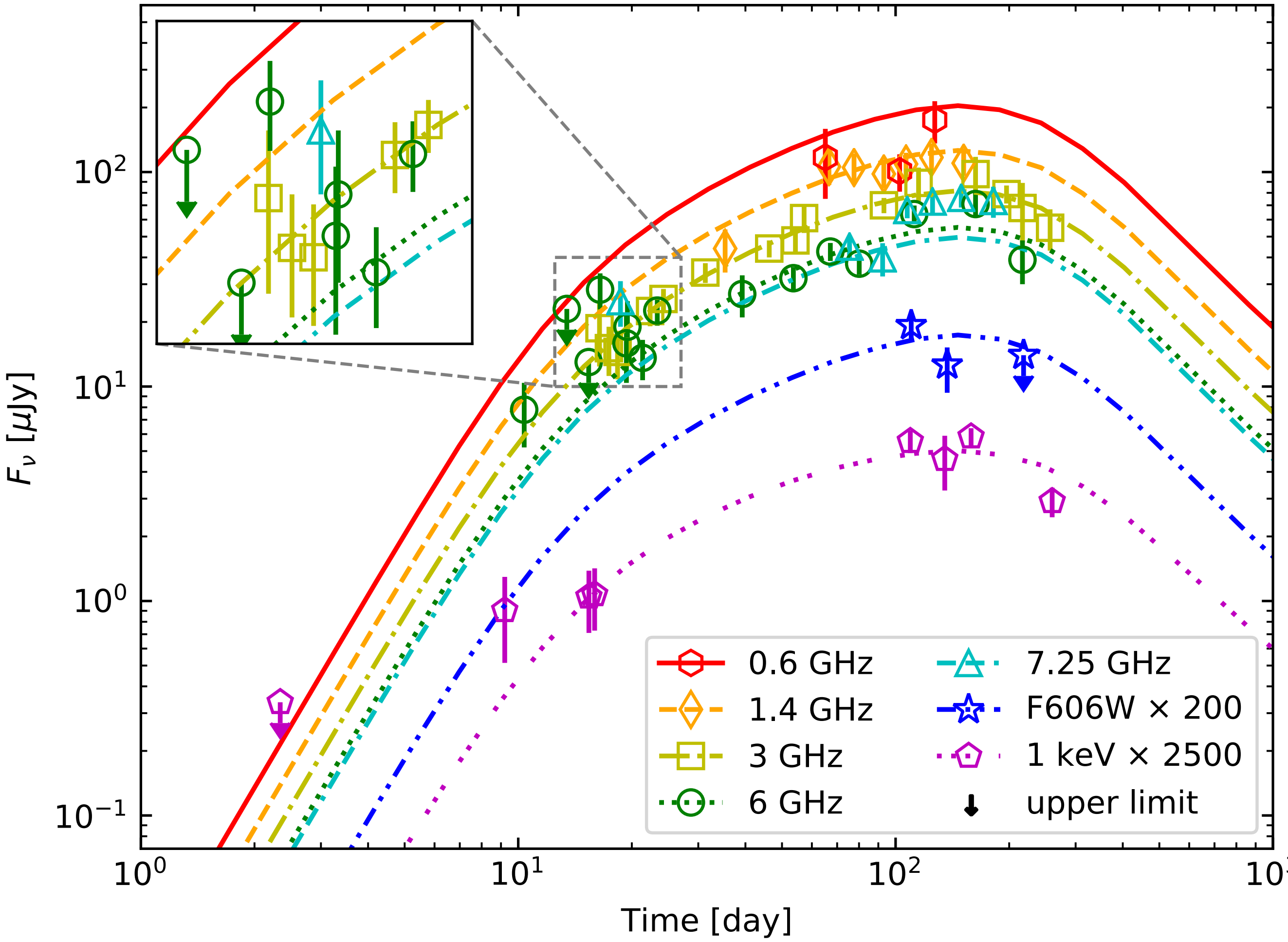
Received flux $F_\nu(\nu, T) = \frac{1}{4\pi d^2}\int_0^{2\pi}\frac{d\varphi}{2\pi}\int_0^{\pi}\frac{\sin\theta d\theta}{2}\delta_D^3(\mu)\, P'_{\nu'}(\nu', t)$

- Doppler effect $\nu'(\mu) = \nu/\delta_D(\mu) \quad \delta_D = [\Gamma_s(1 - \beta_s\mu)]^{-1}$
- Photon arrival time delay $T = t - \frac{R(t)\mu}{c}$
- Emitting angle from observer $\mu = \cos\theta\cos\theta_v + \sin\theta\cos\varphi\sin\theta_v$

Best-fit model (Jet)

$f = 1$ with similar parameters of previous fits

Constant single power-law spectrum ($\nu > \nu_m$)

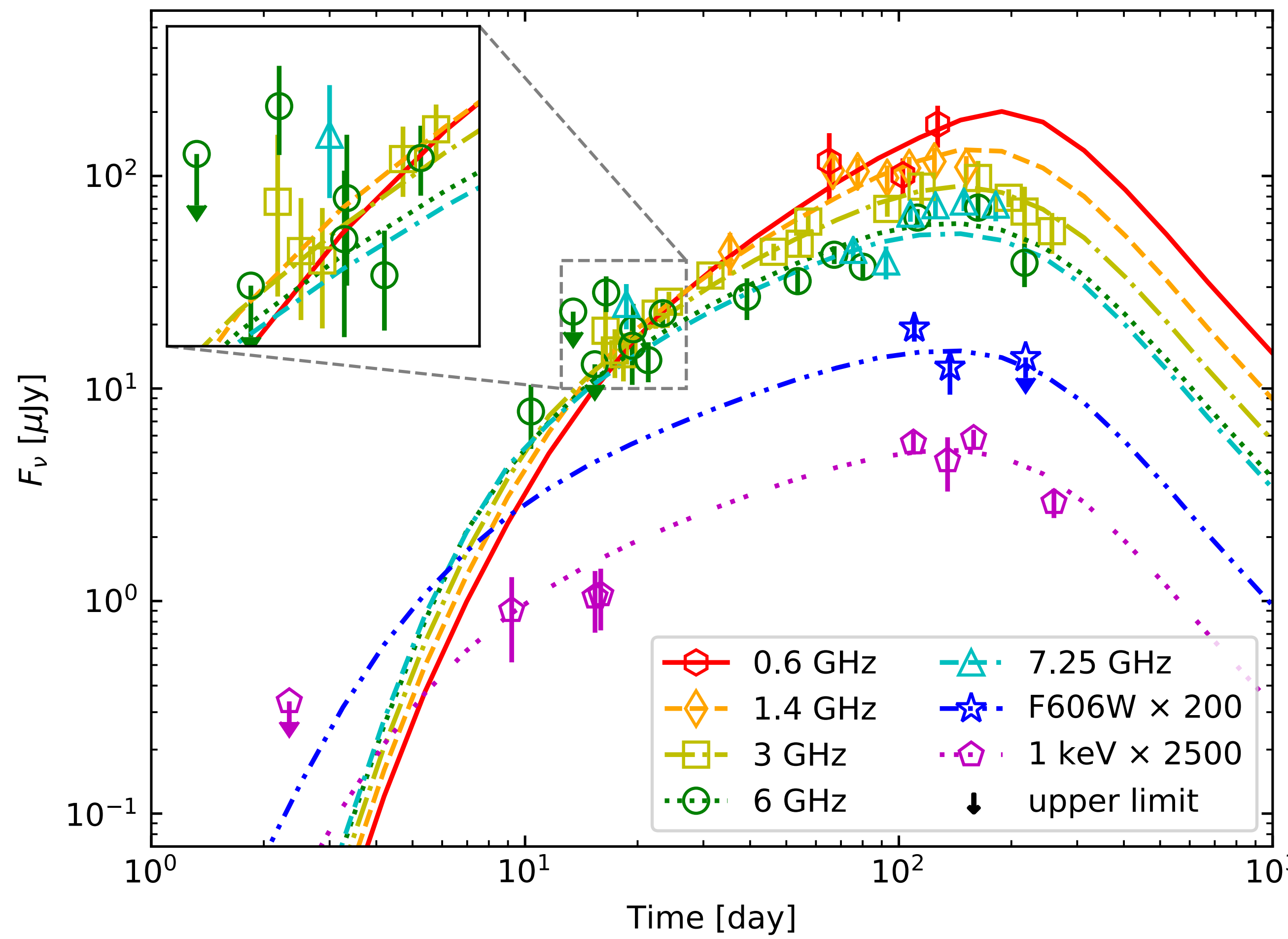


n=10$^{-2.5}$ cm^{-3}, ϵ_B=10$^{-2.3}$, ϵ_e=10$^{-1.5}$, p=2.13,
$E_{c,\mathrm{iso}}$=10$^{51.70}$ erg, Γ_c=229, θ_c=0.11, θ_v=0.58

$\chi^2_{\min} \sim 137$

Best-fit free f model by MCMC

New fit with radio in synchrotron tail ($\nu < \nu_m$)



n=10$^{-2.5}$ cm^{-3}, ϵ_B=10$^{-4.7}$, ϵ_e=10$^{-1.2}$, p=2.18,
$E_{c,\mathrm{iso}}$=10$^{52.59}$ erg, Γ_c=242, θ_c=0.09, θ_v=0.45, η_e=0.14

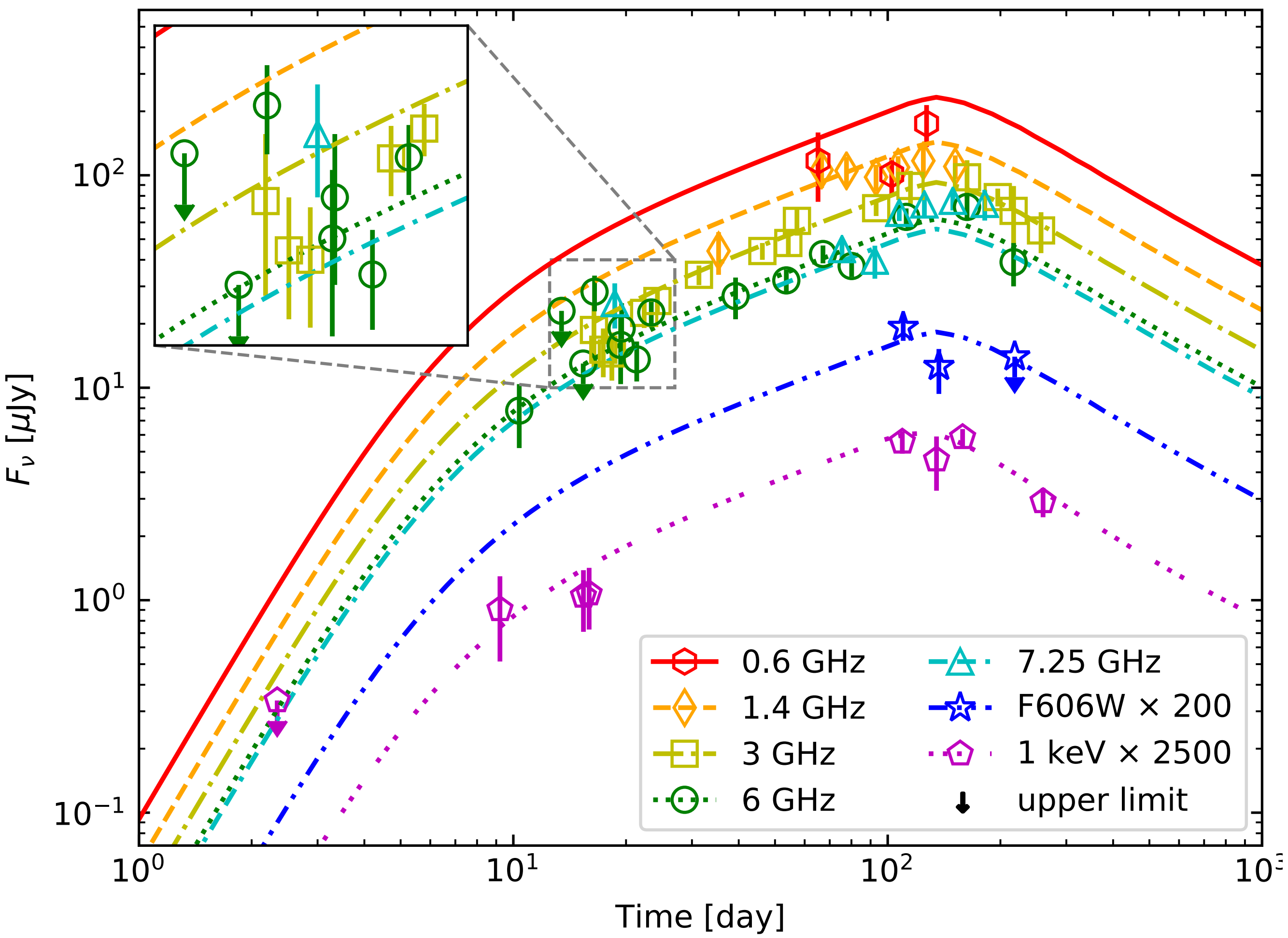
$\chi^2_{\min} \sim 108$

$N_{\mathrm{data}} = 54$ $\quad \Delta\chi^2_{\min} \sim 29\,!$ with 1 more model parameter

Best-fit model (Sph)

$f = 1$ with similar parameters of previous fits

Constant single power-law spectrum ($\nu > \nu_m$)

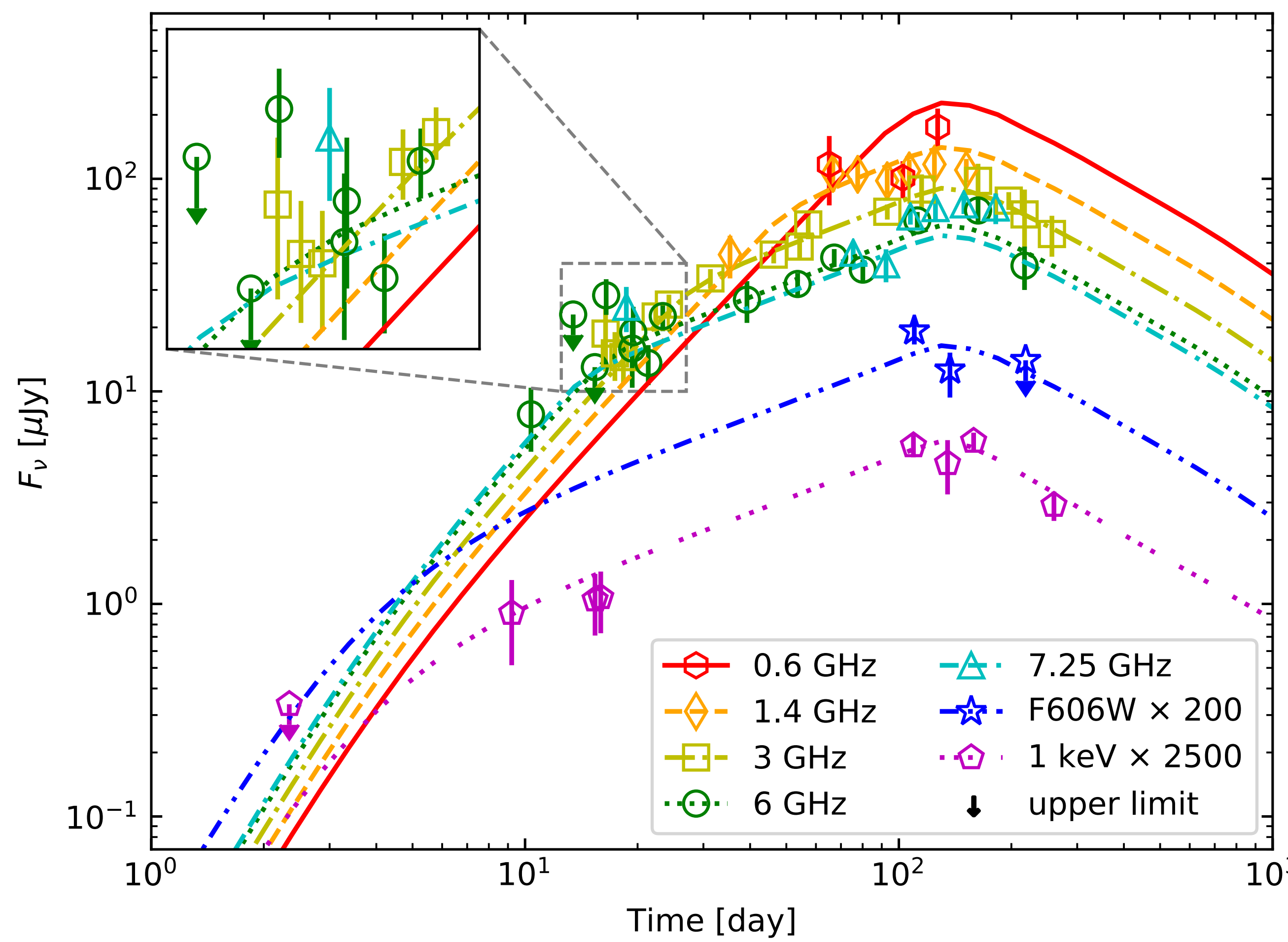


n=10$^{-3.2}$ cm^{-3}, ϵ_B=10$^{-1.1}$, ϵ_e=10$^{-1.2}$, p=2.15,
$E_{k,\mathrm{iso}}$=10$^{49.68}$ erg, $u_{\max}$=2.9, $u_{\min}$=1.7, k=5.7

$\chi^2_{\min} \sim 127$

Best-fit free f model by MCMC

New fit with radio in synchrotron tail ($\nu < \nu_m$)



n=10$^{-1.8}$ cm^{-3}, ϵ_B=10$^{-2.2}$, ϵ_e=10$^{-1.3}$, p=2.16,
$E_{k,\mathrm{iso}}$=10$^{49.71}$ erg, $u_{\max}$=2.6, $u_{\min}$=1.2, k=6.2, η_e=0.3

$\chi^2_{\min} \sim 115$

$N_{\mathrm{data}} = 54$ $\Delta\chi^2_{\min} \sim 12$ with 1 more model parameter

Synchrotron tail fit

New fit with early radio flux in the regime of synchrotron tail!

$$P'_{\nu'}/P'_m = \begin{cases} (\nu'_c/\nu'_m)^{-(p-1)/2}(\nu'/\nu'_c)^{-p/2} & (\nu'_c < \nu') \\ (\nu'/\nu'_m)^{-(p-1)/2} & (\nu'_m < \nu' < \nu'_c) \\ (\nu'/\nu'_m)^{1/3} & (\nu' < \nu'_m) \end{cases}$$

Emitted by low-energy electrons ~ γ_m

Q: why new fit is found by allowing free f?

$$\gamma_m(f=1) = \epsilon_e \frac{p-2}{p-1}\frac{m_p}{m_e}(\Gamma_s - 1) \longrightarrow \nu'_m(f=1) \simeq 2.2\,\mathrm{MHz}\,\epsilon_{e,-1}^2 \epsilon_{B,-2}^{1/2} n_{-3}^{1/2} \Gamma_s^3 \quad (p=2.17)$$

ν_m is far below GHz with $f = 1$ fixed

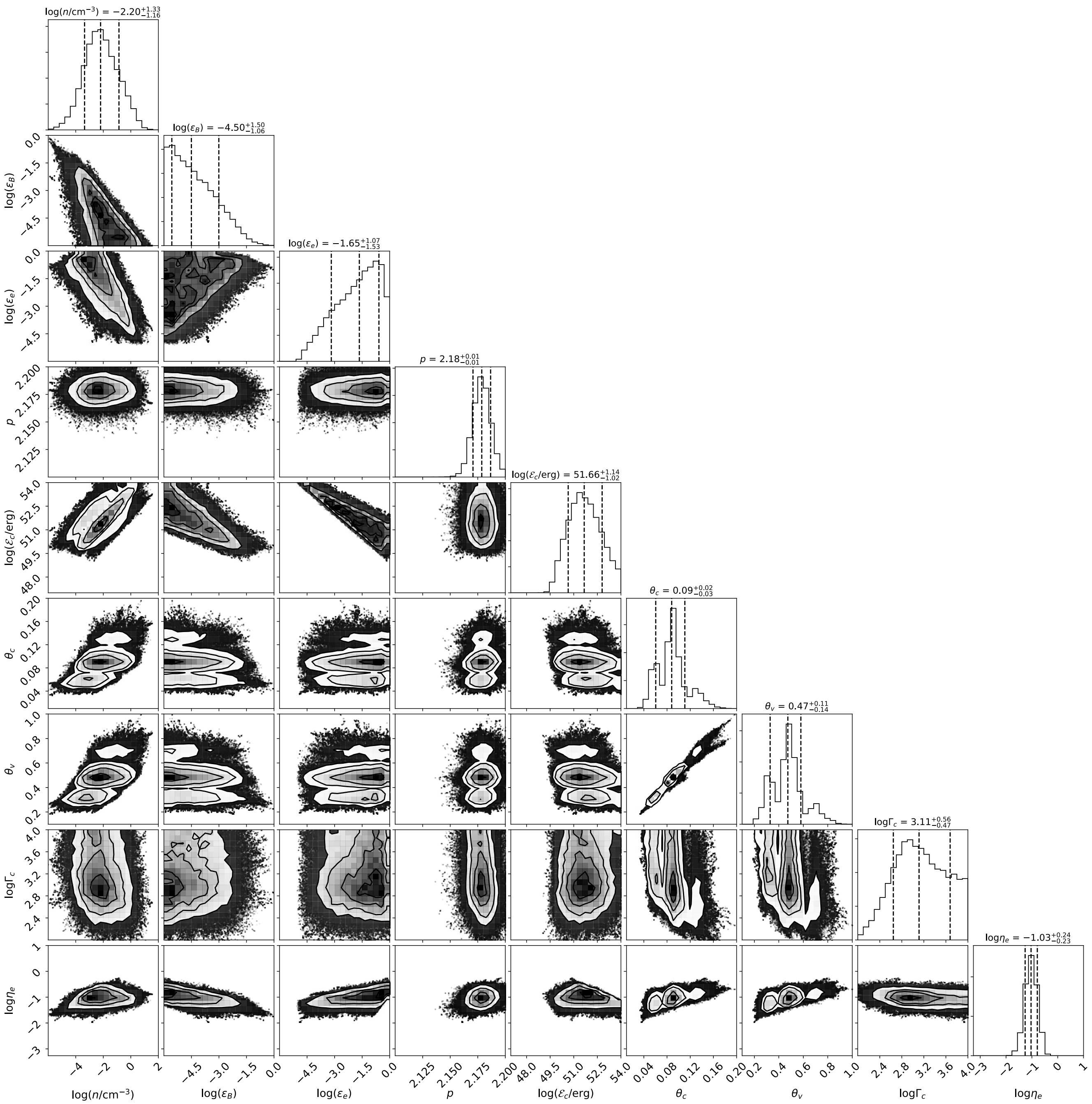
$$\nu'_m = \frac{1}{2\pi}\frac{eB'}{m_e c}\gamma_m^2$$

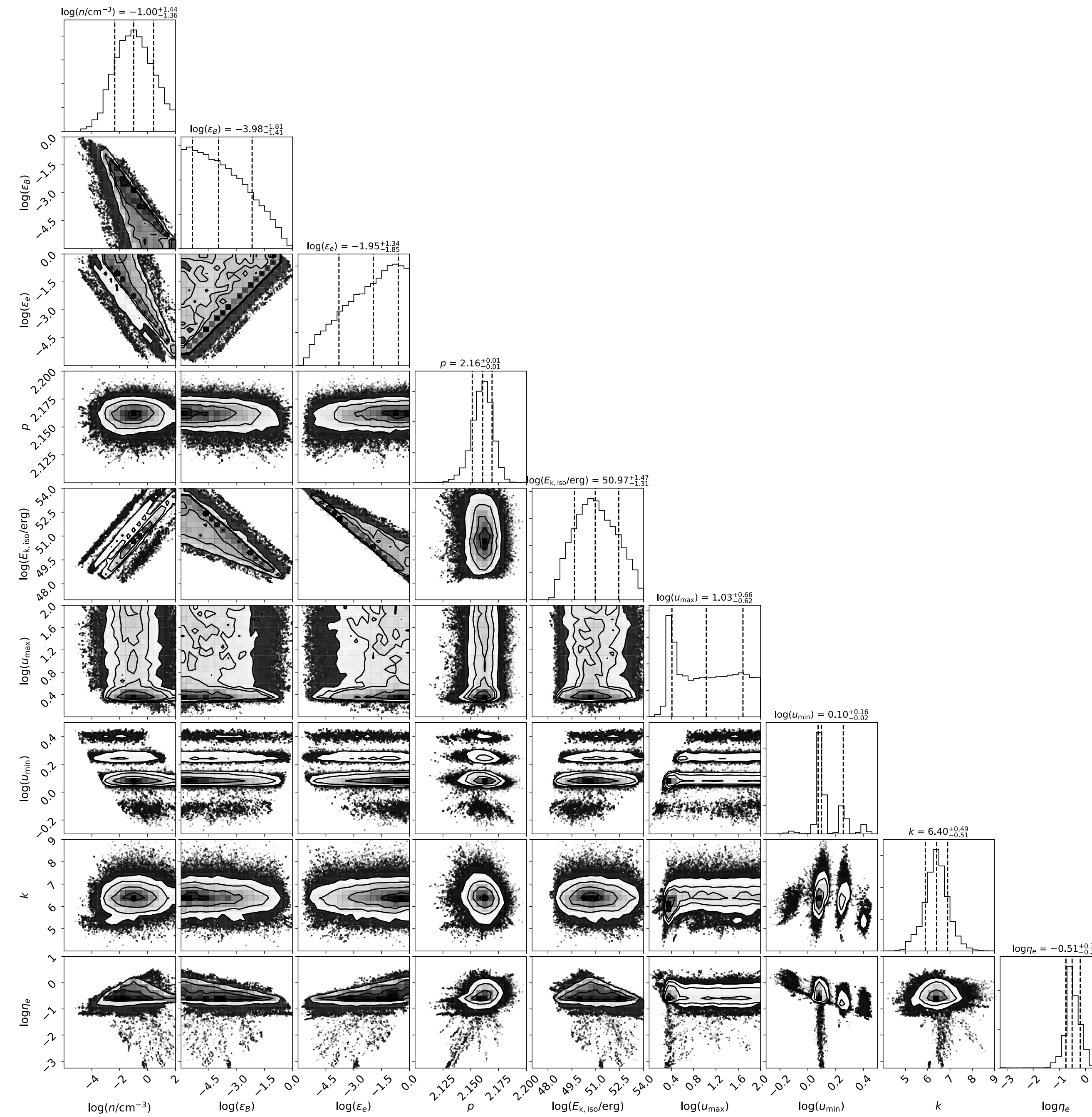
$$\gamma_m(\text{free } f) = \eta_e \frac{m_p}{m_e}(\Gamma_s - 1) \longrightarrow \nu'_m(\text{free } f) \simeq 11.6\,\mathrm{GHz}\,\eta_e^2 \epsilon_{B,-2}^{1/2} n_{-3}^{1/2} \Gamma_s^3 \quad (p=2.17)$$

Possible early ν_m passage in GHz!

For reasonable range of parameters (e.g. $\epsilon_e < 1$),
synchrotron tail fit is uniquely found by free f model

MCMC





MCMC: Table

Jet model

Parameter	Prior	$f=1$ 1D dist.[a]	$f=1$ maximum[b]	Free f 1D dist.[a]	Free f maximum[b]
$\log(n/\mathrm{cm}^{-3})$	-6 – 2	$-3.72^{+0.77}_{-0.97}$	-2.92	$-2.20^{+1.33}_{-1.16}$	-2.48
$\log\epsilon_B$	-6 – 0	$-2.48^{+0.89}_{-1.15}$	-2.32	$-4.50^{+1.50}_{-1.06}$	-4.69
$\log\epsilon_e$	-6 – 0	$-0.23^{+0.17}_{-0.28}$	-0.06	$-1.65^{+1.07}_{-1.53}$	-1.15
p	2 – 2.5	$2.18^{+0.01}_{-0.01}$	2.18	$2.18^{+0.01}_{-0.01}$	2.18
$\log(E_{c,\mathrm{iso}}/\mathrm{erg})$	47 – 54	$51.12^{+0.74}_{-0.48}$	50.53	$52.76^{+1.14}_{-1.02}$	52.59
θ_c	0.01 – 0.2	$0.09^{+0.04}_{-0.03}$	0.13	$0.09^{+0.02}_{-0.03}$	0.09
θ_v	0.1 – 1	$0.47^{+0.20}_{-0.16}$	0.70	$0.47^{+0.11}_{-0.14}$	0.45
Γ_c	4 – 8	$3.15^{+0.52}_{-0.45}$	2.85	$3.11^{+0.56}_{-0.47}$	2.38
$\log\eta_e$	-3 – 1	—	—	$-1.03^{+0.24}_{-0.23}$	-0.84
$\log f$	—	0	0	$-1.45^{+0.99}_{-1.41}$	-1.13

Spherical model

Parameter	Prior	$f=1$ 1D dist.	$f=1$ maximum	Free f 1D dist.	Free f maximum
$\log(n/\mathrm{cm}^{-3})$	-6 – 2	$-2.06^{+1.52}_{-1.31}$	-2.42	$-1.00^{+1.44}_{-1.36}$	-1.84
$\log\epsilon_B$	-6 – 0	$-3.20^{+2.00}_{-1.83}$	-2.00	$-3.98^{+1.81}_{-1.41}$	-2.24
$\log\epsilon_e$	-6 – 0	$-1.57^{+1.02}_{-1.91}$	-1.89	$-1.95^{+1.34}_{-1.85}$	-1.29
p	2 – 2.5	$2.15^{+0.01}_{-0.01}$	2.16	$2.16^{+0.01}_{-0.01}$	2.16
$\log(E_{\mathrm{k,iso}}/\mathrm{erg})$	47 – 54	$50.87^{+1.51}_{-1.32}$	50.45	$50.97^{+1.47}_{-1.31}$	49.71
$\log u_{\max}$	0 – 2	$1.09^{+0.61}_{-0.53}$	0.59	$1.03^{+0.66}_{-0.62}$	0.41
$\log u_{\min}$	-1 – 1	$0.26^{+0.02}_{-0.02}$	0.25	$0.10^{+0.16}_{-0.02}$	0.08
k	4 – 9	$6.68^{+0.48}_{-0.42}$	6.40	$6.40^{+0.49}_{-0.51}$	6.17
$\log\eta_e$	-3 – 1	—	—	$-0.51^{+0.31}_{-0.24}$	-0.52
$\log f$	—	0	0	$-2.33^{+1.13}_{-1.70}$	-1.63

[a]Medians with symmetric 68% uncertainties; [b]Maximum of the posterior probability density function

Common constraints by light curve turnover
(Jet: $\Gamma_c \sim 1/(\theta_v - \theta_c)$, Sph: $u_s \sim u_{\min}$)

Jet: energy ~ other SGRBs (10^50–53 erg); Jet size (θ_c) & viewing angle (θ_v) strictly constrained θ_c ~ 0.09 (5°) and θ_v ~ 0.47 (27°). If + LIGO & VLBI, θ_c ~ 0.06 (3.4°) and θ_v ~ 0.3 (17°)

Sph: energy ~ kilonova (10^51 erg), high $u_{\min} \sim 1$–2 (0.7–0.9c) disfavors dynamical ejecta ($u_{\min} \sim 0.4$, 0.4–0.5c) while cocoon remains as an possible option ($u_{\min} >\sim 1$, 0.7c)

MCMC: Highlights

Synchrotron tail solution

Degree of electron-proton coupling η_e is strictly constrained, $\eta_e \sim 0.1$ (jet) and ~ 0.3 (sph)

Close to simulation of non- or trans-relativistic shock acceleration (Park et al. 2015): $\eta_e \sim 0.4$

Other differences by Free f model

Higher density **by 1—2 orders of magnitude**

in tension with HI observation (< 0.04 cm^{-3})

Make sense if hot gas is taken into account (0.01—0.1 cm^{-3} for offset ~ $0.64 r_e$* of GW170817 in hot gas dominated giant elliptical galaxies like NGC 4993)

*Half-light radius r_e

			$f = 1$		Free f	
	Parameter	Prior	1D dist.[a]	maximum[b]	1D dist.[a]	maximum[b]
Jet	$\log(n/\mathrm{cm}^{-3})$	-6 – 2	$-3.72^{+0.77}_{-0.97}$	-2.92	$-2.20^{+1.33}_{-1.16}$	-2.48
Sph	$\log(n/\mathrm{cm}^{-3})$	-6 – 2	$-2.06^{+1.52}_{-1.31}$	-2.42	$-1.00^{+1.44}_{-1.36}$	-1.84

Jet: higher energy by **nearly 2 order of magnitudes**!

$\log(E_{c,\,\mathrm{iso}}/\mathrm{erg})$	47 – 54	$51.12^{+0.74}_{-0.48}$	50.53	$52.76^{+1.14}_{-1.02}$	52.59

Lower and higher end of SGRB observations (1e50-53 erg)

One more thing…

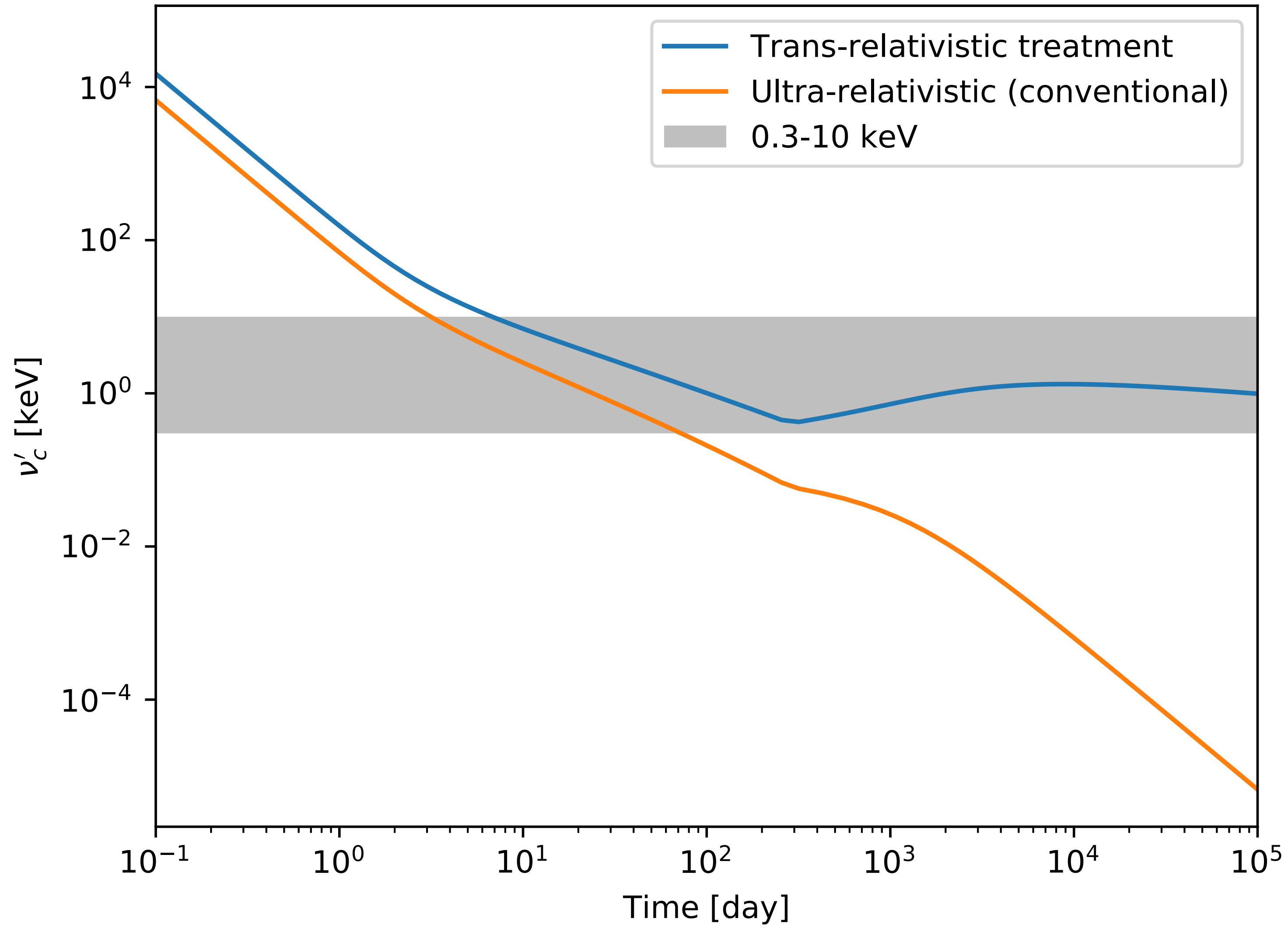
$$B' = (32\pi\epsilon_B n m_p)^{1/2}\Gamma_s c$$

Relativistic limit (conventional GRB theory)

→

$$B' = \left[8\pi\epsilon_B \frac{\hat{\gamma}\Gamma_s + 1}{\hat{\gamma} - 1} n m_p c^2 (\Gamma_s - 1)\right]^{1/2}$$

Trans-relativistic



Comoving cooling frequency ν_c' v.s. time

*best-fit parameters to GW170817 by **sph** model

Previous predictions: very early cooling passage in X-ray for **Sph** model

Non-passage for 1 yr disfavors **Sph** model

But in non-relativistic limit, ν_c almost stops evolving (~ $t^{-0.2}$) after the shock becomes Newtonian ~ O(100d)

Even if ν_c already reaches X-ray band, it stops to further decrease and cooling break becomes not observable

↓

Time of cooling break cannot be diagnostic btw jet/sph models

Summary

A natural model for nonthermal electron spectrum is formulated:

- Injection efficiency f is allowed to vary freely
- Minimum energy γ_m is independently determined by electron-proton coupling level

A **new fit** to afterglow of GW170817 with **early radio in synchrotron tail**, in contrast to previous fits

Parameter of GW170817 by free f model:

- **Higher** ambient density by 1 – 2 orders of magnitude (but consistent with hot gas)
- Jet energy **increased** by nearly 2 orders of magnitude

Time of cooling break at ν_c cannot be diagnostic btw jet/sph models

Our prediction of "early radio in the synchrotron tail" may be tested in the future events by early multi-frequency radio observations