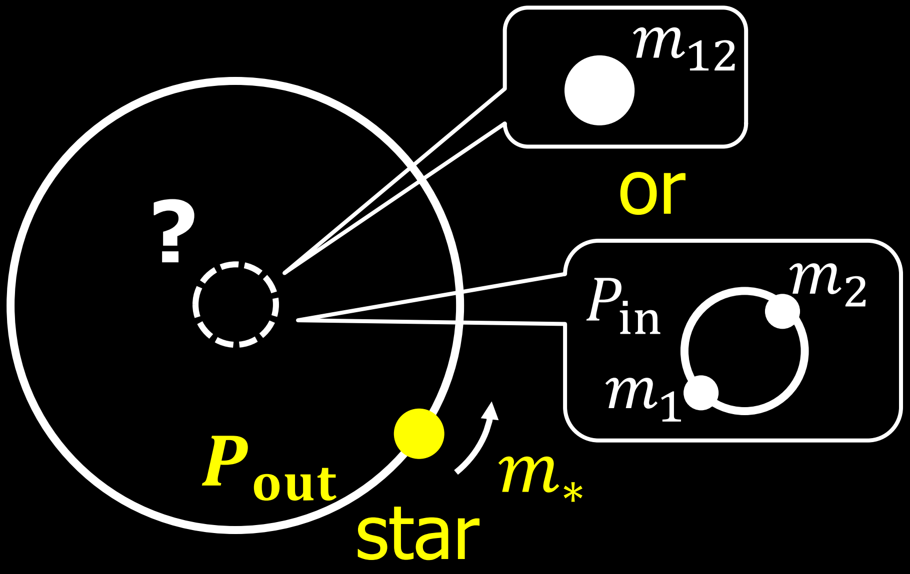
A single black hole or a binary black hole:
how to detect a star orbiting a binary black hole



Yasushi Suto

Department of Physics & Research Center for the Early Universe, The University of Tokyo

11:00 July 12, 2023. Astrophysics Colloquium@Ludwig-Maximilians-Universität München

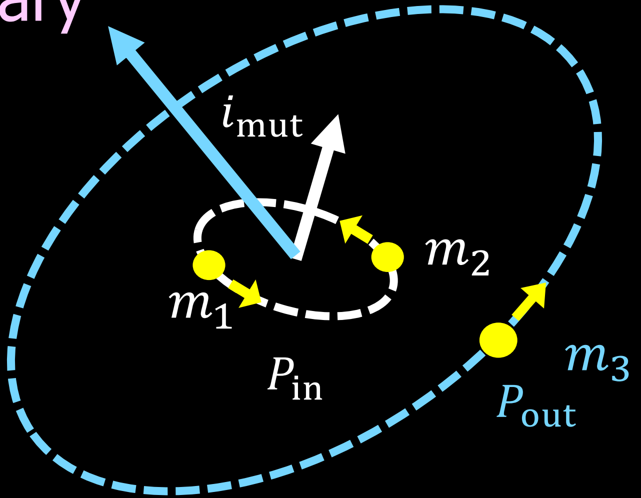
Today's talk

1. Hierarchical triple systems
2. von Zeipel-Lidov-Kozai (ZKL) oscillations
   (or Kozai, Kozai-Lidov oscillations)
3. Search for blackhole binaries in optical:
   a star-blackhole binary
   or a star-binary blackhole triple?
4. Feasibility study with Gaia BH1 and BH2
5. Summary and outlook

1 Hierarchical triple systems

Hierarchical three-body systems

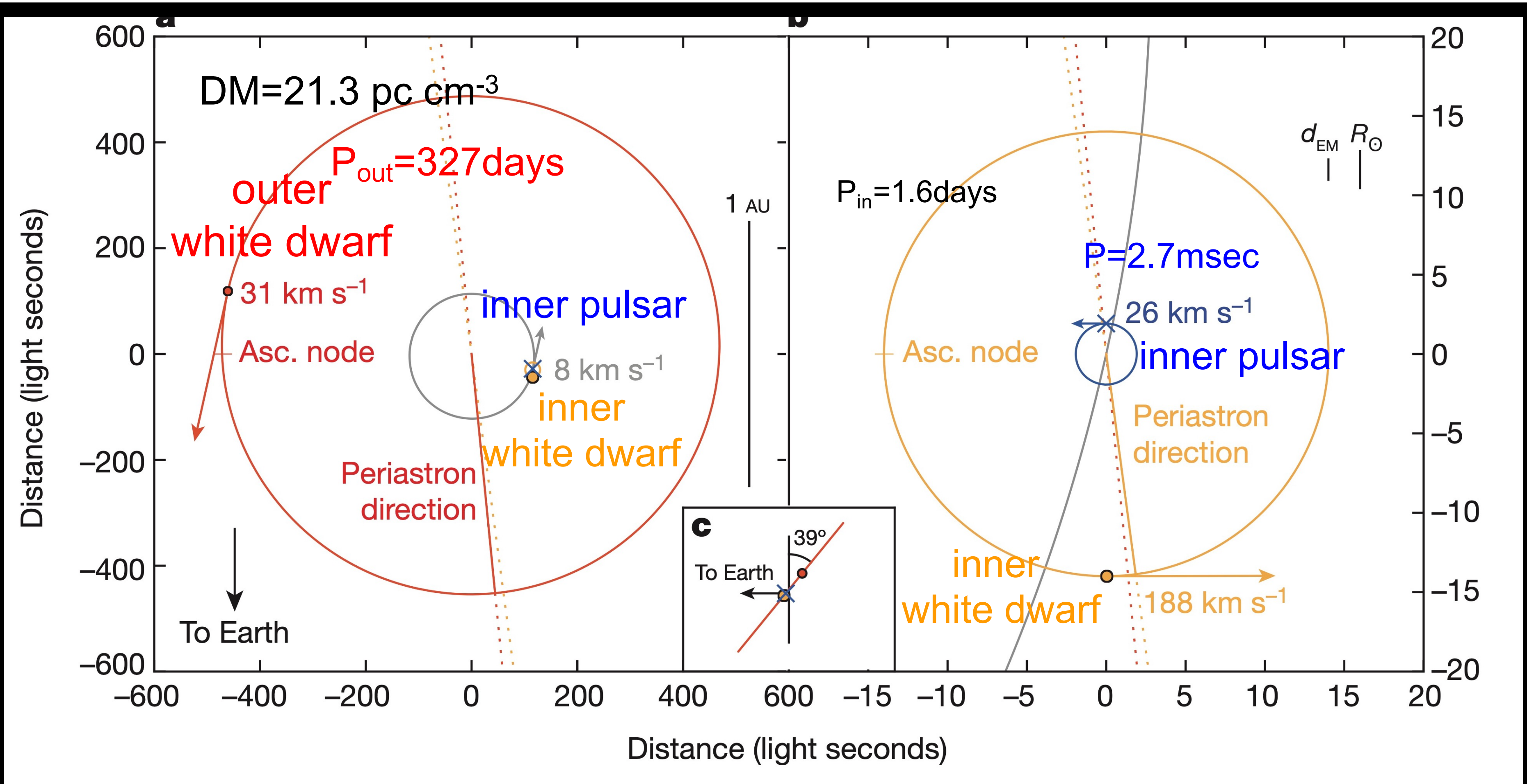
- Gravitational three-body systems are unstable in general
  - stable three-body systems are mostly hierarchical: tight binary + distant tertiary orbiting the center-of-mass of the inner binary
  - observed three-body systems are likely to be hierarchical
- Stable systems are inevitably associated with (undemocratic) hierarchies
  - quite universal in biological, astronomical and social systems
    - quarks and leptons – atoms – molecules – DNAs – cells – organs – animals – villages – cities – nations – planets – stars – star clusters – galaxies – galaxy clusters – universe(s) – multiverse(s)



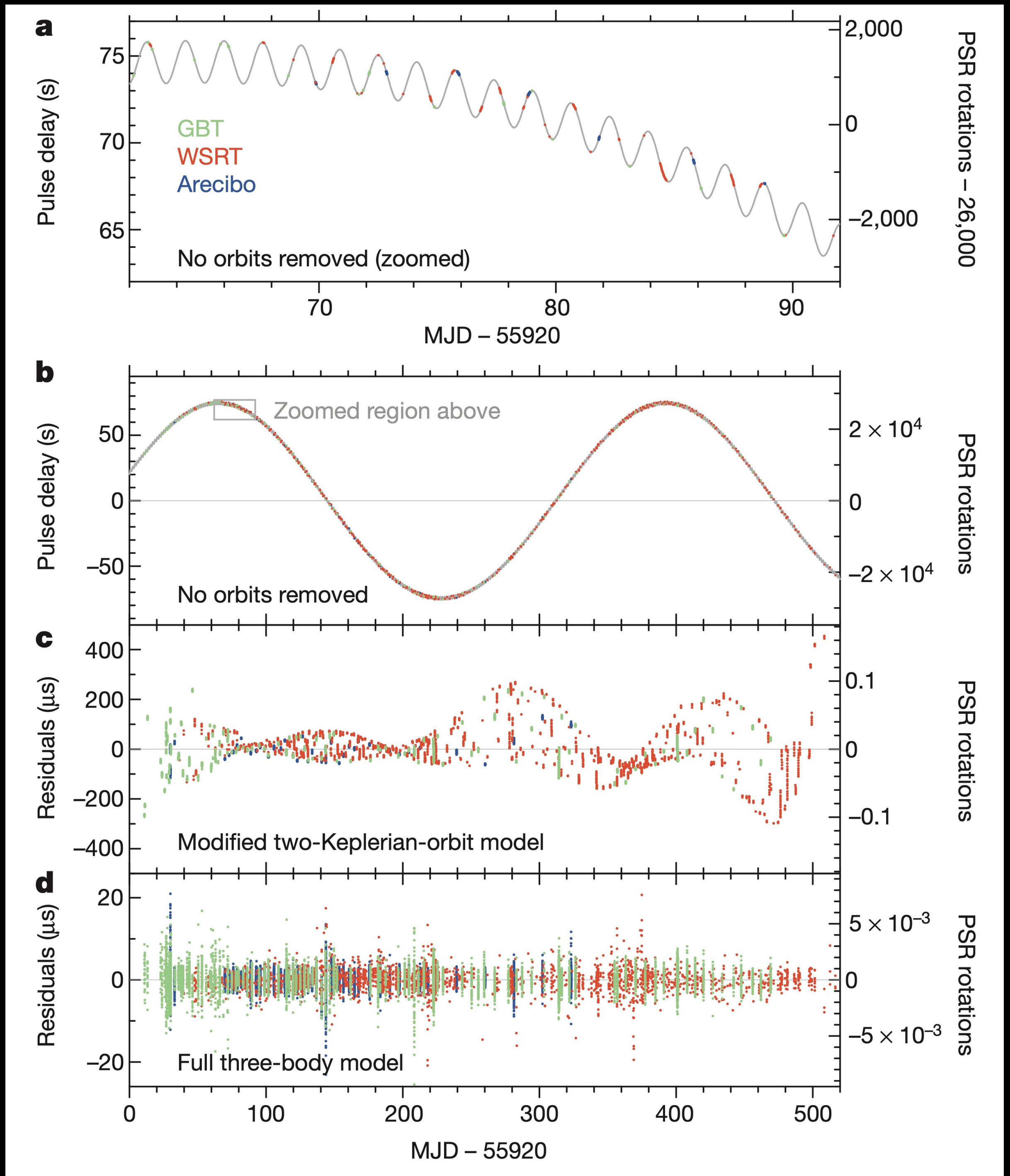
- non-intuitive (counter-intuitive) dynamical behavior of hierarchical triples triggers unexpectedly broad diversities in astronomical phenomena (e.g., ZKL effect)

A millisecond pulsar in a stellar triple system

S. M. Ransom[1], I. H. Stairs[2], A. M. Archibald[3,4], J. W. T. Hessels[3,5], D. L. Kaplan[6,7], M. H. van Kerkwijk[8], J. Boyles[9,10], A. T. Deller[3], S. Chatterjee[11], A. Schechtman-Rook[7], A. Berndsen[2], R. S. Lynch[4], D. R. Lorimer[9], C. Karako-Argaman[4], V. M. Kaspi[4], V. I. Kondratiev[3,12], M. A. McLaughlin[9], J. van Leeuwen[3,5], R. Rosen[1,9], M. S. E. Roberts[13,14] & K. Stovall[15,16]



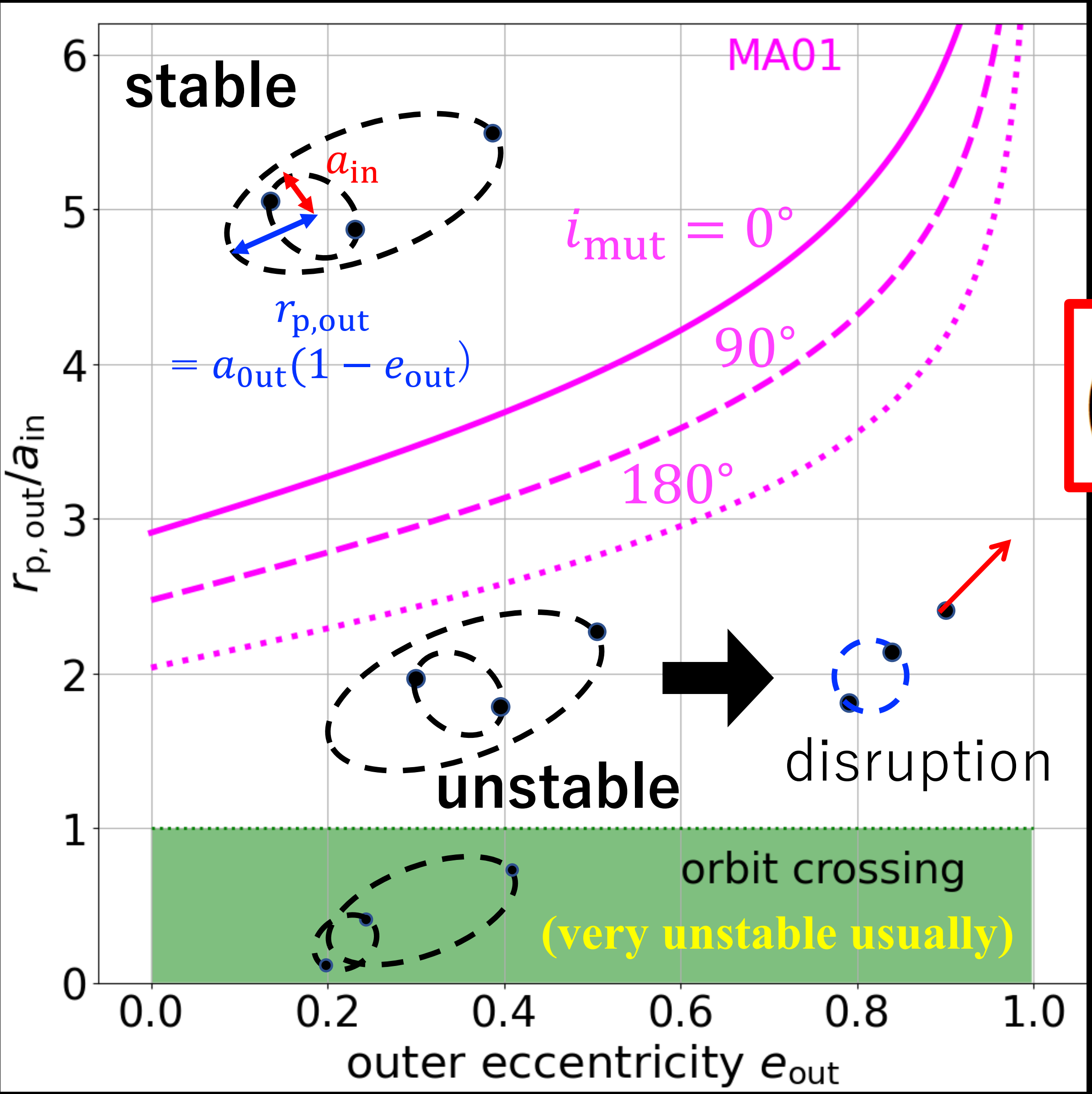
Ransom et al.
Nature
505 (2014)
520



Ransom et al.
Nature 505(2014)520
NS-WD binary + WD

PSR J0337+1715 parameters	
inner orbital period (pulsar+WD)	1.629401788(5) day
outer orbital period (WD)	327.257541(7) day
pulsar spin period	2.73258863244(9) msec
mutual orbital inclination	0.0120(17) deg.
	highly circular & coplanar !
Pulsar mass	1.4378(13) $M_\odot$
Inner WD mass	0.19751(15) $M_\odot$
Outer WD mass	0.4101(3) $M_\odot$

Triples are unstable in general ⇔ diversity



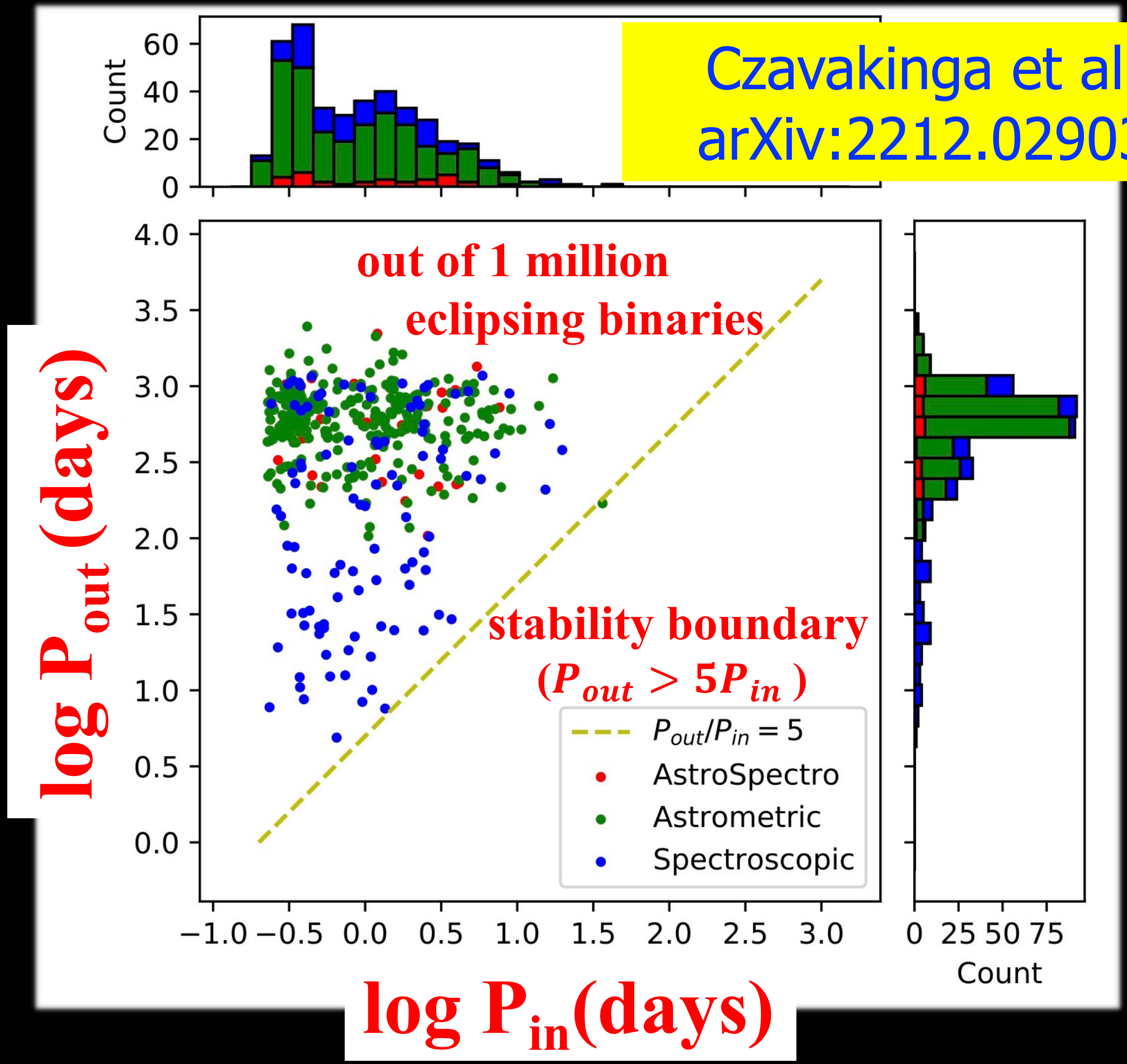
- Stability criterion (Mardling & Aarseth 2001)

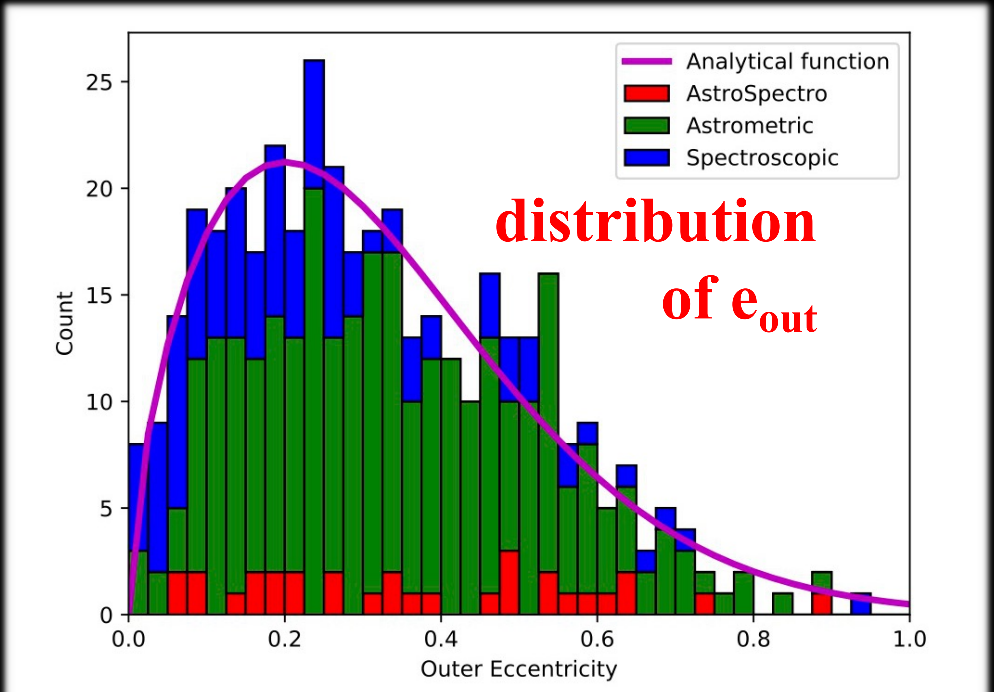
$$\left(\frac{r_{\rm p,out}}{a_{\rm in}}\right)_{\rm MA} \equiv 2.8\left(1 - 0.3\frac{i_{\rm mut}}{\pi}\right)\left[\left(1 + \frac{m_3}{m_{12}}\right)\frac{(1 + e_{\rm out})}{\sqrt{1 - e_{\rm out}}}\right]^{2/5}$$

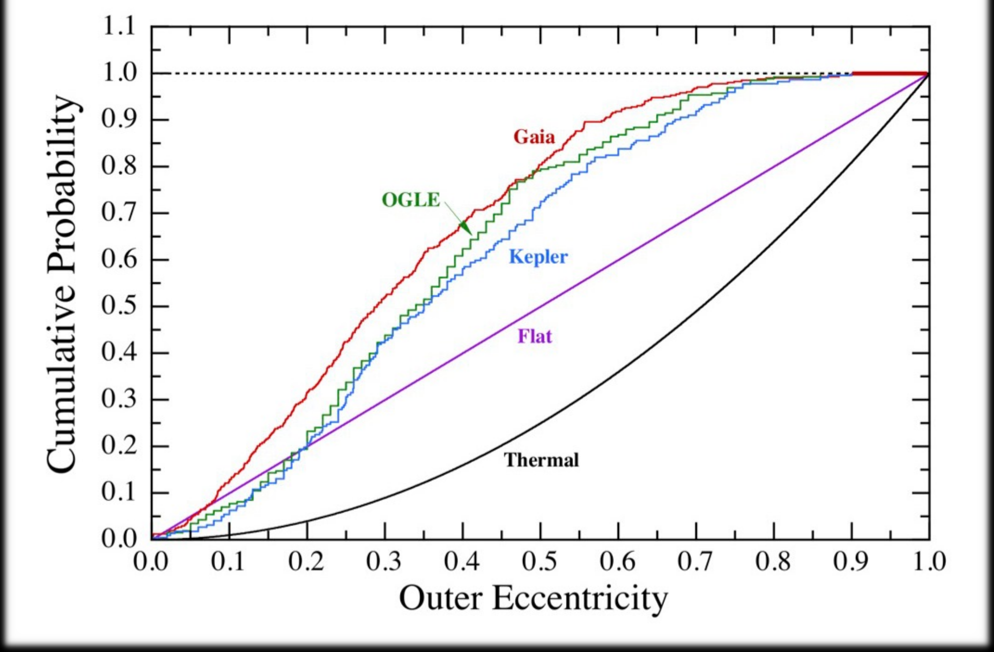
  - Well-known and widely used, but its implication is often misinterpreted…
  - Lyapunov (chaoticity of local trajectory) vs. Lagrange (escape of a body from the system) stability
- Hayashi, Trani & YS; ApJ 939(2022)81
  ApJ 943(2023)58

Hierarchical stellar-triple candidates in Gaia DR3 (403 in total)

Czavakinga et al.
arXiv:2212.02903



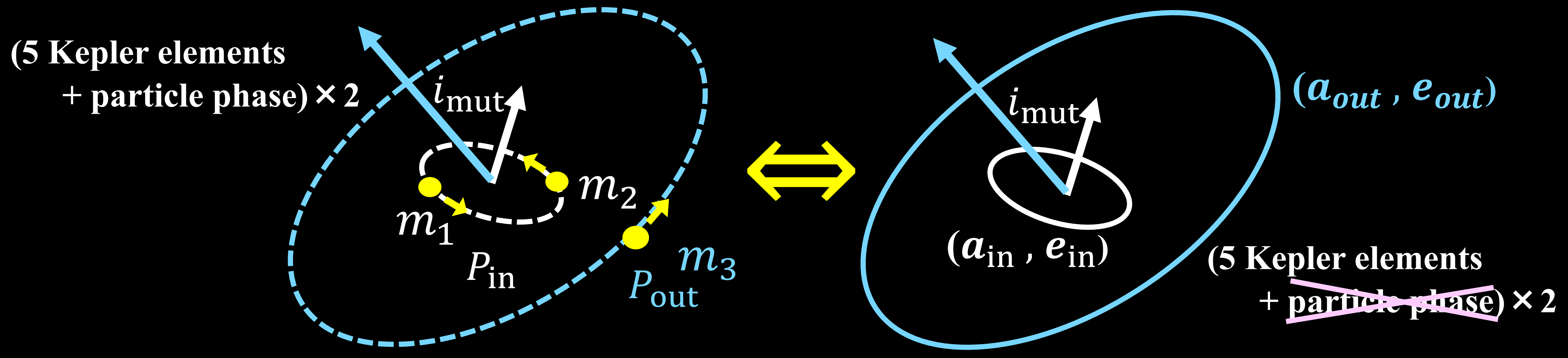




2 von Zeipel-Lidov-Kozai (ZKL) oscillations
(or Kozai, Kozai-Lidov oscillations)

Secular approximation to triple dynamics

- Very different timescales involved: $P_{in} \ll P_{out}$
  - time-consuming for accurate numerical integration
- perturbative expansion in terms of $a_{in}/\, a_{out}$ ($\ll 1$)
  - long-time numerical integration by approximating the particle-particle interaction with the ring-ring interaction over appropriate time-averaging of particles on their orbits



Legendre expansion of Hamiltonian

Kepler motion for inner orbit

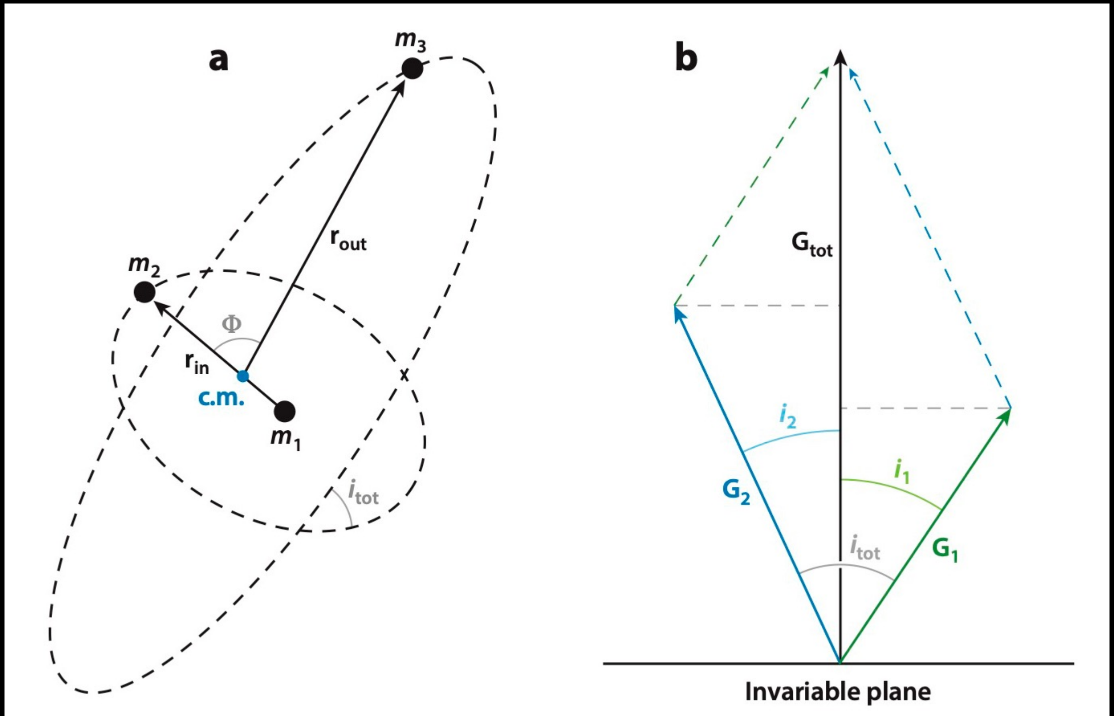
interaction between inner and outer orbits

$$\mathcal{H} = \frac{k^2 m_1 m_2}{2a_1} + \frac{k^2 m_3 (m_1 + m_2)}{2a_2} + \frac{k^2}{a_2} \sum_{n=2}^{\infty} \left(\frac{a_1}{a_2}\right)^n M_n \left(\frac{\mathbf{r}_{\rm in}}{a_1}\right)^n \left(\frac{a_2}{\mathbf{r}_{\rm out}}\right)^{n+1} P_n(\cos\Phi)$$

Kepler motion for outer orbit

coupling constant

$$M_n = m_1 m_2 m_3 \frac{m_1^{n-1} - (-m_2)^{n-1}}{(m_1 + m_2)^n}$$

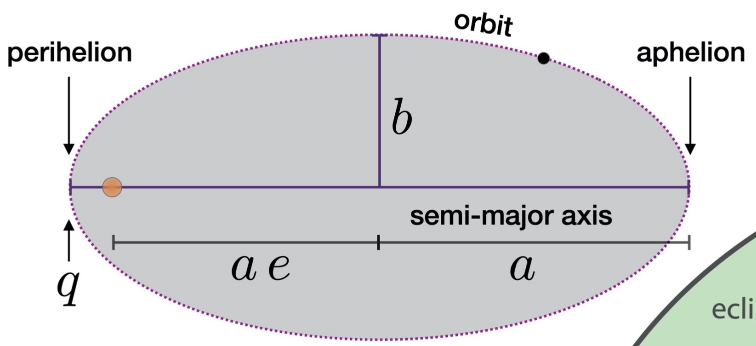


$$\mathcal{H} = \mathcal{H}_{\rm Kep,in} + \mathcal{H}_{\rm Kep,out} + \mathcal{H}_{\rm int}$$

$$\mathcal{H}_{\rm int} = \mathcal{H}_{\rm quad} + \mathcal{H}_{\rm oct} + \cdots$$

⇒ **approximation by double-averaging of the Hamiltonian over the inner and outer orbits**

Kepler orbital elements



a : semimajor axis

e : eccentricity

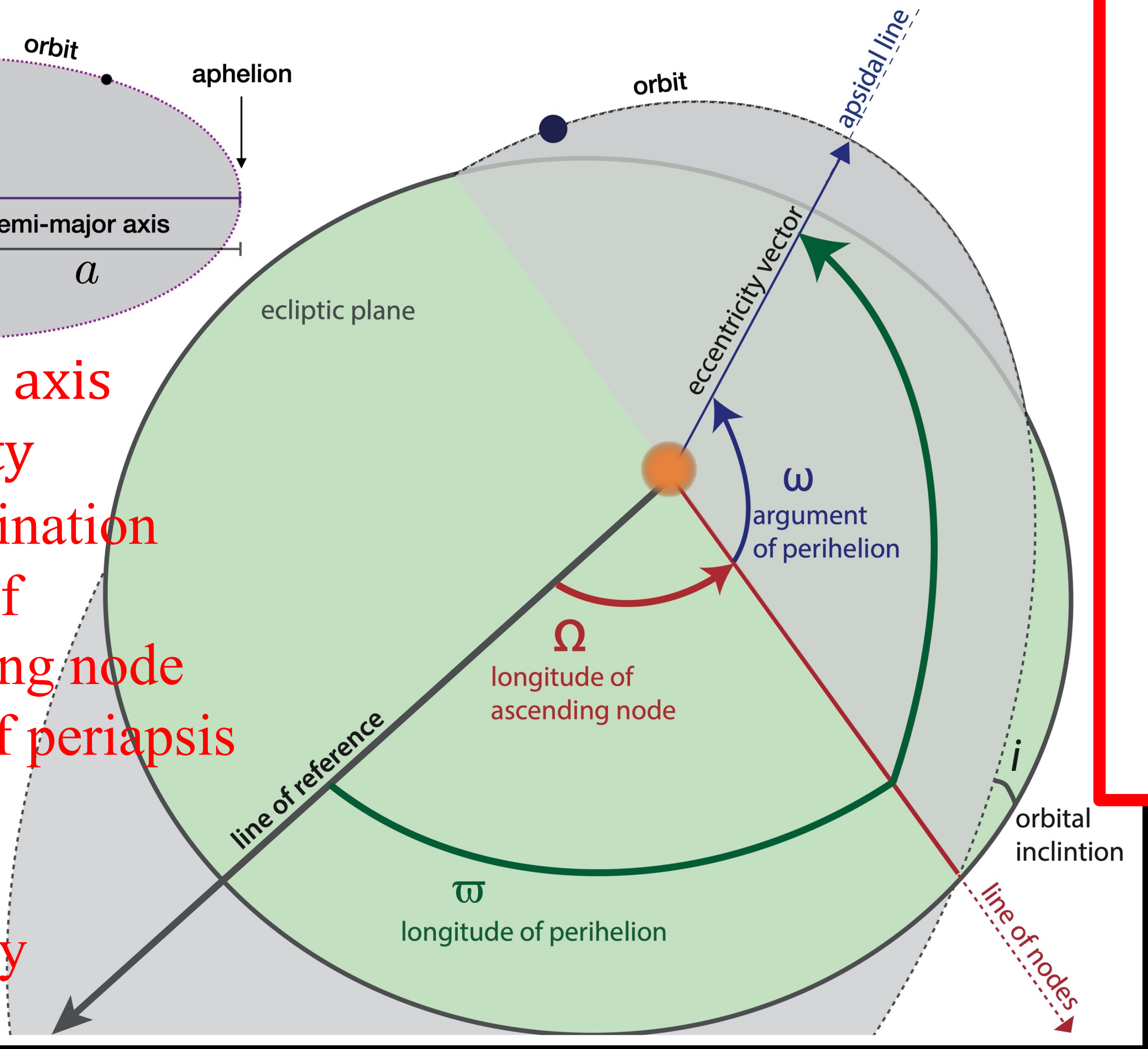
i : orbital inclination

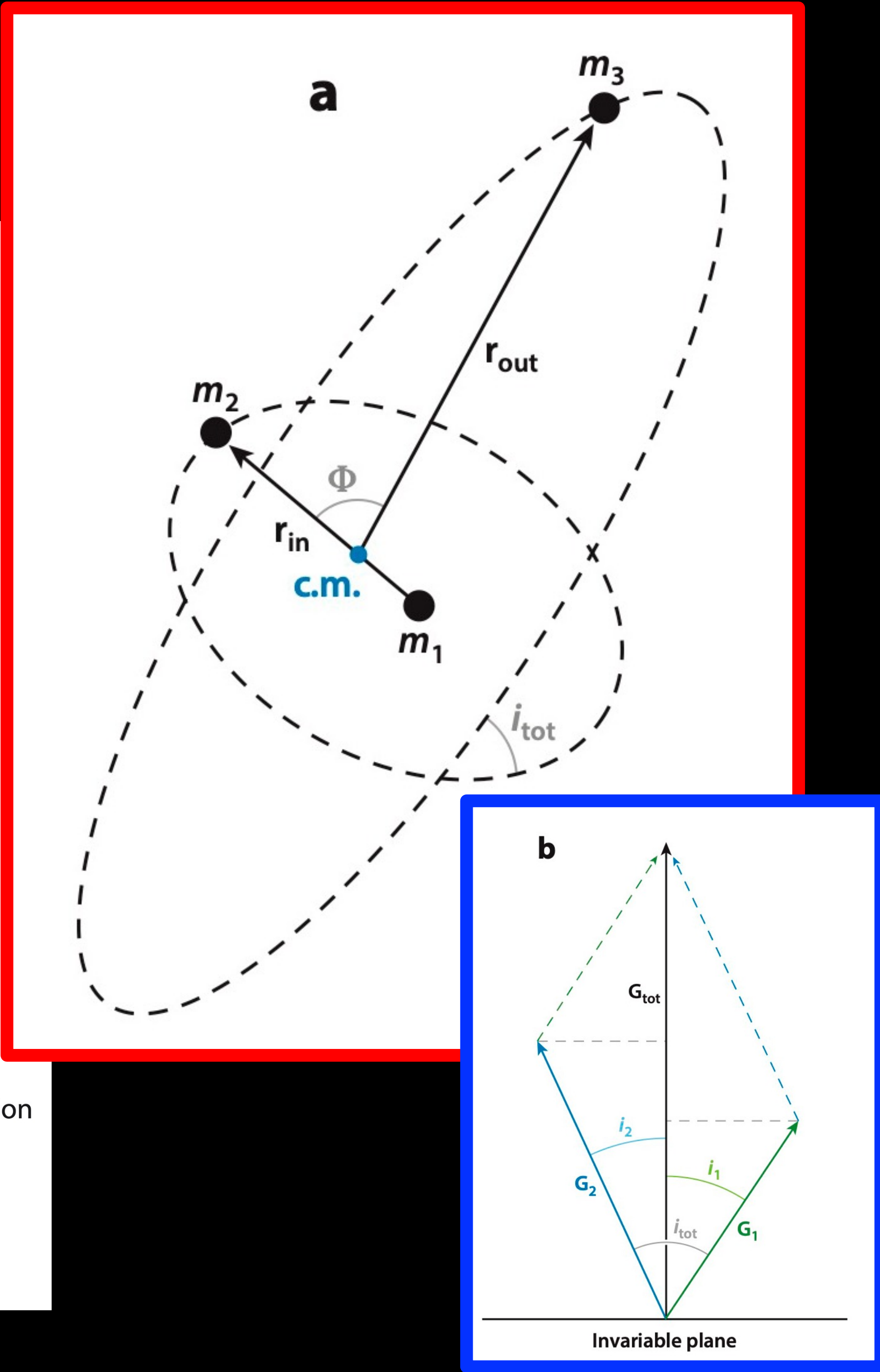
Ω : longitude of ascending node

ω : argument of periapsis

+

ν : true anomaly





Double-averaged quadrupole Hamiltonian

$$\langle \mathcal{H}_{\rm quad} \rangle = -\frac{m_1 m_2}{m_1+m_2}\frac{G m_3}{8 a_2 (1-e_2^2)^{3/2}}\left(\frac{a_1}{a_2}\right)^2 \times \left[2+3e_1^2 - 3\sin^2 i_{\rm tot}(5e_1^2\sin^2\omega_1 + 1 - e_1^2)\right]$$

- Delaunay variables (canonical coordinates and momenta)

$$\begin{cases} \ell &= n(t-t_0) \\ g &= \omega \\ h &= \Omega \end{cases}$$

$$\begin{cases} L &= \sqrt{\mathcal{G}Ma} \\ G &= \sqrt{\mathcal{G}Ma(1-e^2)} \\ H &= \sqrt{\mathcal{G}Ma(1-e^2)}\cos I \end{cases}$$

- L_1, L_2, G_2, H_1, and H_2 are conserved

$\Rightarrow$ a_1, a_2, and e_2 are conserved

if m_1=0 (test particle), i_{tot}=i_1

$$j_{1,z} = \sqrt{1-e_1^2}\cos i_1 = \sqrt{1-e_{1,\rm init}^2}\cos i_{1,\rm init}$$

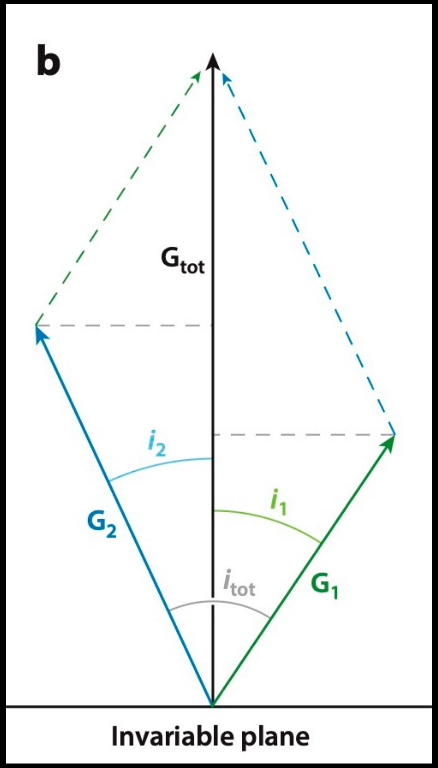
standard and eccentric ZKL effects

- standard ZKL
  - quadrupole Hamiltonian and test particle limit
  - $i_{tot}=i_1$ $$j_{1,z}=\sqrt{1-e_1^2}\cos i_1=\sqrt{1-e_{1,\mathrm{init}}^2}\cos i_{1,\mathrm{init}}$$

$$\text{if } e_{1,\mathrm{init}}=0 \text{ and } |\cos i_{1,\mathrm{init}}|<\sqrt{3/5}$$

  - inner eccentricity and inclination periodically change with

$$e_{1,\max}=\sqrt{1-\frac{5}{3}\cos^2 i_{1,\mathrm{init}}} \quad \text{if} \quad 39.2^\circ < i_{1,\mathrm{init}} < 140.8^\circ$$



- eccentric ZKL
  - $e_{out}\sim 1 \Rightarrow$ more drastic effect due to the octupole term

examples of ZKL effects in secular approximation

Naoz, S: ARAA 54(2016)441

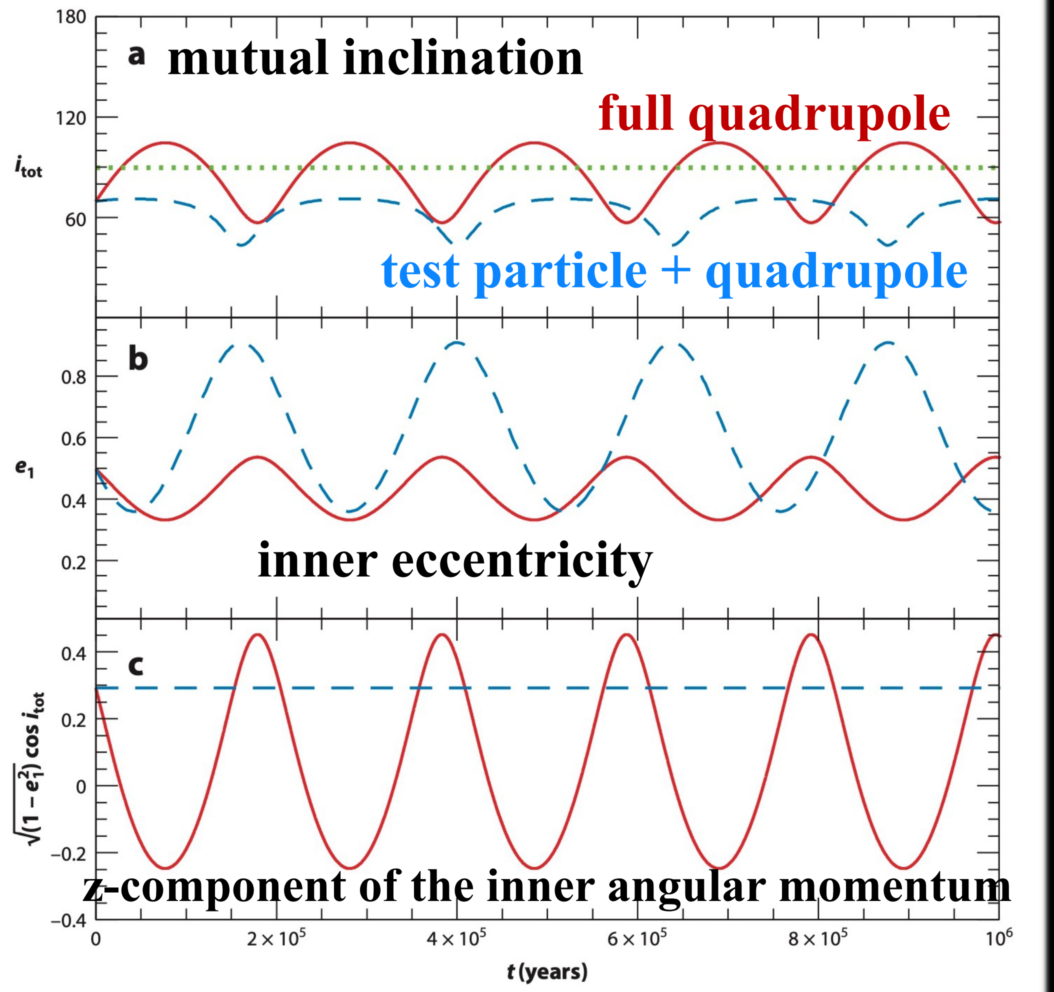


Figure 3

Comparison between the test particle quadrupole (TPQ) formalism (*dashed blue lines*) and the full quadrupole calculation (*solid red lines*). The system has an inner binary with $m_1 = 1.4 M_\odot$ and $m_2 = 0.3 M_\odot$, and the outer body has mass $m_3 = 0.01 M_\odot$. The orbit separations are $a_1 = 5$ AU and $a_2 = 50$ AU. The system was set initially with $e_1 = 0.5$ and $e_2 = 0$, $\omega_1 = 120°$ and $\omega_2 = 0$, and relative inclination $i_{tot} = 70°$. The panels

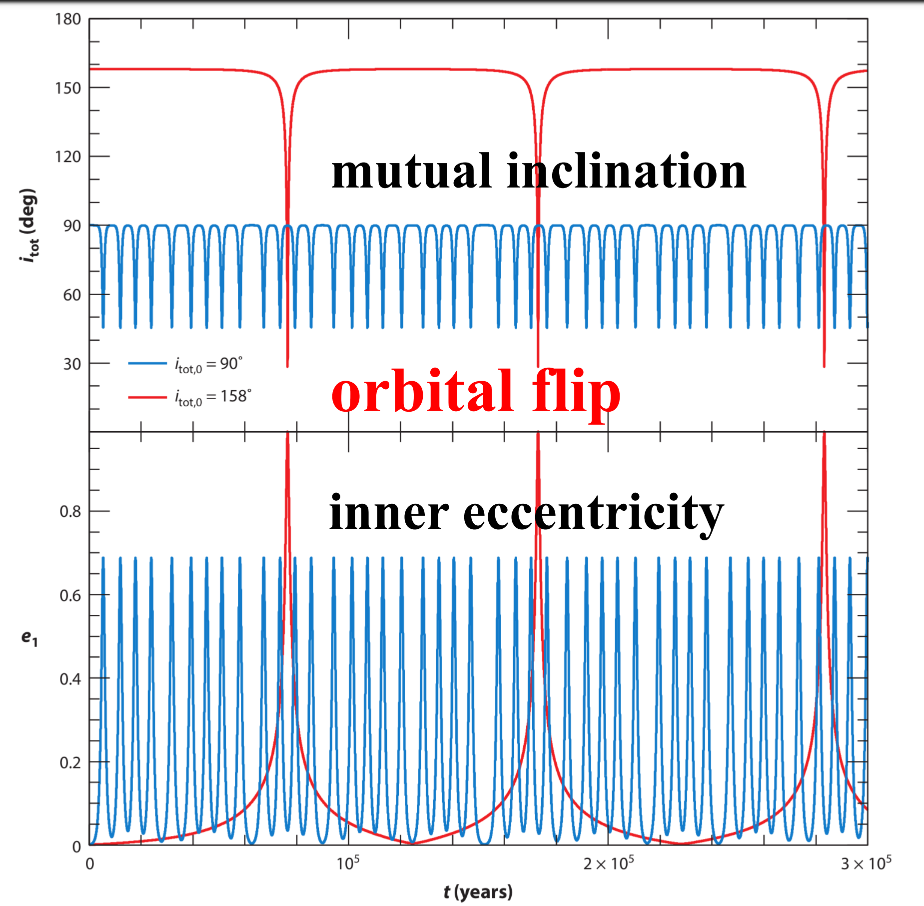
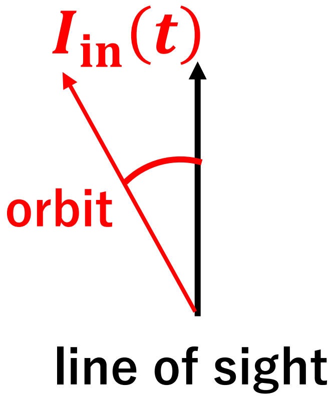


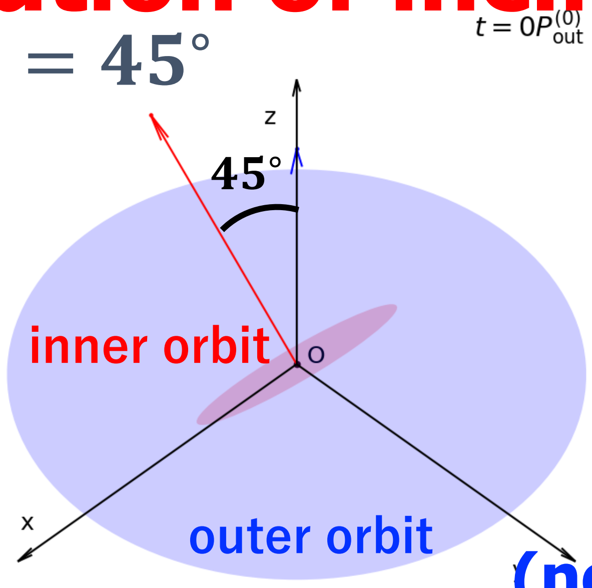
Figure 4

Small-mass outer perturber that induces large eccentricity excitation away from the nominal range of the Kozai angles of 39.2°–140.77°. We consider $m_1 = 1 M_\odot$, $m_2 = 0.5 M_\odot$, $m_3 = 0.05 M_\odot$, $a_1 = 0.5$ AU, and $a_2 = 5$ AU. Both outer and inner eccentricities are set initially to zero, and also set initially are $\omega_1 = 90°$ and $\omega_2 = 0°$. We show two examples: The first shows the eccentricity excitations for as expected initial mutual inclination of $i_{tot} = 90°$, where in this case $i_1 = 25.01°$ and $i_2 = 64.99°$. This produces eccentricity excitation with $e_{1,max} = 0.689$. We also consider an example for which the mutual inclination is set initially to be $i_{tot} = 158°$. In this case $i_1 = 17.12°$ and $i_2 = 140.88°$. The latter parameters are adapted from Martin & Triaud (2015b), which leads to maximum inner eccentricity of $e_{1,max} = 0.99$. Note that in both examples i_2 is close to the nominal Kozai angles range.

Evolution of inclination for non-coplanar triples



$i_{mut} = 45°$



$P_{out} = 78.9$ days

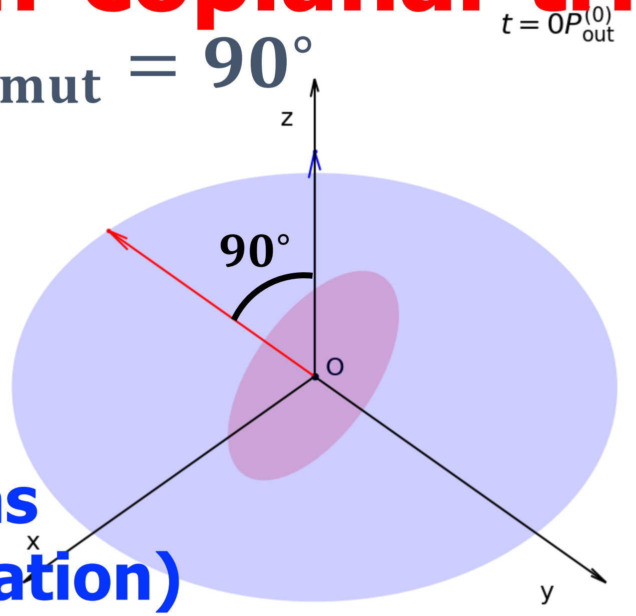
$P_{in} = 10$ days

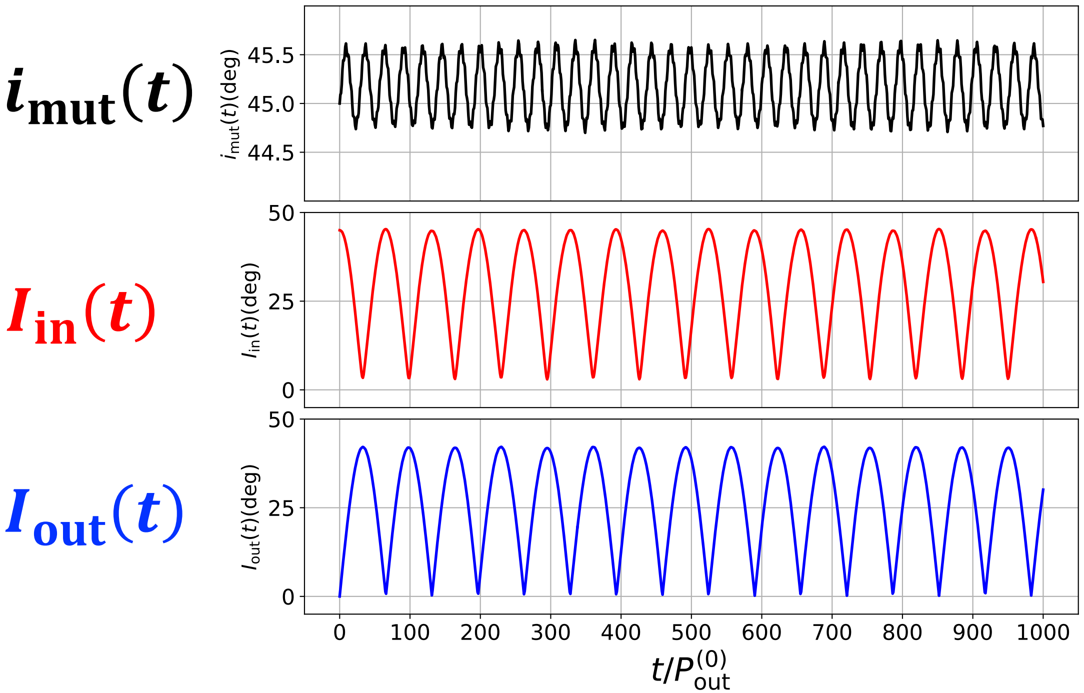
$m_1 = m_2 = 10M_\odot$

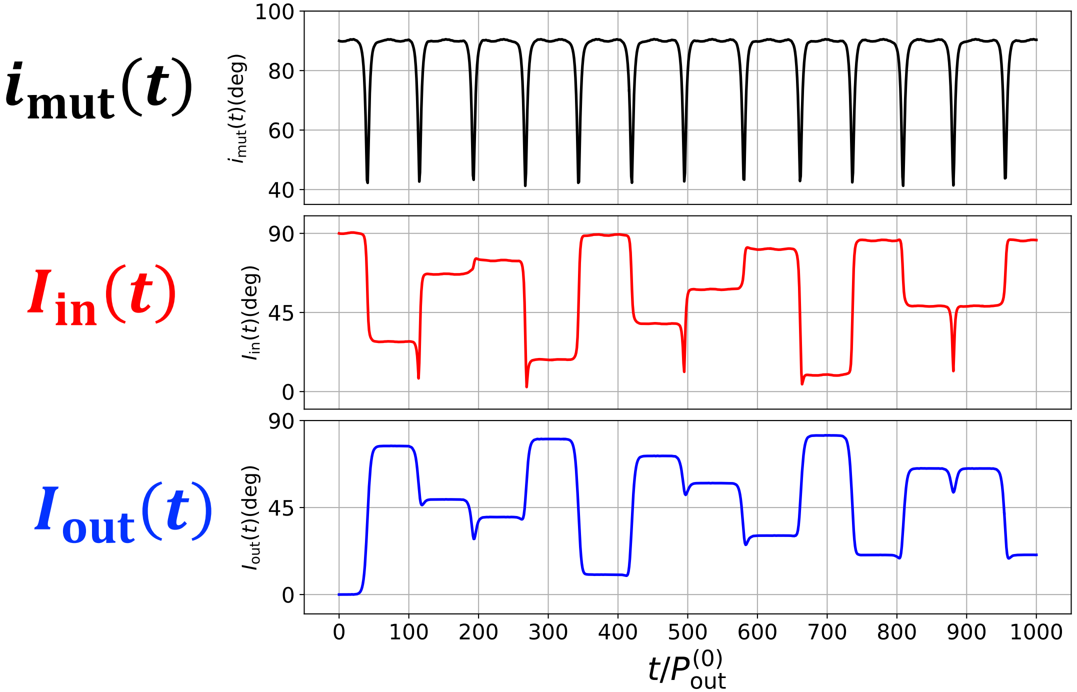
$m_* = 3M_\odot$

3-body simulations (not secular approximation)

$i_{mut} = 90°$





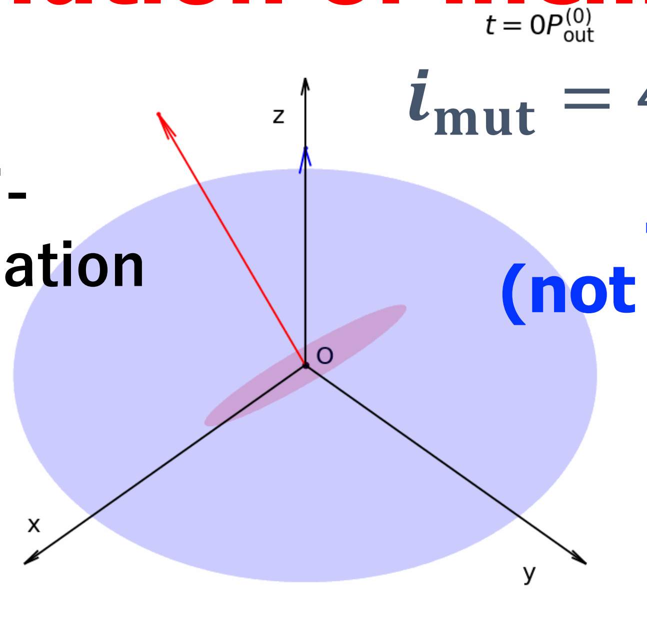


Evolution of inclination for non-coplanar triples

$i_{\rm mut} = 45^\circ$

weak Kozai-Lidov oscillation

⇒ small-amplitude nodal precession

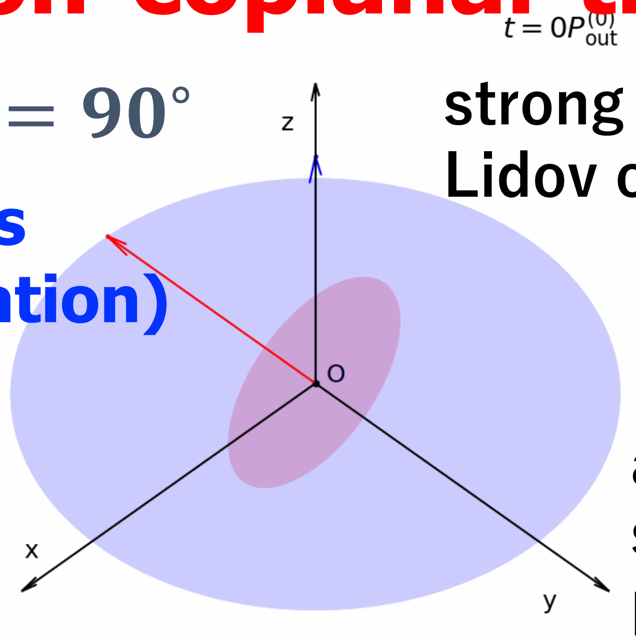


3-body simulations (not secular approximation)

Hayashi + YS: ApJ 897(2020)29

$$K_{\rm Kep} = K_0 \sin I_{\rm out}(t)$$

$i_{\rm mut} = 90^\circ$



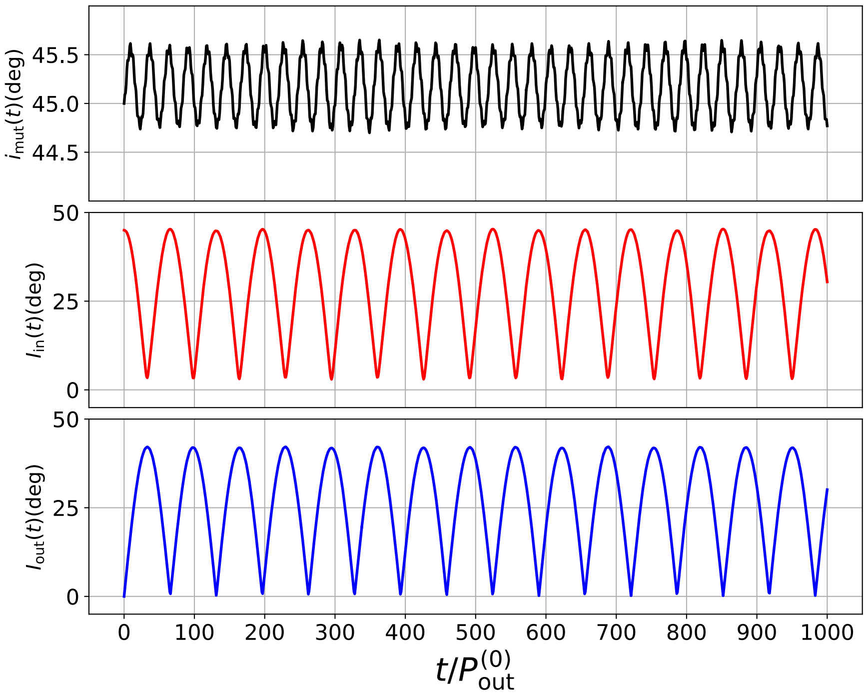
strong Kozai-Lidov oscillation

⇒ large-amplitude sporadic precession

$i_{\rm mut}(t)$

$I_{\rm in}(t)$

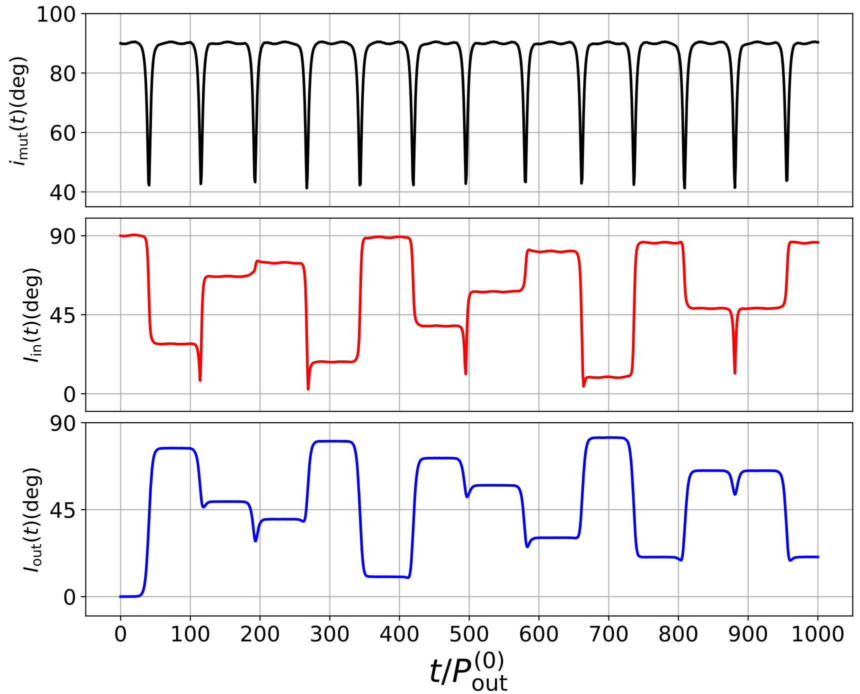
$I_{\rm out}(t)$



$i_{\rm mut}(t)$

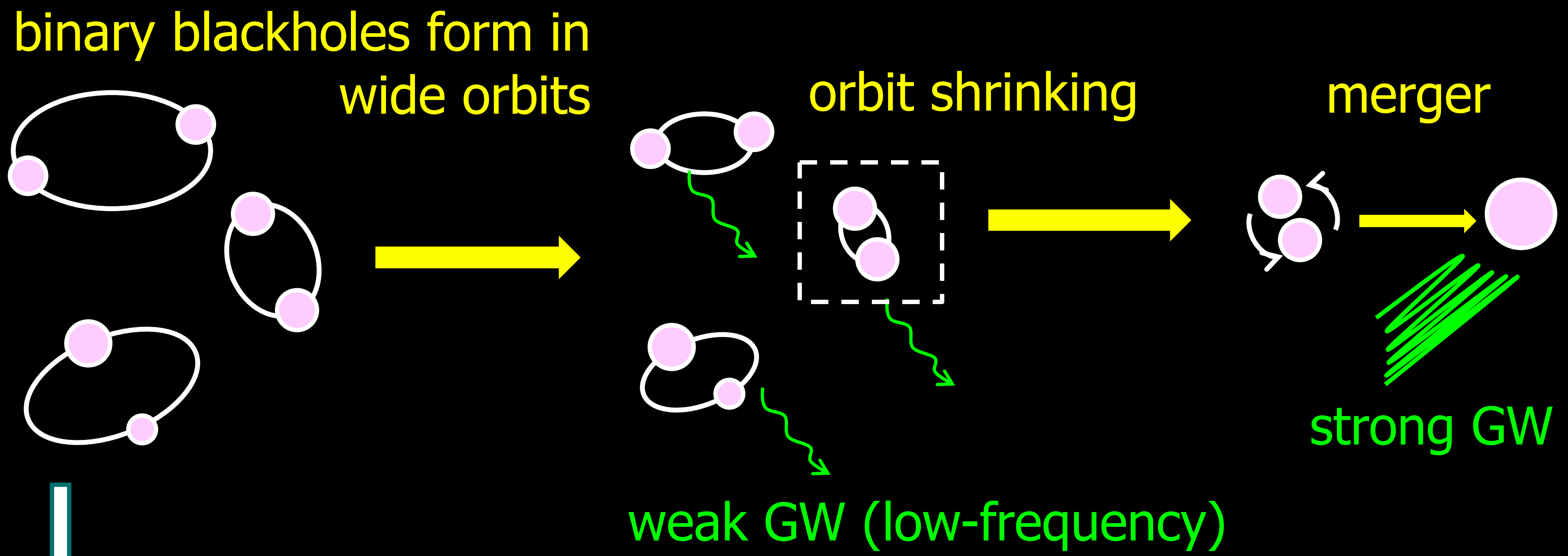
$I_{\rm in}(t)$

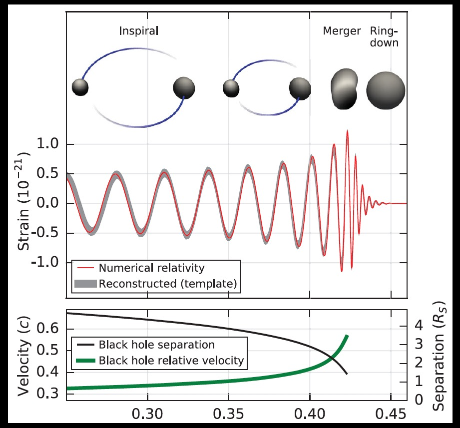
$I_{\rm out}(t)$



3 Search for blackhole binaries in optical:
a star-blackhole binary
or
a star-binary blackhole triple?

Generic picture of binary BH evolution





LIGO/Virgo

BBHs would spend longer time in wide orbits before merging

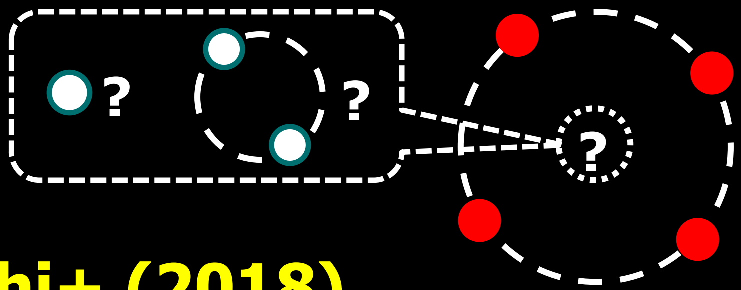
Abundant longer orbital-period BBHs may remain undetected (e.g. ~10day orbital period~$10^{-6}\mathrm{Hz}$).

Other detection strategies complementary to GW ?

Proposals to search for star-BH binaries

Gaia mission (2013-)

Astrometry of stars in Galaxy
$\sim 10^9$ stars eventually
RV with 200-350m/s precision
for brightest stars (Katz 2018)



Yamaguchi+ (2018)

5-year mission may detect
200-1000 star-BH binaries

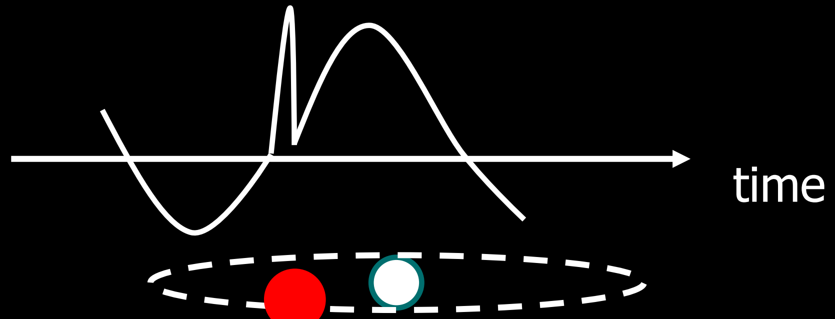
Tanikawa+(2023)

TESS mission (2018-)

photometry of nearby stars ($\sim$ 12mag)
transit planets

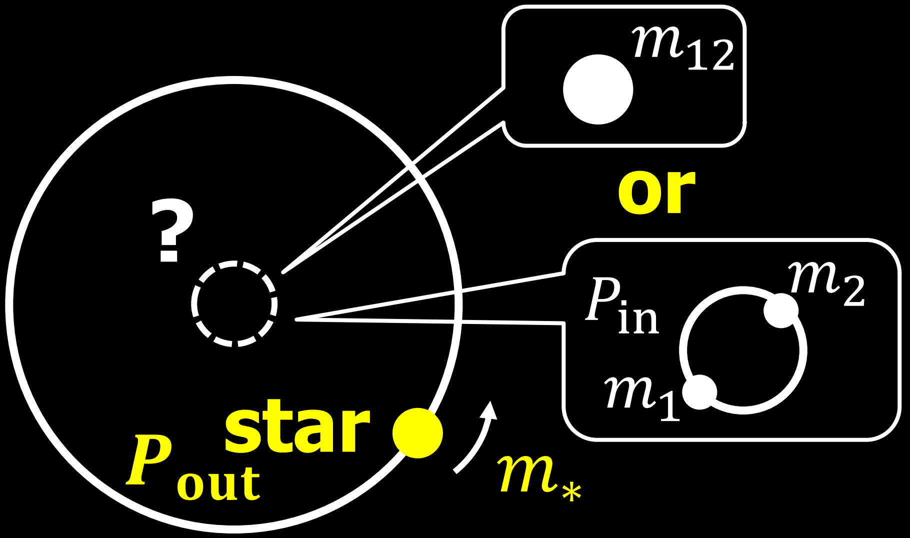
Masuda & Hotokezaka (2019)

Light curve modulation
(relativistic effects, tidal deformation)
$\Rightarrow (10-100)$ star-BH binaries may be identified



Some of them may be indeed a star-binary BH triple!
Can precise radial velocity follow-ups unveil the inner BBH?

RV (radial velocity) modulations of a tertiary star



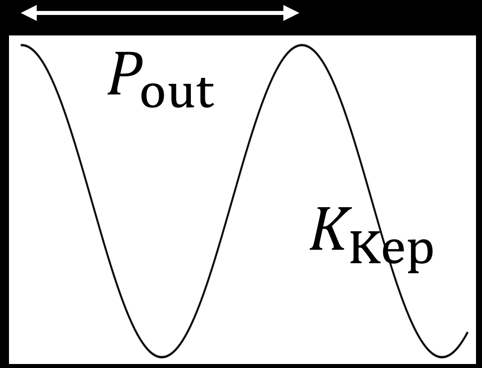
Keplerian motion RV
+ RV modulations of a tertiary star due to a hidden inner binary

Hayashi, Wang + YS: ApJ 890(2020)112
Hayashi + YS: ApJ 897(2020)29
Hayashi, YS + Trani (2023): arXiv:2307.01793

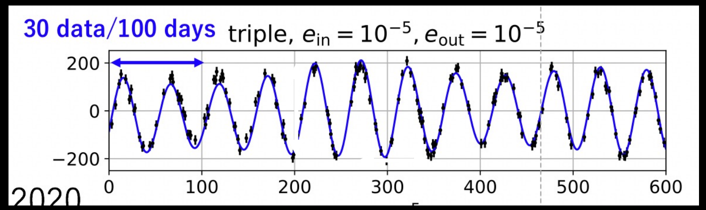
Morais & Correia (2008, 2012)

(i) short-term

$$\mathrm{Amp} \sim K_{\mathrm{Kep}} \left(\frac{P_{\mathrm{in}}}{P_{\mathrm{out}}}\right)^{\frac{7}{3}}$$



\+



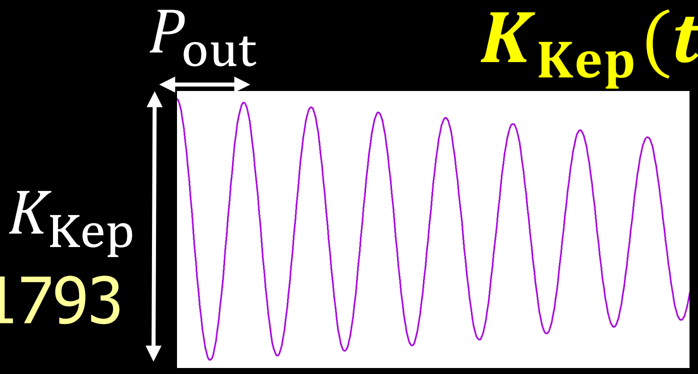
$$\text{period} \sim P_{\mathrm{in}}/2$$

Kepler motion **+ Short-term RV variations (inner-binary perturbation)**

(ii) long-term for non-coplanar triples

Inclination $I_{out}(t)$ modulated in the ZKL timescale

$$K_{\mathrm{Kep}}(t) = K_0 \sin I_{\mathrm{out}}(t)$$



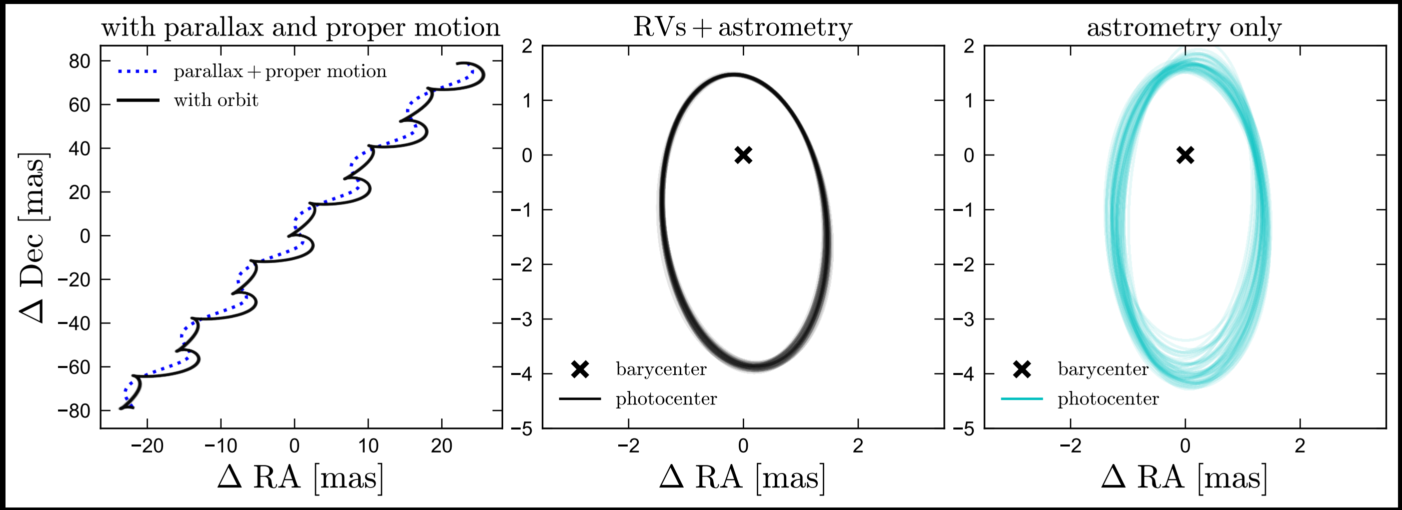
semi-amplitude of Kepler RV varies over longer timescales

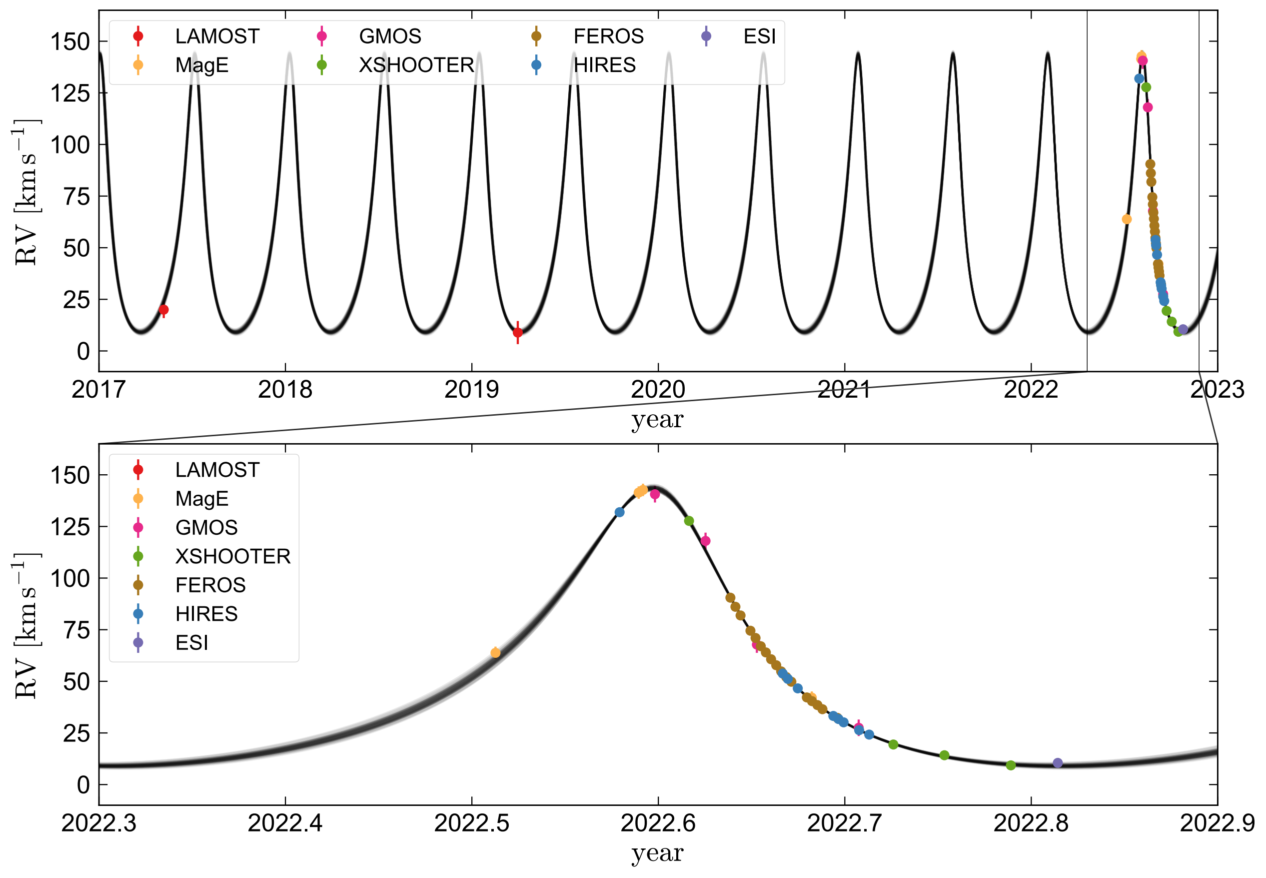
Gaia BH-1

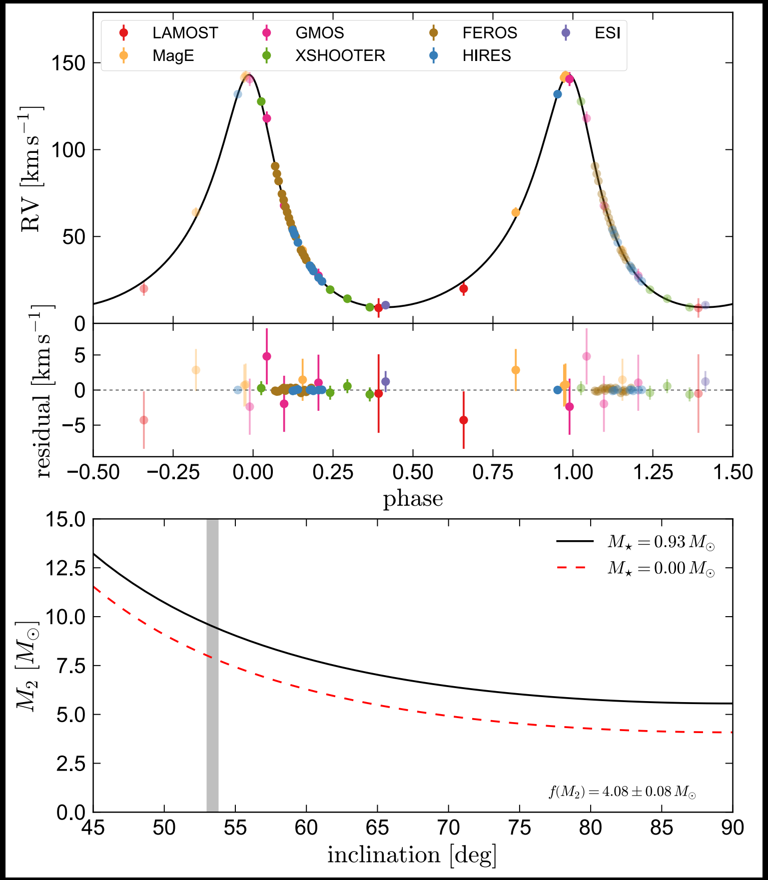
$0.93 M_{\odot}$ G star + $9.6 M_{\odot}$ BH

(P_{orb}=186days) at d=477pc

eccentricity ~ 0.45



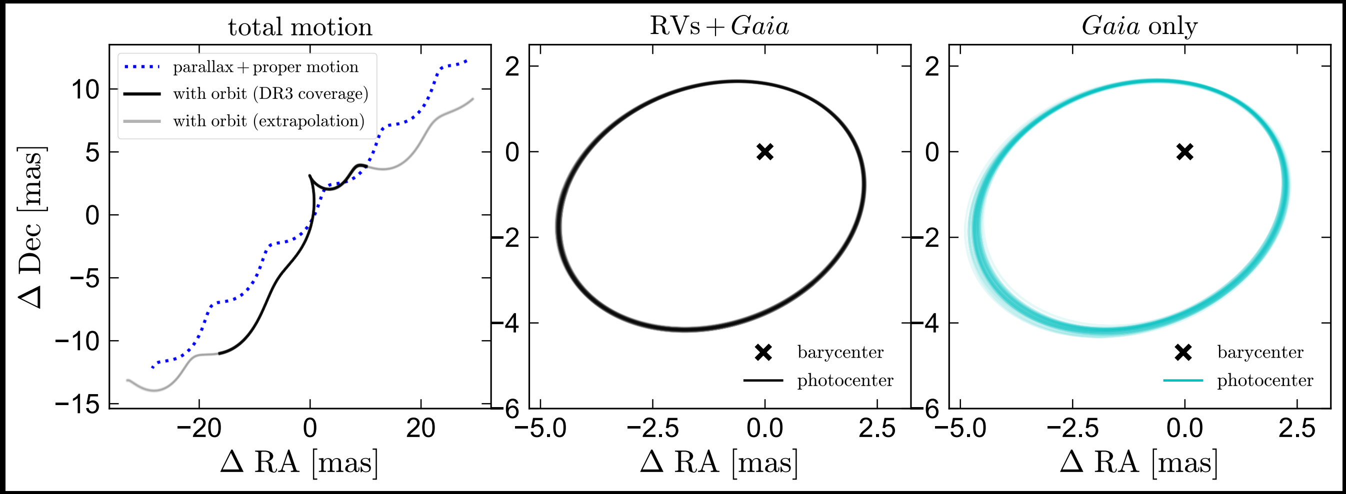


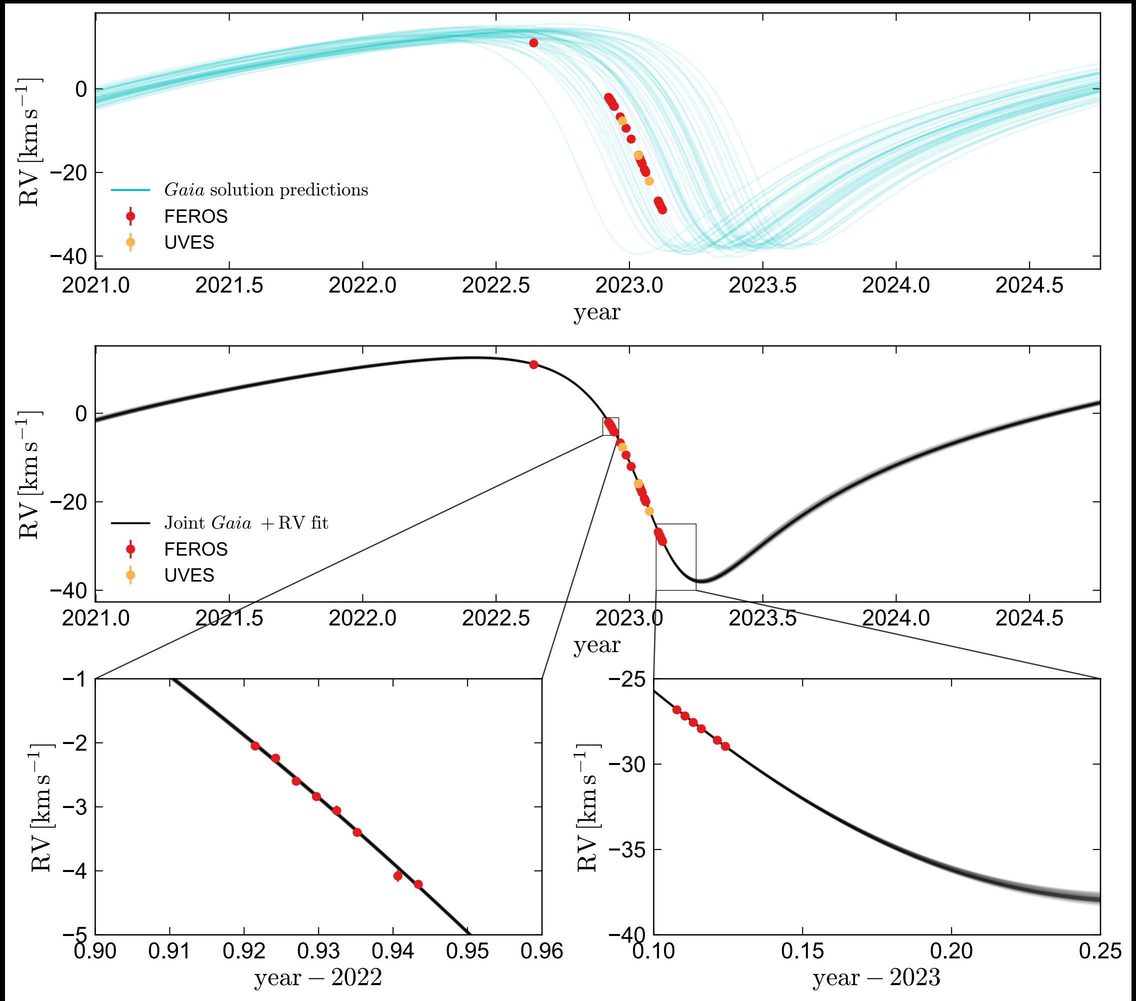


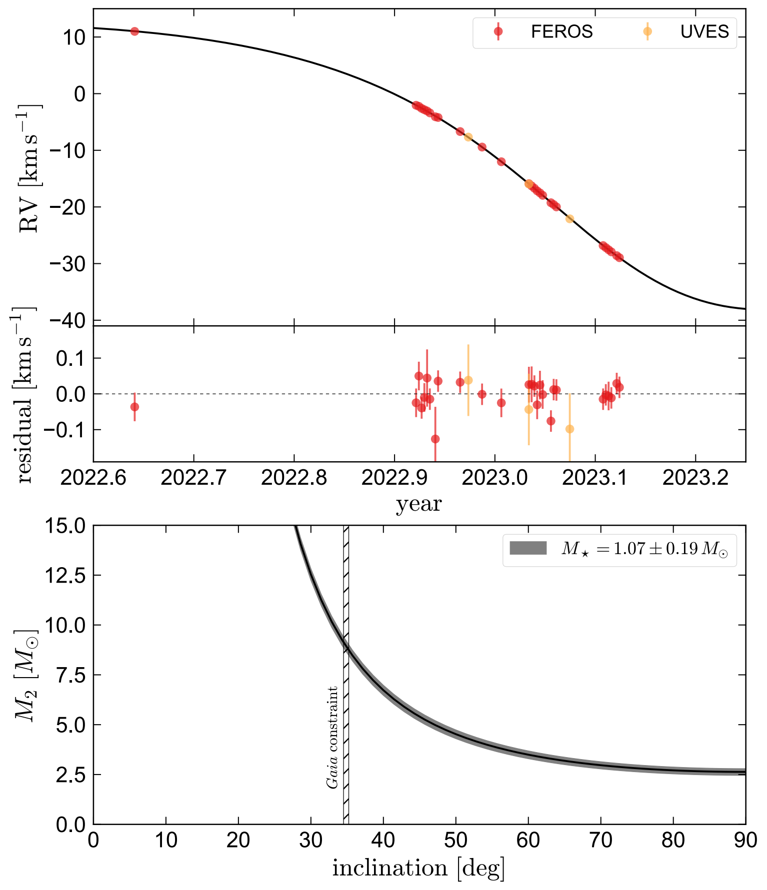
El-Badry et al.
MNRAS
518 (2023)1057

Gaia BH-2

$1M_\odot$ red giant + $9M_\odot$BH
(P_{orb}=1277days) at d=1.16kpc
eccentricity ~ 0.52

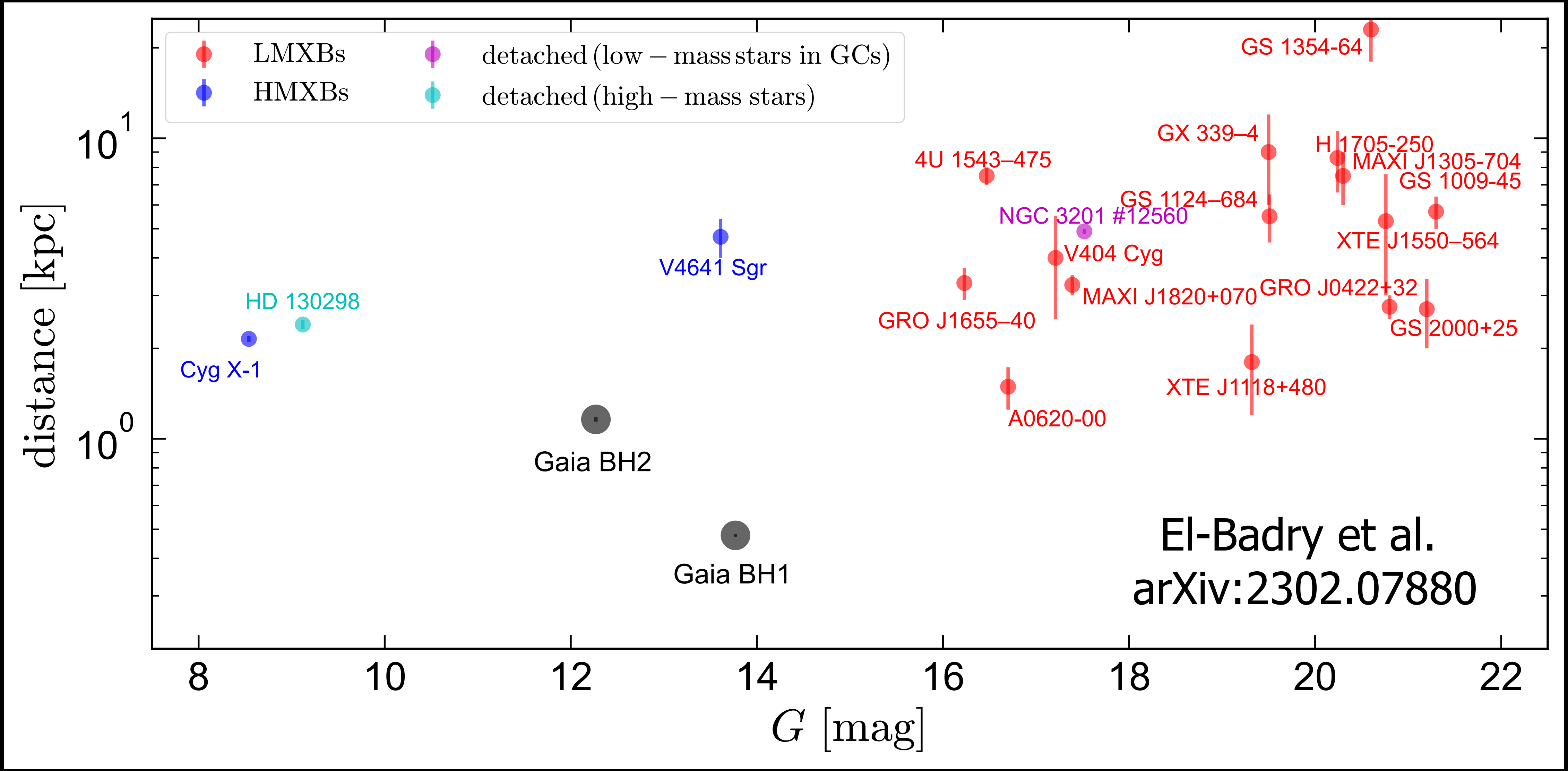






discovered first by Tanikawa et al. ApJ 946(2023)79

confirmed and RV follow-up by El-Badry et al. arXiv:2302.07880



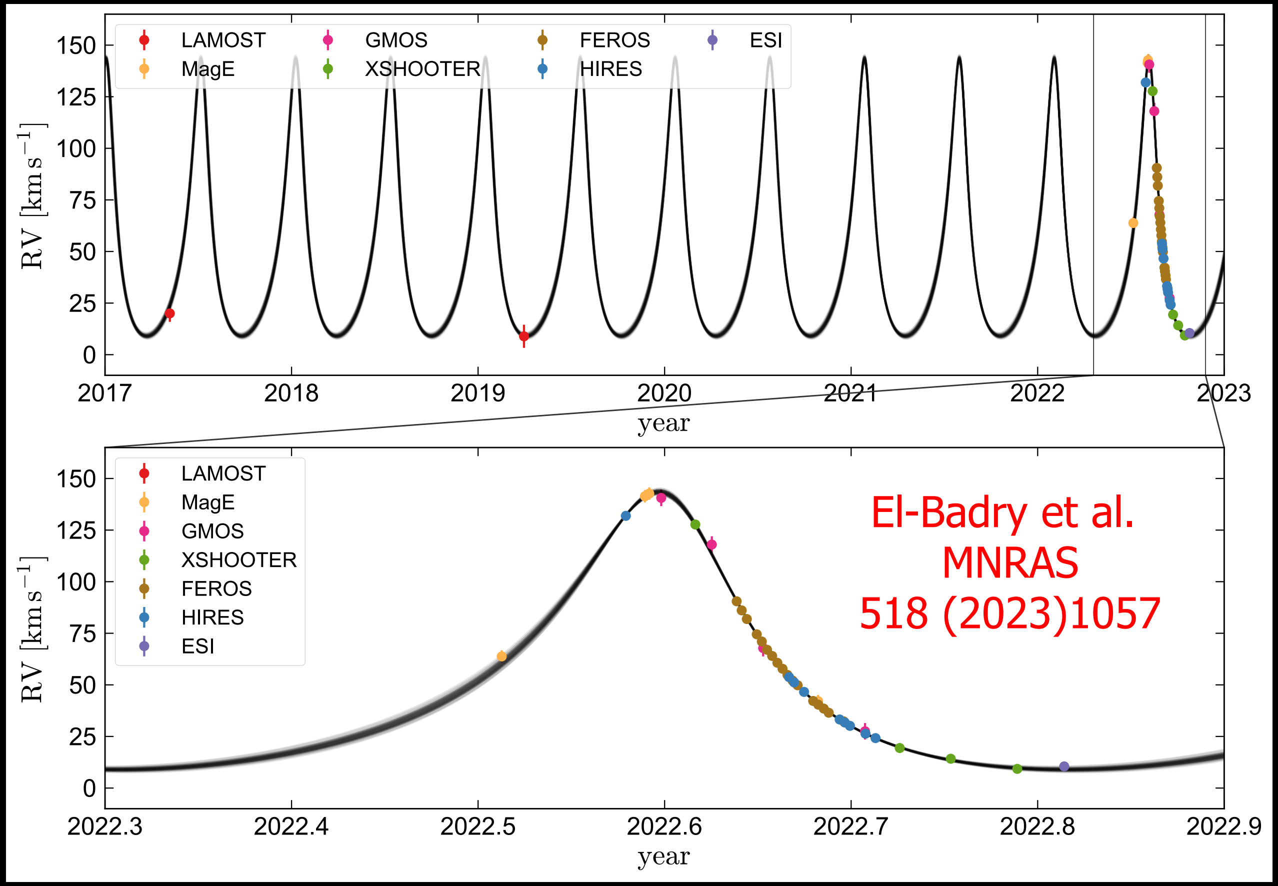
Comparison of Gaia BH1 and BH2 (black points) to known Galactic BHs in the plane of distance and quiescent optical magnitude

4 Feasibility study with Gaia BH1 and BH2

Gaia star-BH candidates

Gaia BH-1

$0.93M_\odot$ G star + $9.6M_\odot$BH
(P_{orb}=186days) at d=477pc
eccentricity ~ 0.45



Gaia BH-2

$1M_\odot$ red giant + $9M_\odot$BH
(P_{orb}=1277days) at d=1.16kpc
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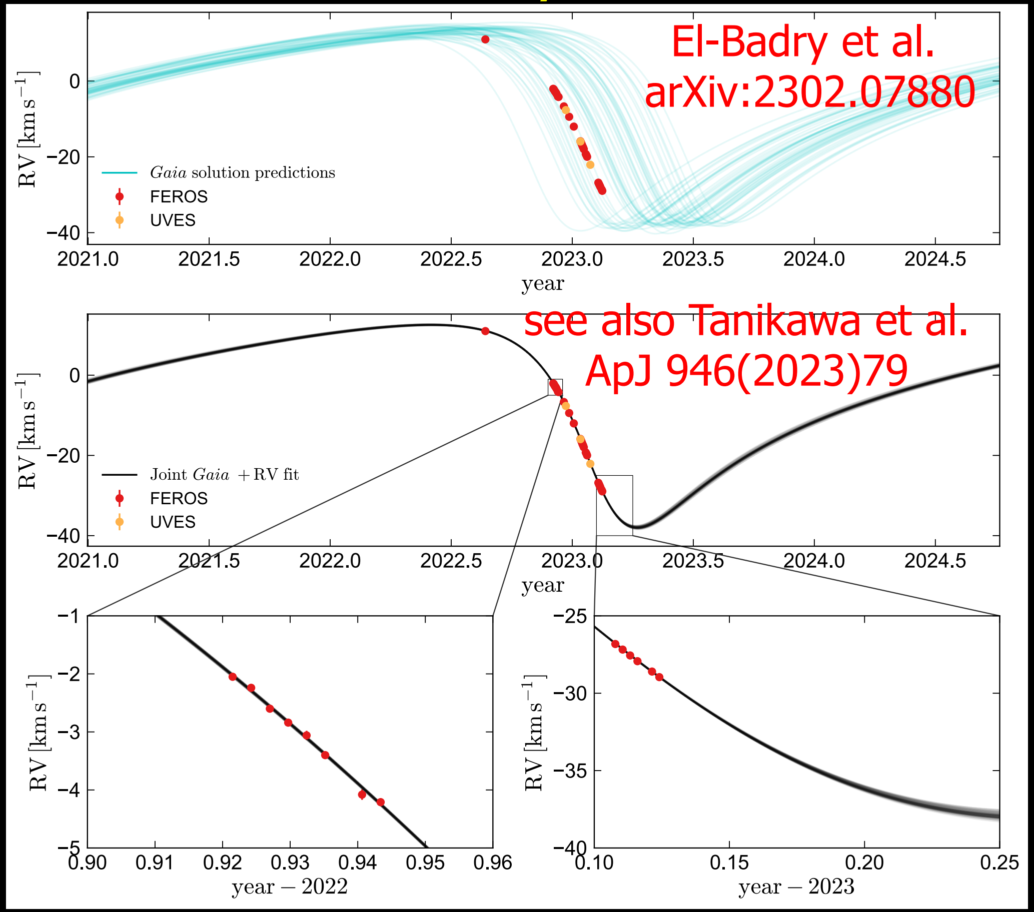
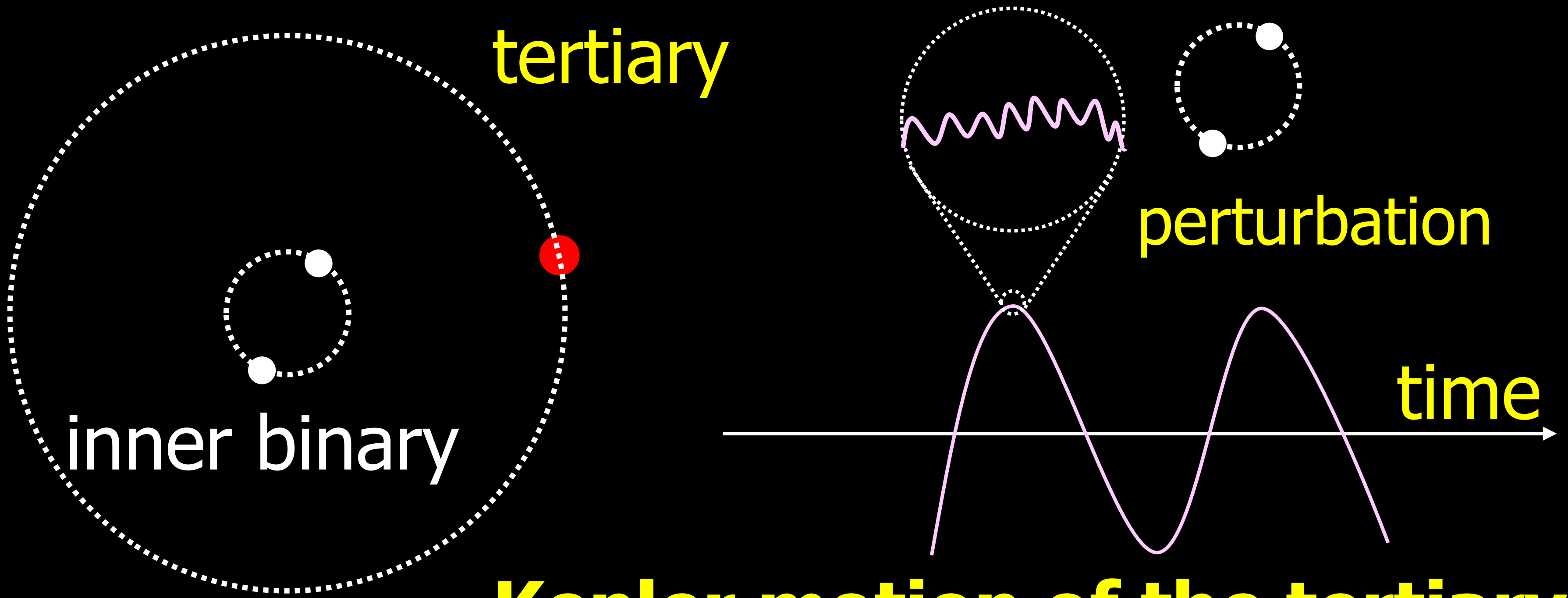


Short-term Radial velocity modulation of a tertiary star due to an inner binary



Kepler motion of the tertiary

Hayashi, Wang + YS: ApJ 890(2020)112
Hayashi + YS: ApJ 897(2020)29
Hayashi, YS + Trani (2023): arXiv:2307.01793

Approximate analytic expressions for short-term RV modulations

- **Semi-amplitude of the short-term RV modulation (coplanar and circular orbits)**

$$V_{0,0} \equiv \frac{m_{12}}{m_{123}} a_{\rm out} \nu_{\rm out} = \left(\frac{2\pi \mathcal{G} m_{12}^3}{m_{123}^2 P_{\rm out}} \right)^{1/3}$$

$$K_{\rm short} \equiv \frac{m_1 m_2}{m_{12}^2} \sqrt{\frac{m_{123}}{m_{12}}} \left(\frac{a_{\rm in}}{a_{\rm out}} \right)^{7/2} V_{0,0} \sin I_{\rm obs} = \frac{m_1 m_2}{m_{12}^2} \left(\frac{m_{123}}{m_{12}} \right)^{2/3} \left(\frac{P_{\rm in}}{P_{\rm out}} \right)^{7/3} V_{0,0} \sin I_{\rm obs}$$

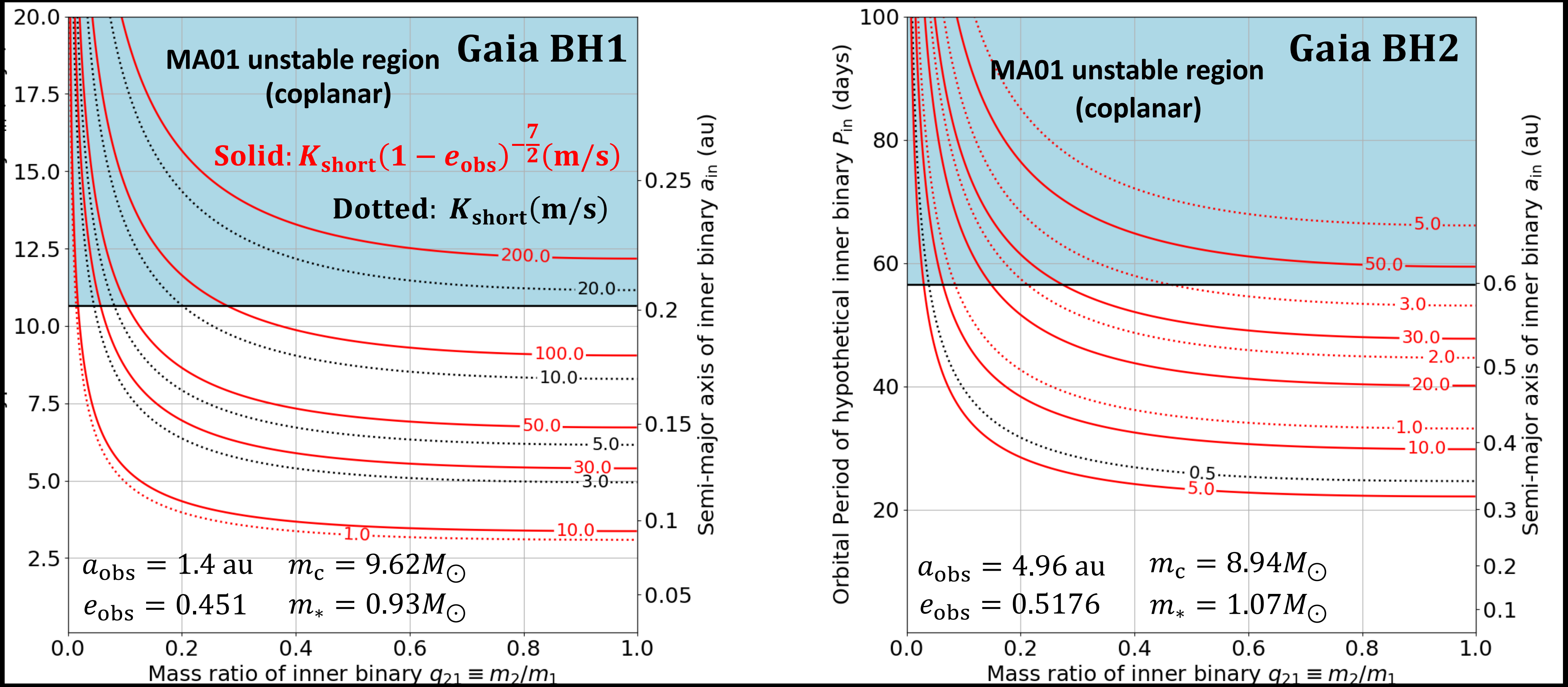
- **Dynamical instability condition (MA01)**

$$\frac{r_{\rm p,out}}{a_{\rm in}} > 2.8 \left(1 - 0.3 \frac{i_{\rm mut}}{180^\circ} \right) \left[\left(1 + \frac{m_*}{m_{12}} \right) \frac{(1+e_{\rm out})}{\sqrt{1-e_{\rm out}}} \right]^{2/5}$$

Short-term RV modulations expected from analytic approximation (coplanar + circular tertiary)

Contours of expected semi-amplitudes of short-term RV modulations:
~ (1-100) m/s for coplanar outer orbits (Hayashi, YS + Trani 2023)

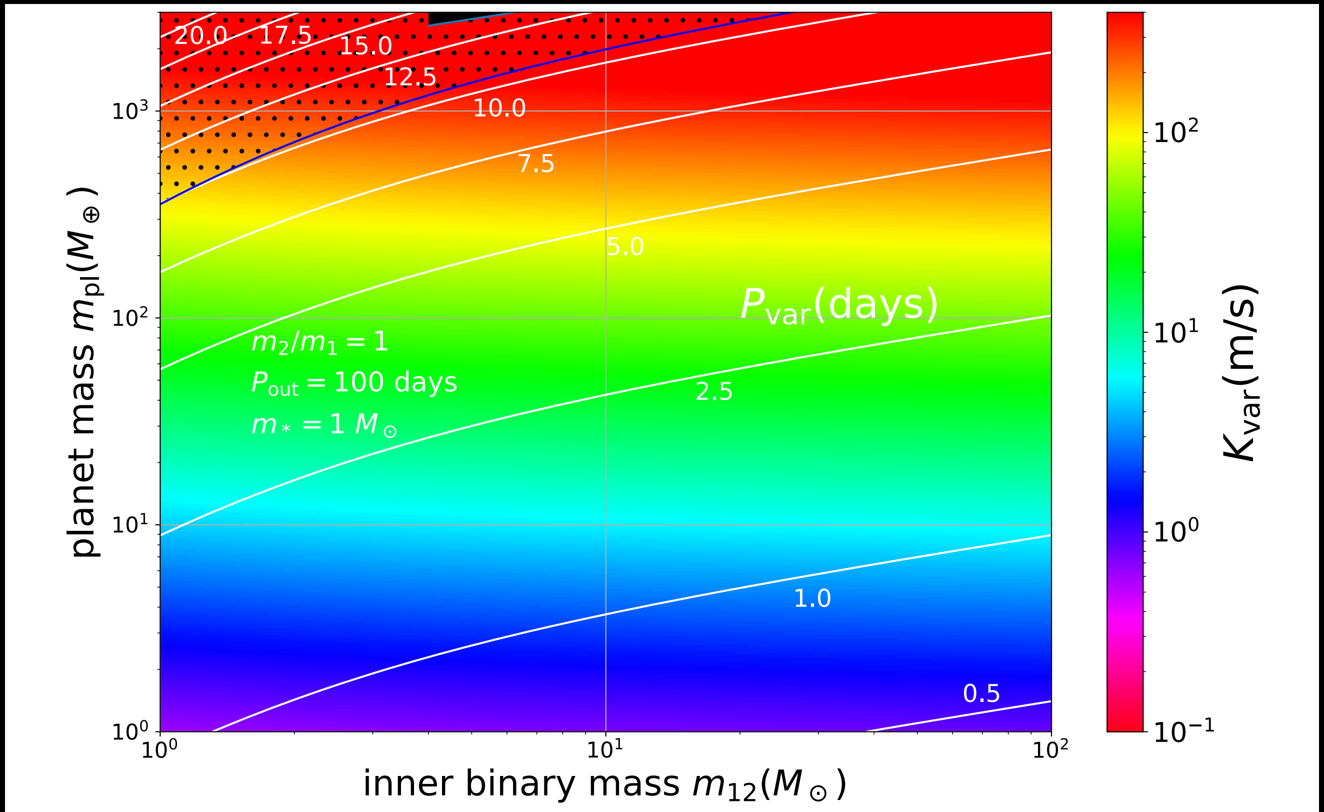
P_{in}(days)



m_2/m_1 m_2/m_1

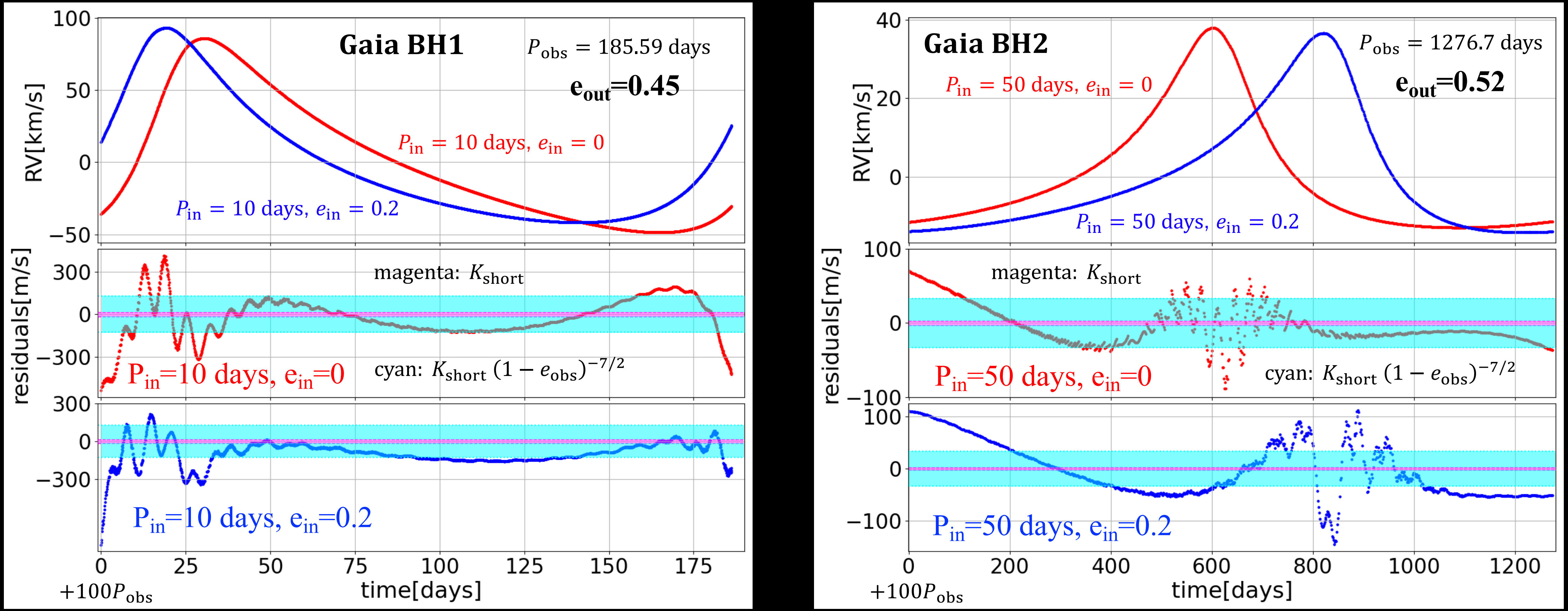
Degeneracy between a binary BBH and a planet on the short-term RV modulations

- Similar short-term modulation is produced if the star has a planet. Given the modulation amplitude and period, the relation between the planet mass and BBH mass is computed. The degeneracy can be broken by the precise RV data.



Hayashi, Wang & YS (2020)
see also Morais & Correia (2008, 2012)

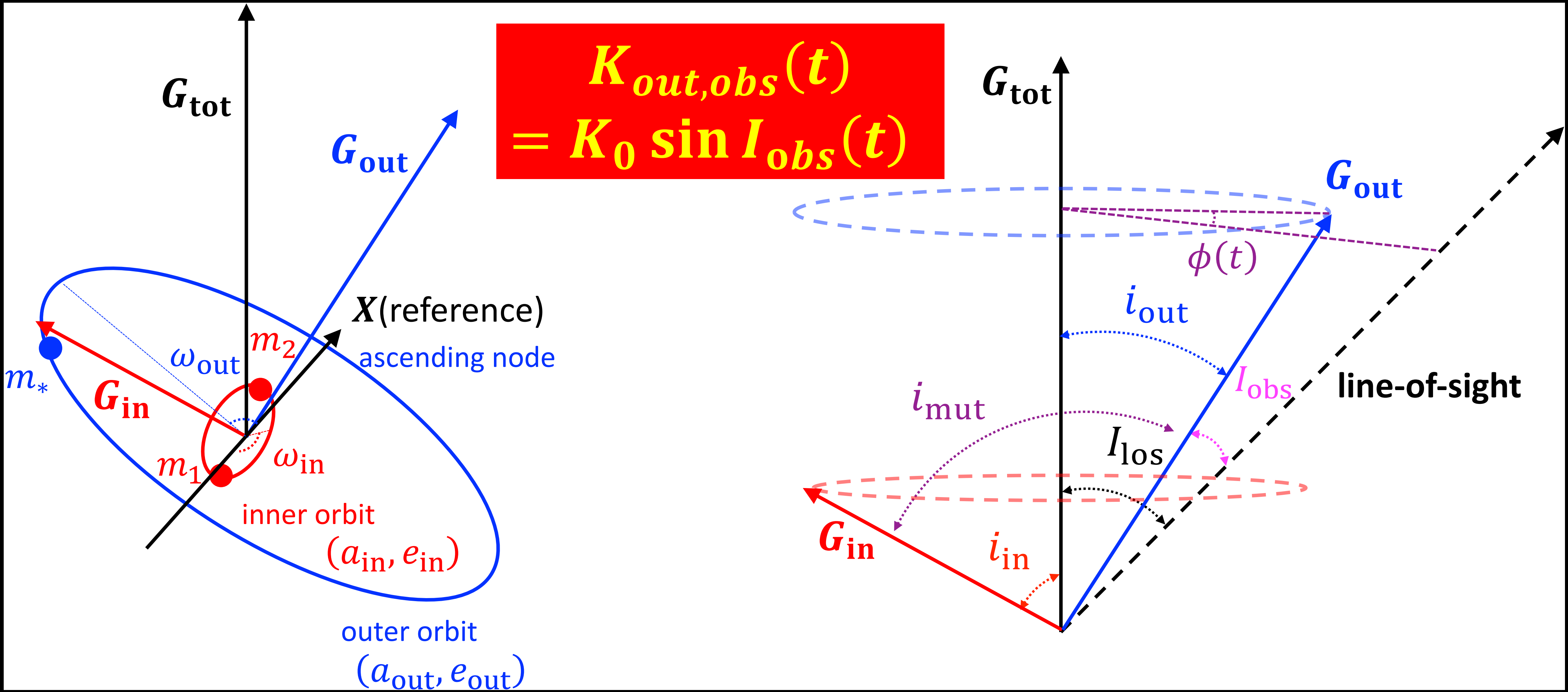
Short-term RV modulations from direct three-body simulation (coplanar + tertiary with observed eccentricity)



- Due to the outer eccentricity, the amplitude of the short-term RV modulations becomes (10-100) times larger at the pericenter passage than the analytic estimate for circular outer orbits

Long-term RV modulations due to nodal precession and ZKL (von Zeipel-Kozai-Lidov) oscillations for non-coplanar inclined triples

$$K_{out,obs}(t) = K_0 \sin I_{obs}(t)$$



Long-term modulation timescales

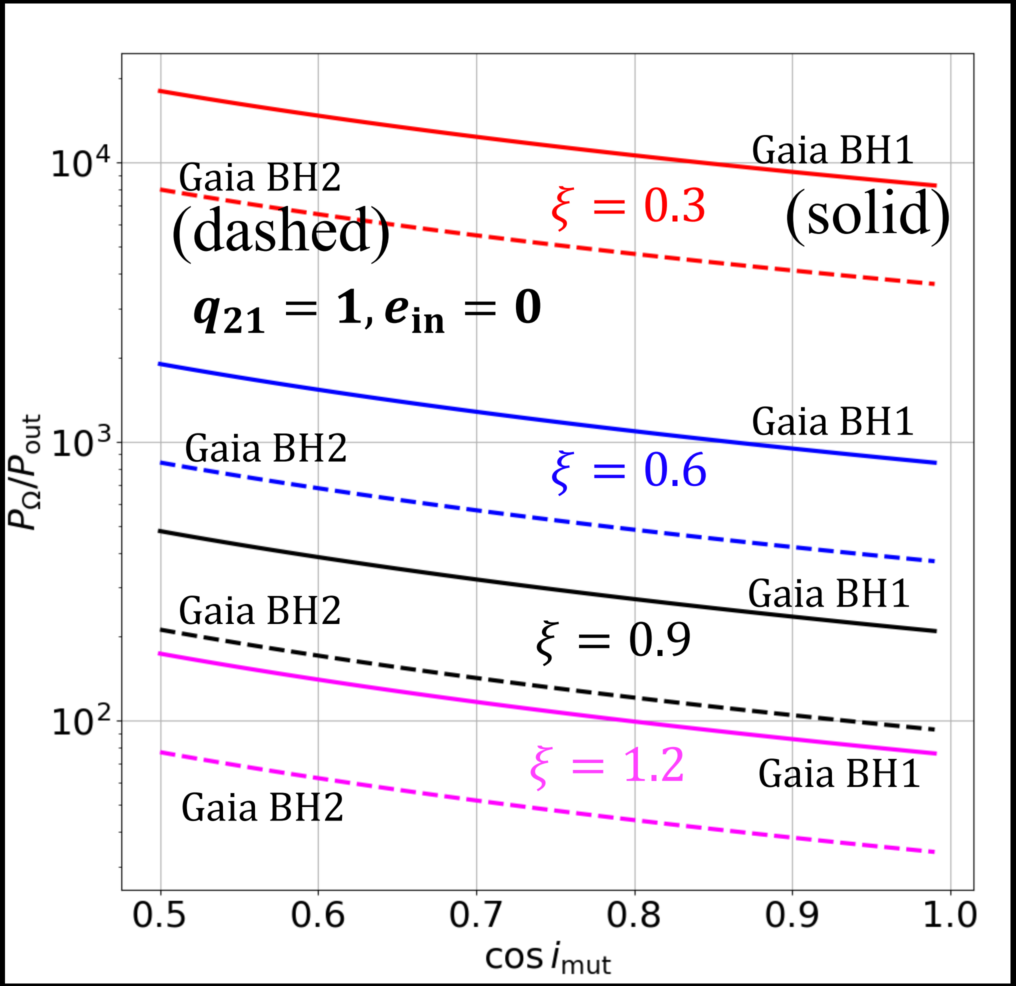
- **Nodal precession**

$$\frac{P_\Omega}{P_{\rm out}} = \frac{4q_{21}^3}{3(1+q_{21})^6}\left(\frac{m_{12}^2 m_{123}^2}{m_*^4}\right)\frac{(1-e_{\rm in}^2)^2}{\xi^3\cos i_{\rm mut}}\frac{1}{\sqrt{1+2\xi\cos i_{\rm mut}+\xi^2}}.$$

$$\xi \equiv \frac{G_{\rm in}}{G_{\rm out}} = \frac{q_{21}}{(1+q_{21})^2}\sqrt{\frac{1-e_{\rm in}^2}{1-e_{\rm out}^2}}\left(\frac{m_{12}}{m_*}\right)\left(\frac{m_{123}P_{\rm in}}{m_{12}P_{\rm out}}\right)^{1/3}$$

- **ZKL oscillation**

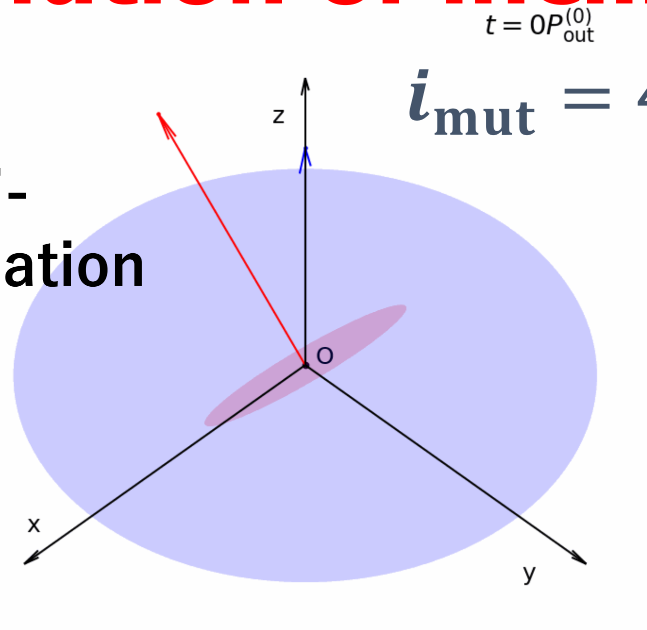
$$\frac{T_{\rm ZKL}}{P_{\rm out}} = \frac{m_{123}P_{\rm out}}{m_3 P_{\rm in}}(1-e_{\rm out}^2)^{3/2} \approx 130\left(\frac{m_{123}}{10M_\odot}\right)\left(\frac{m_3}{1M_\odot}\right)^{-1}\left(\frac{P_{\rm out}/P_{\rm in}}{20}\right)\left[1-\left(\frac{e_{\rm out}}{0.5}\right)^2\right]^{3/2}$$



Evolution of inclination for non-coplanar triples

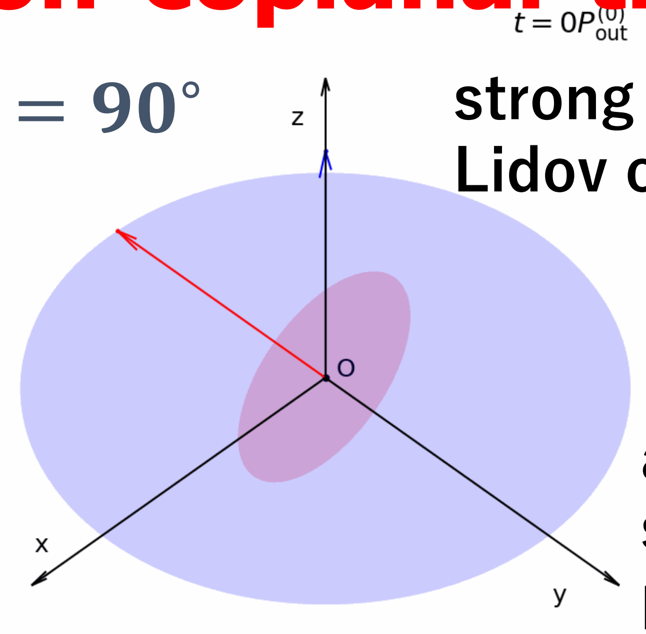
weak Kozai-Lidov oscillation

⇒ small-amplitude nodal precession



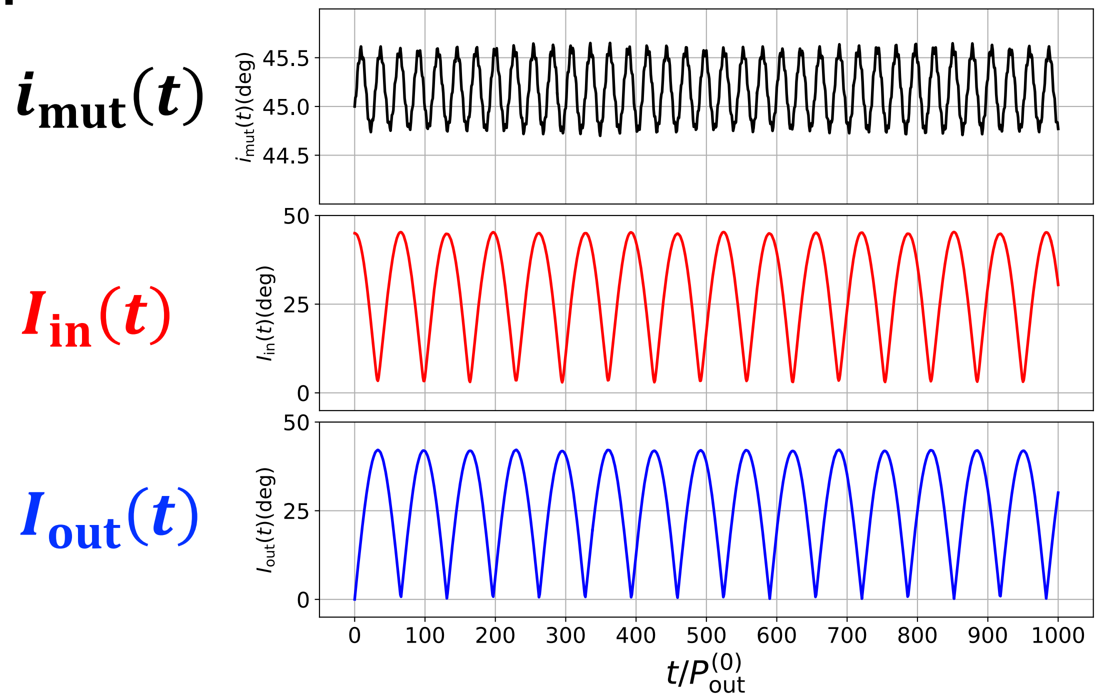
Hayashi + YS: ApJ 897(2020)29

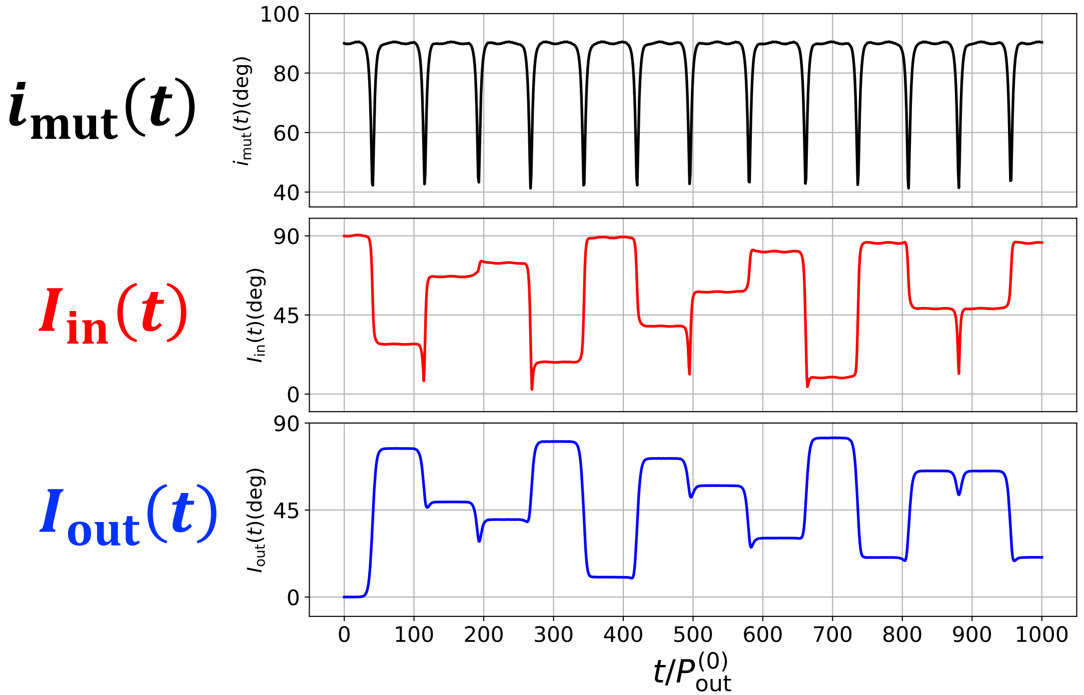
$$K_{\rm Kep} = K_0 \sin I_{\rm out}(t)$$



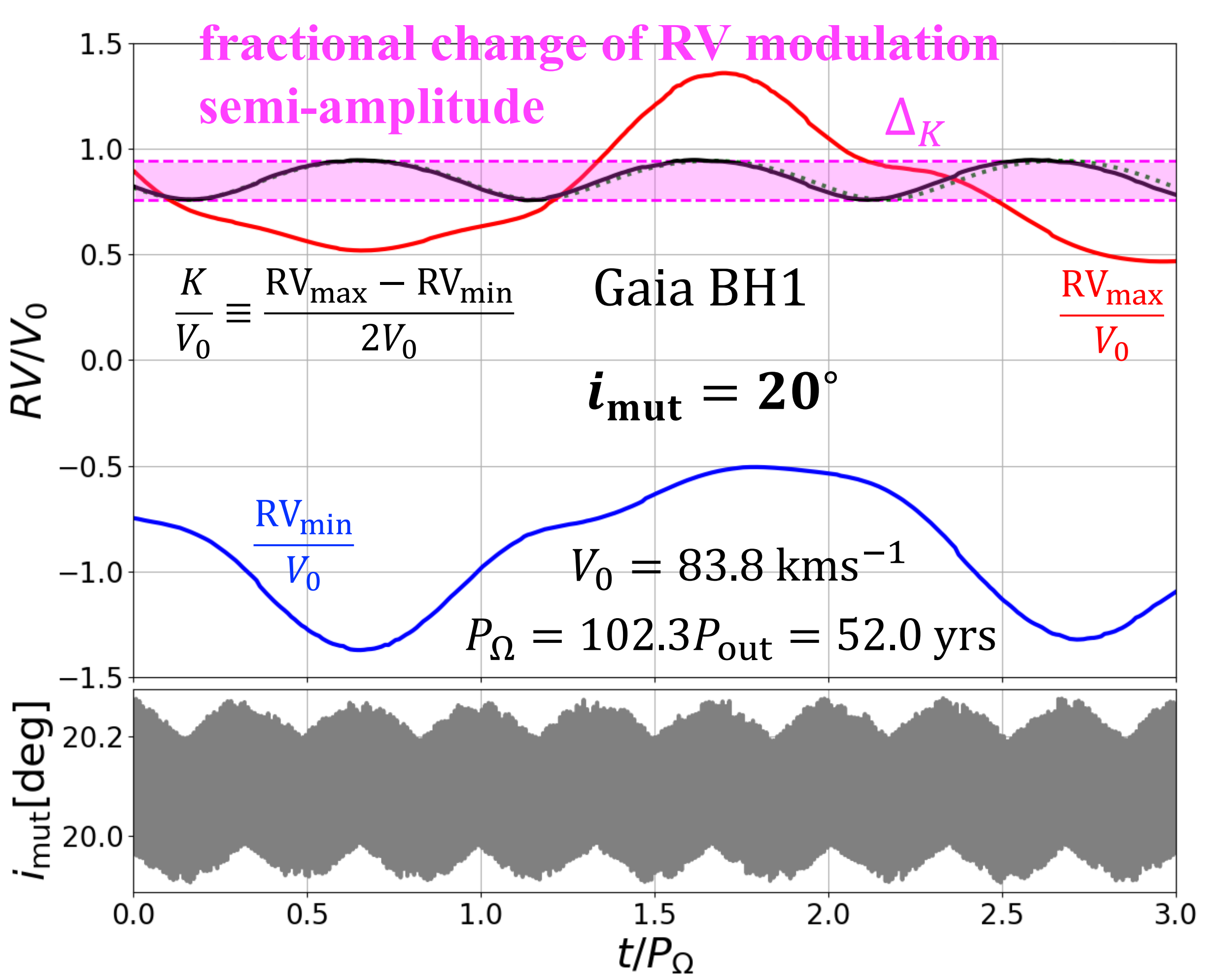
strong Kozai-Lidov oscillation

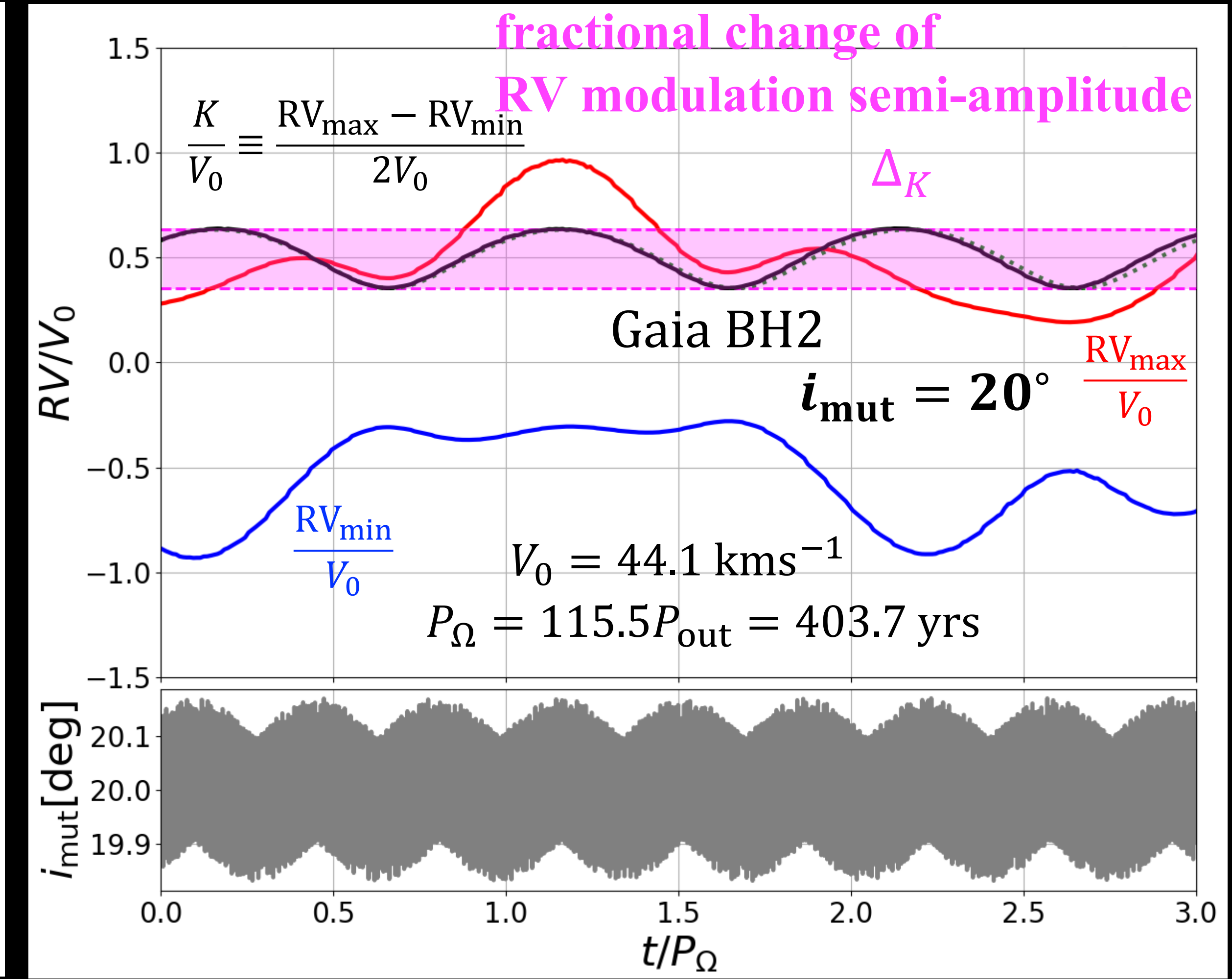
⇒ large-amplitude sporadic precession





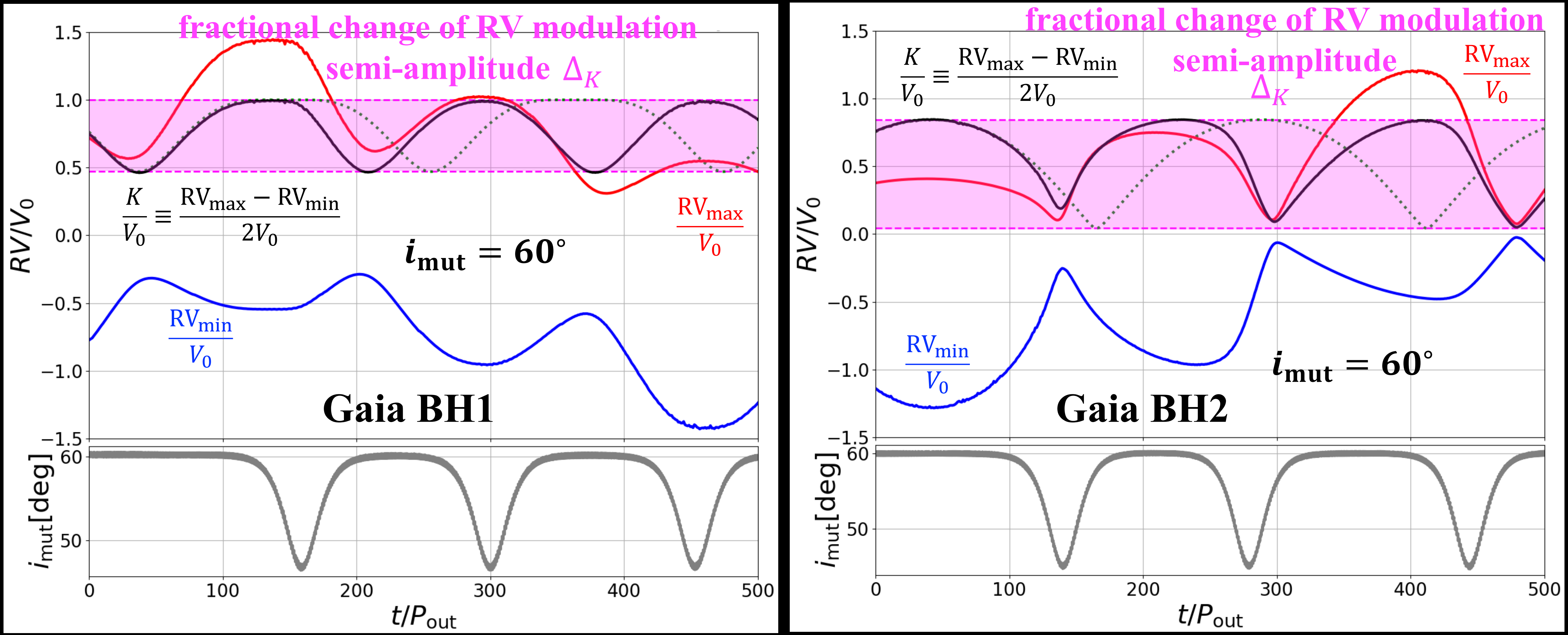
Long-term RV modulations due to nodal precession (i_{mut}=20°)





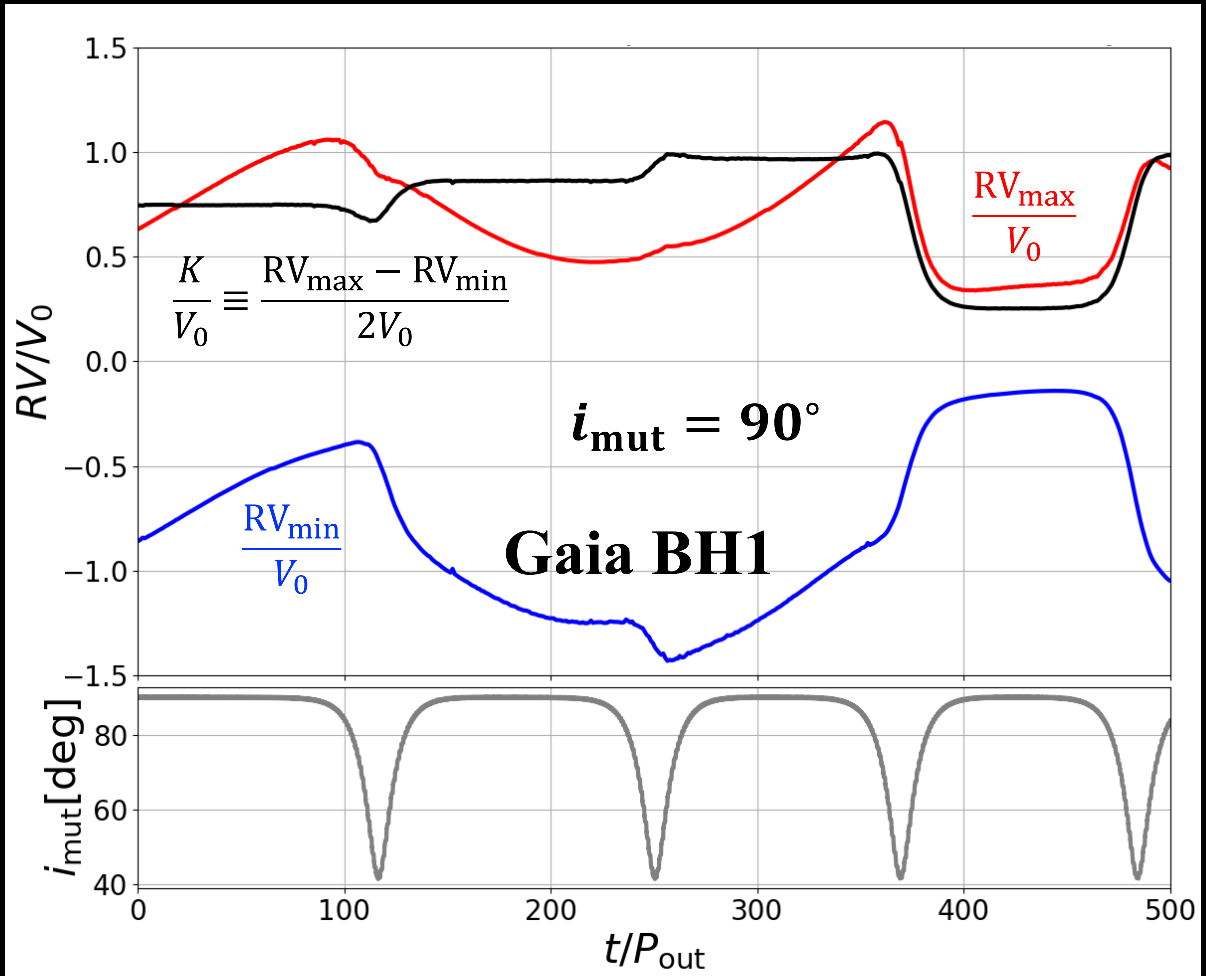
Hayashi, YS + Trani (2023)

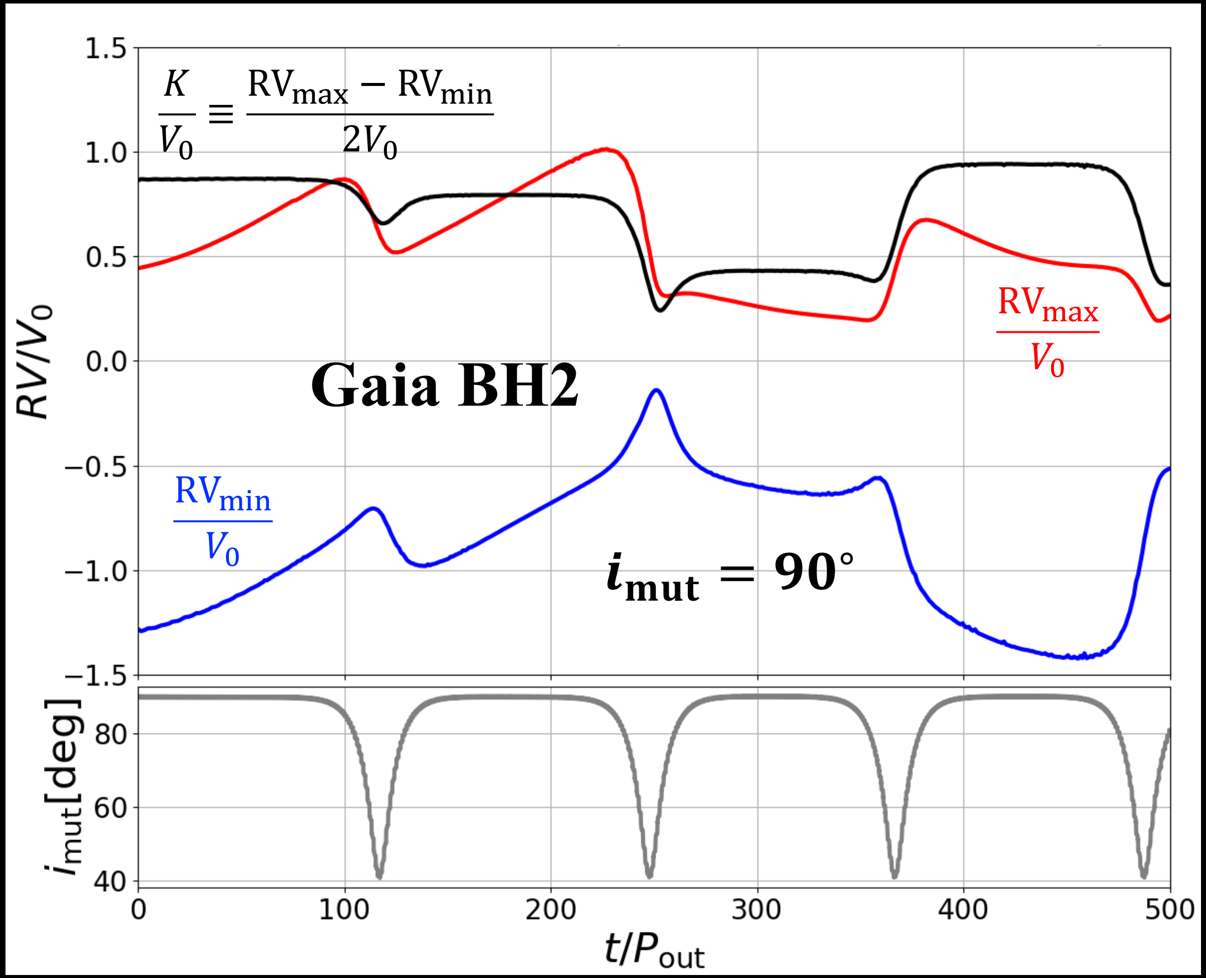
Long-term RV modulations due to moderate ZKL oscillations (i_{mut}=60°)



Hayashi, YS + Trani (2023)

Long-term RV modulations due to strong ZKL oscillations (i_{mut}=90°)





Hayashi, YS + Trani (2023)

5 Summary and outlook

Conclusion: RV signatures of inner binary black holes in triple systems may be detectable

- Radial velocity (RD) monitoring of star-black hole binary candidates may reveal inner binary black holes if exist at all
  - short-term RD variations Hayashi, Wang + YS: ApJ 890(2020)112
    - periodic modulations of O(0.1) percent of the Kepler orbital velocity amplitude with a half inner orbital period
  - long-term RD variations in inclined triples Hayashi + YS: ApJ 897(2020)29
    - the semi-amplitude of the Kepler orbital velocity modulated quasi-periodically by the nodal precession and/or the ZKL oscillations of the inner and outer orbits over O(100) years. detectable from astrometry as well.
  - A proof-of-concept study for Gaia BH1 and BH2 systems
    - may be even detectable for Gaia BH1! Hayashi, YS + Trani: arXiv:2307.01793

False positive? No, a nice discovery!

- *Suppose that Gaia BH1 exhibits a short-term RV modulation of O(10) m/s with period P_{short}*
  - Signal (=false positive for planet hunters)
    - the first discovery of a stellar-mass binary blackhole (orbital period of $2P_{short}$) in a triple system
  - False positive (=signal for planet hunters)
    - the first discovery of a planetary system (orbital period of P_{short}) orbiting a stellar-mass blackhole

see Morais & Correia (2008, 2012), Hayashi, Wang & YS (2020)

Everything not forbidden by the laws of nature is mandatory

Nobody's guaranteeing success.
But can you think of a more important question?
Imagine them out there sending us signals,
and nobody on Earth is listening.
That would be a joke, a travesty.
Wouldn't you be ashamed of your civilization
if we were able to listen
and didn't have the gumption to do it?

— Carl Sagan "Contact"