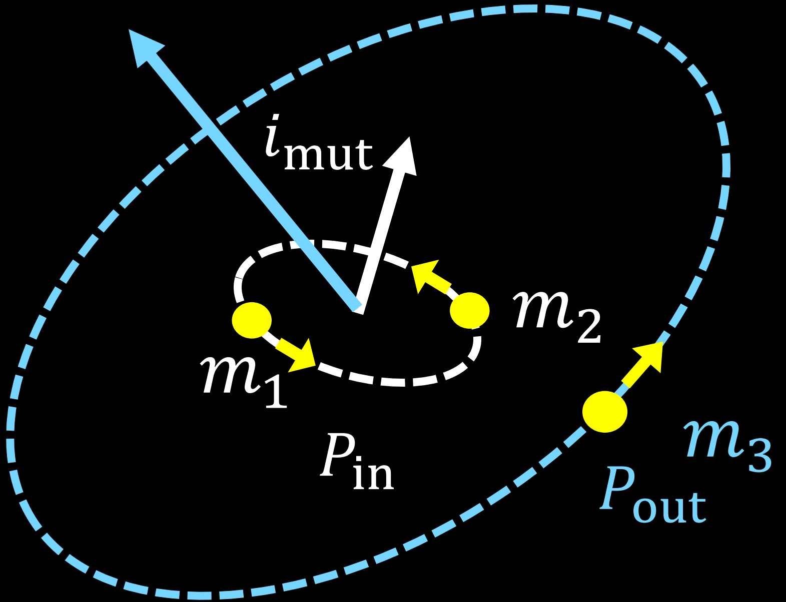
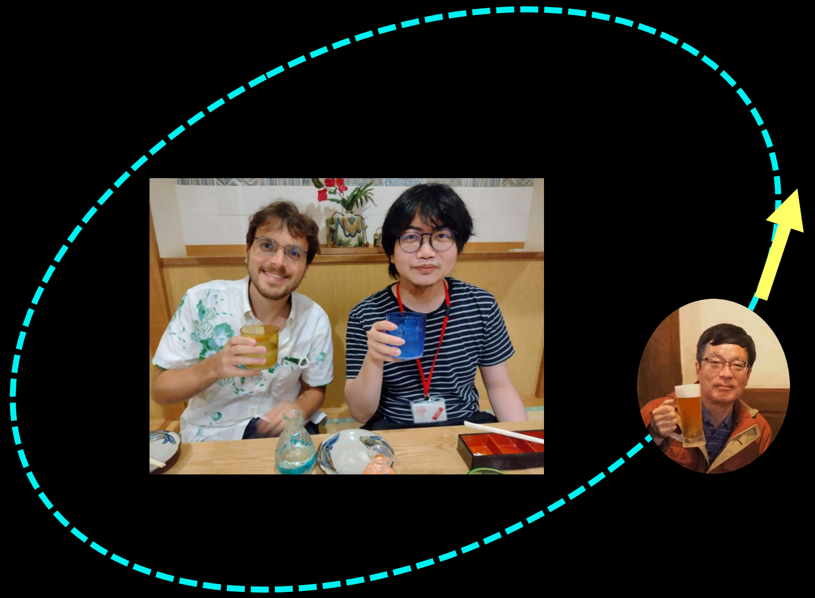
Lagrange and Liapunov stabilities of hierarchical triple systems





Hayashi, Trani & YS: arXiv:2207.12672 ApJ (2022), arXiv:2209.08487

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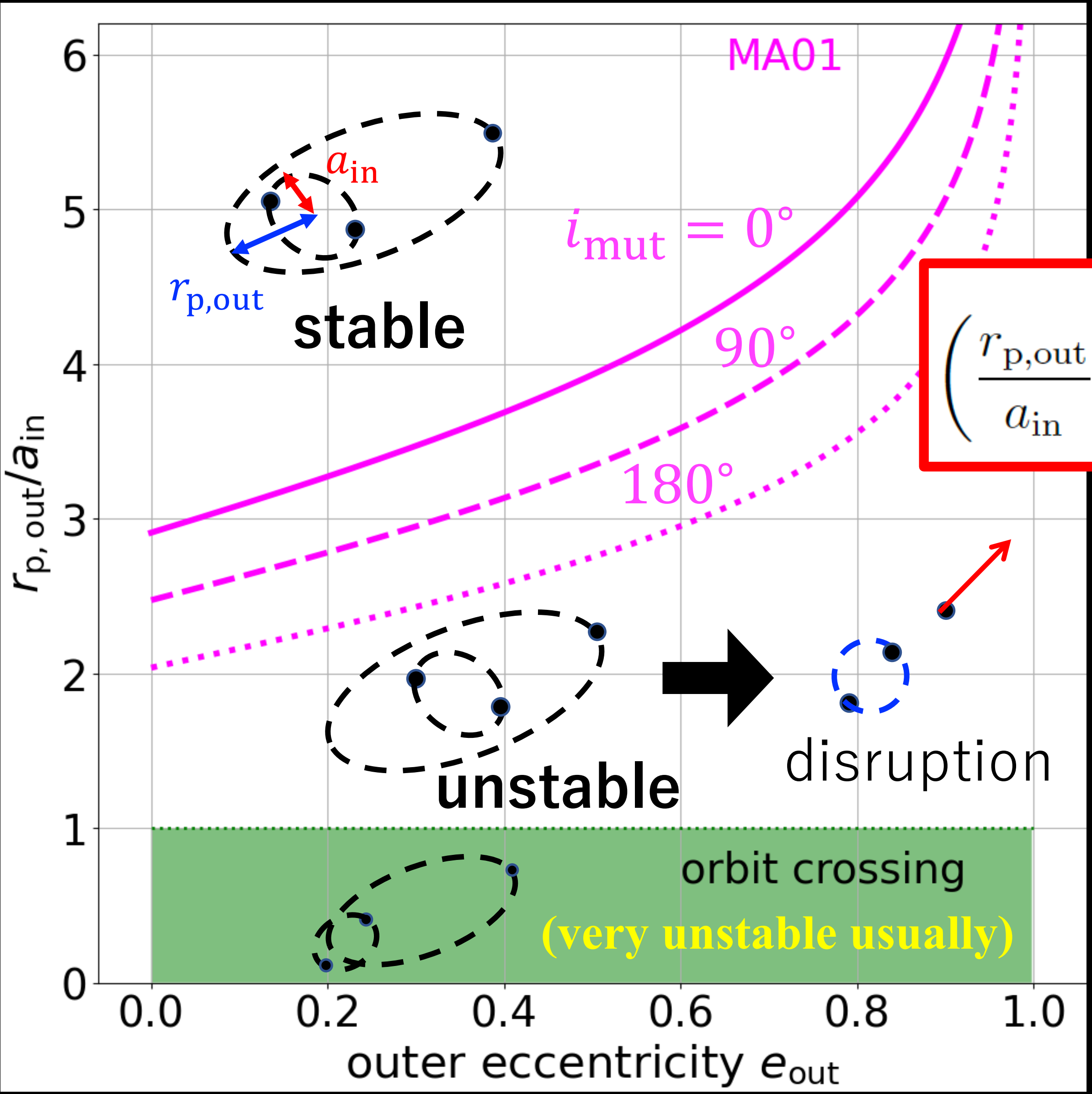
Ubiquity of hierarchical triples

- Stellar systems
  - more than 70% of OBA-type stars and 50% of FGK-type stars a belong to binary/multiple systems (e.g., Alpha Centauri)
- Planetary systems
  - Sun-earth-moon, planet around binary stars, multi-planets
- Black-hole systems
  - Possible pathway towards binary BHs
  - Binaries (stars, BHs) around a supermassive BH in galaxies
  - Triples of compact objects, e.g., pulsar-WD binary + tertiary WD (Ransom et al. 2014)

Triples are unstable in general ⇔ diversity

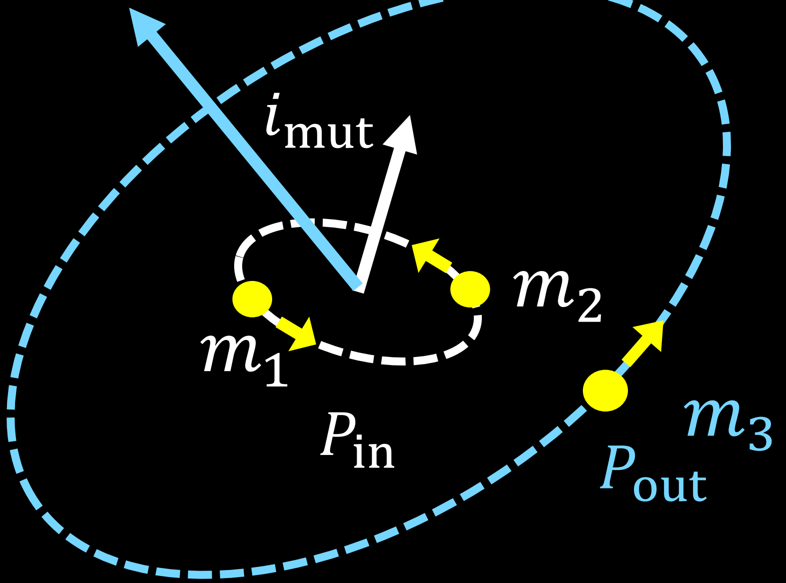


- **Stability criterion (Mardling & Aarseth 2001)**

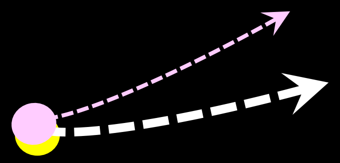
$$\left(\frac{r_{\rm p,out}}{a_{\rm in}}\right)_{\rm MA} \equiv 2.8\left(1-0.3\frac{i_{\rm mut}}{\pi}\right)\left[\left(1+\frac{m_3}{m_{12}}\right)\frac{(1+e_{\rm out})}{\sqrt{1-e_{\rm out}}}\right]^{2/5}$$

- **Well-known and widely used, but its implication is often misinterpreted...**
- **What does it mean?**
- **Liapunov (chaoticity of local trajectory) vs. Lagrange (escape of a body from the system) stability**

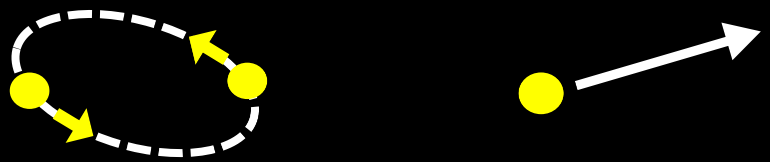
Liapunov vs. Lagrange (in)stability



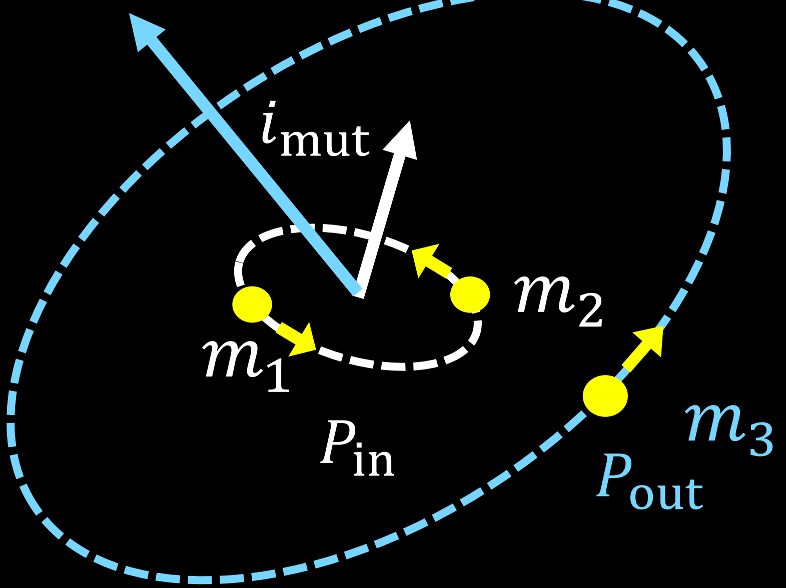
- **Liapunov instability**
  - **Local divergence of trajectories of bodies (chaoticity)**



- **Lagrange instability**
  - **global escape of a body from the triple system (boundedness of an orbit)**



Longer-term 3-body simulations in Newtonian dynamics (neglecting GR effects)



$e_{\rm in} = 10^{-5}$(circular)

N-body code TSUNAMI (Trani and Spera)

$\frac{r_{\rm p,out}}{a_{\rm in}}$ $e_{\rm out}$ ⇒

Integration time up to $10^9 P_{\rm in}$ (roughly $10^{6\text{-}7}$ $P_{\rm out}$)

Disruption time

$$\frac{T_{\rm d}}{P_{\rm in}}(q_{21}, q_{23}, i_{\rm mut}, \cdots)$$

$i_{\rm mut}$: $0°, 90°, 180°$ Coplanar: prograde, retrograde
Orthogonal

$q_{21} \equiv m_2/m_1 (\leq 1)$ Mass ratio (inner binary)

$q_{23} \equiv m_2/m_3$ Mass ratio (tertiary)

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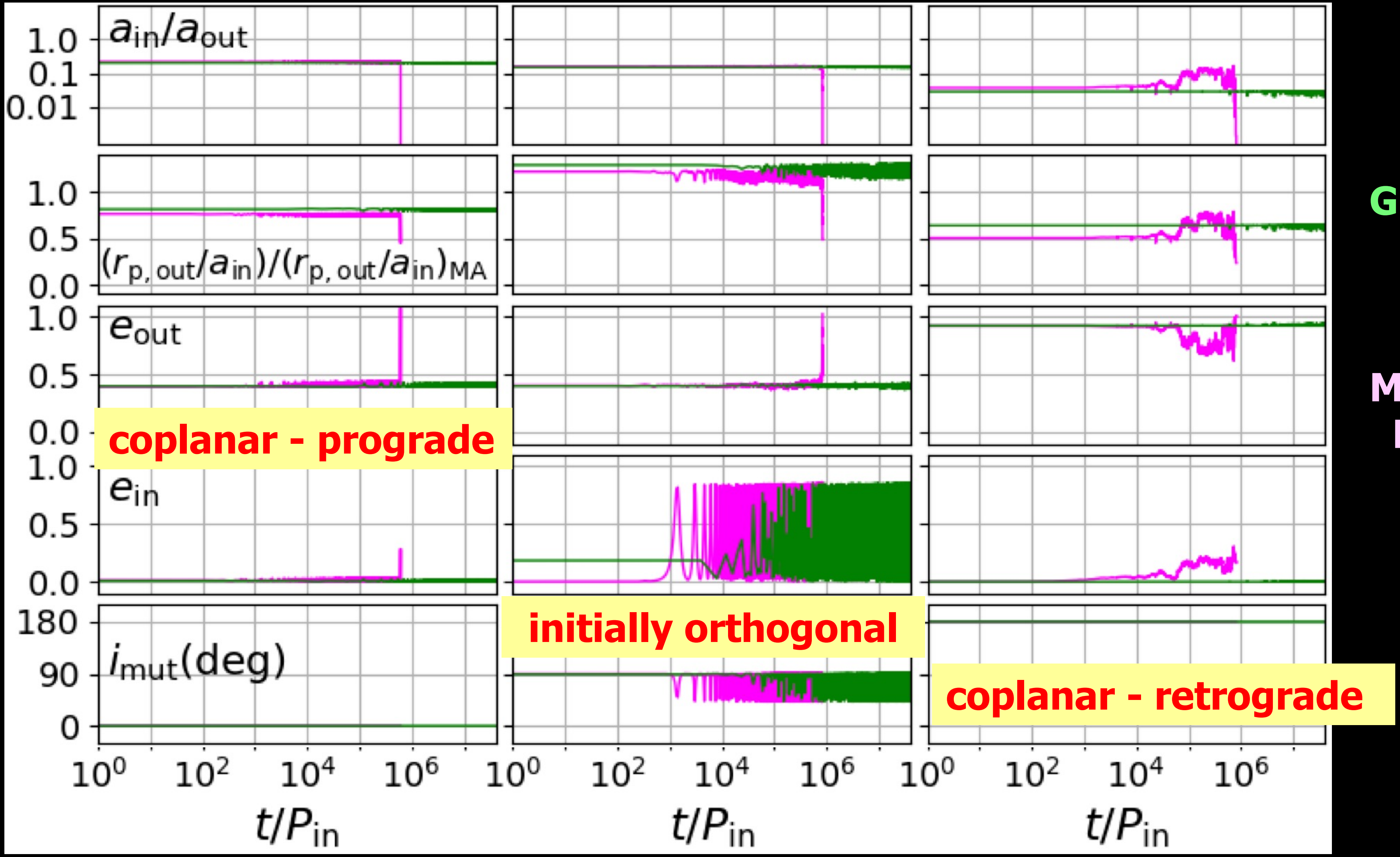
Examples of orbital evolution of triples



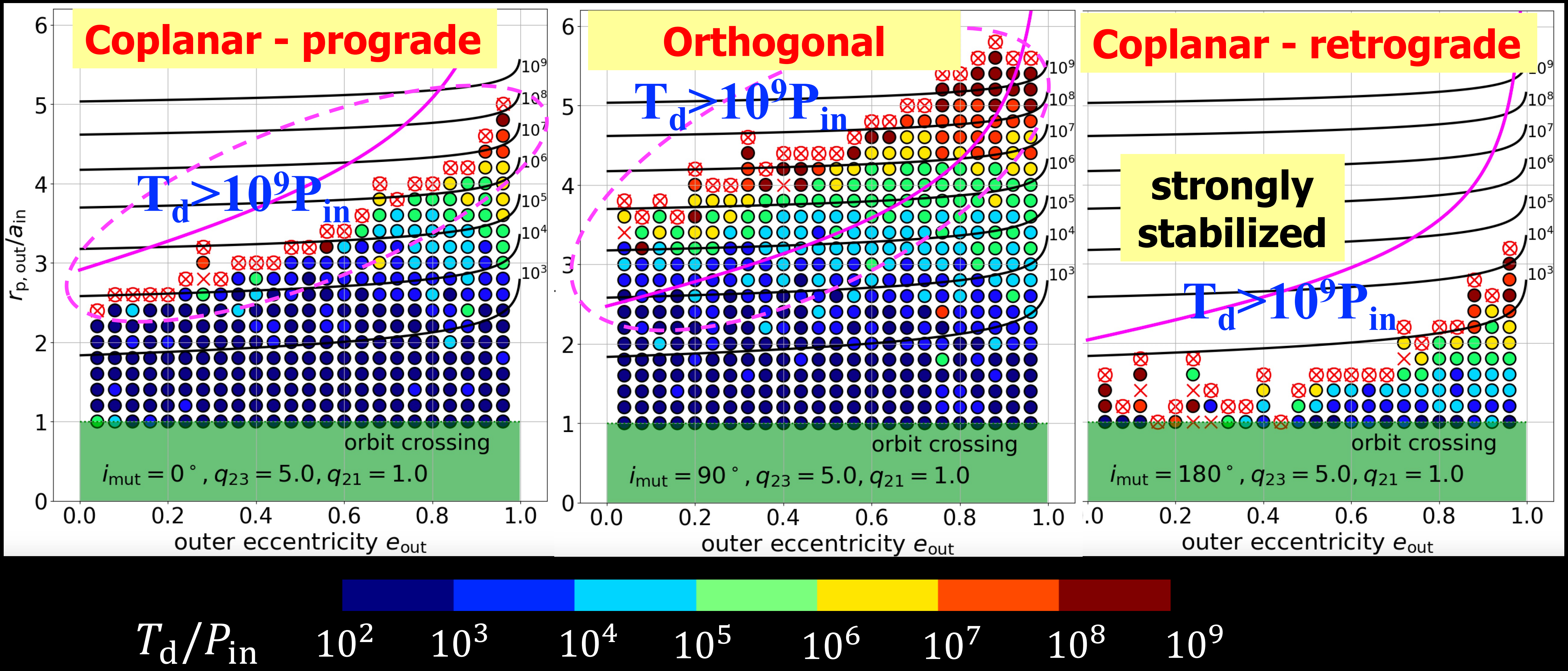
Green:
Lagrange stable up to $t=10^8$ P_{in}

Magenta:
Lagrange unstable around $t=10^6$ P_{in}

Disruption timescales on $a_{out}(1-e_{out})/a_{in} - e_{out}$ plane

$m_1=m_2=5m_3$

Stability boundary by Mardling & Aarseth (2001)

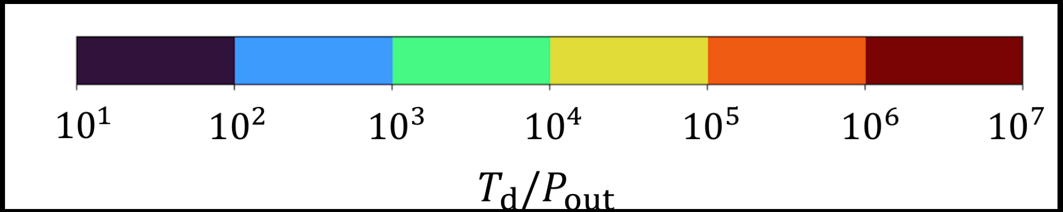


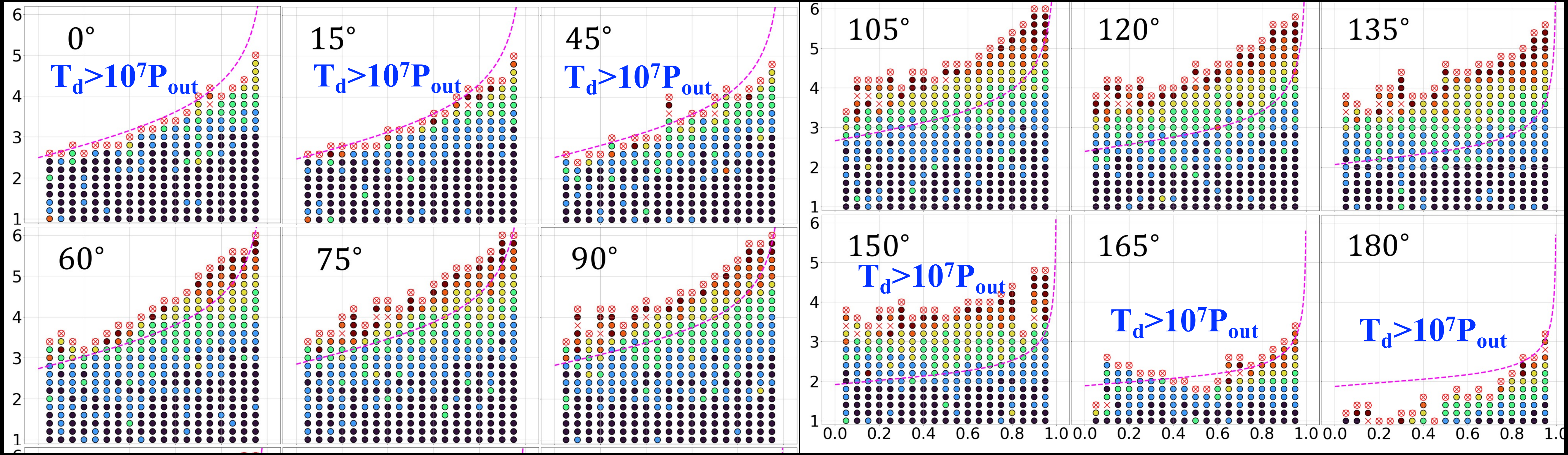
Inclination dependence

T_d/P_{out} on $a_{out}(1\text{-}e_{out})/a_{in}$ – e_{out} plane

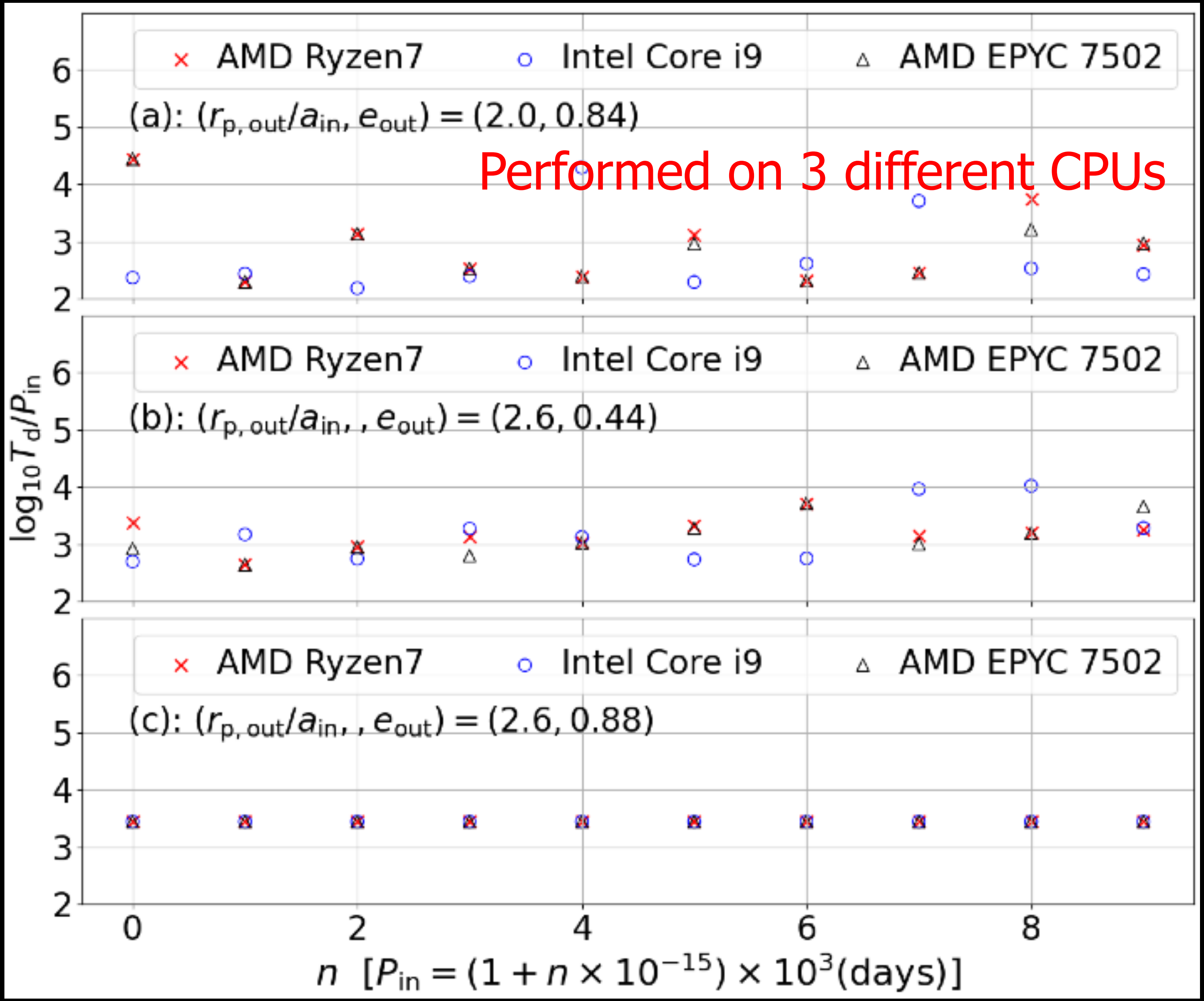
- $60 < i_{mut} < 150$
  - destabilized by the Kozai-Lidov oscillations
- $I_{mut} > 160$
  - significantly stabilized due to inefficient energy transfer between inner and outer orbits

Very different from the stability boundary by Mardling & Aarseth (2001) -----------



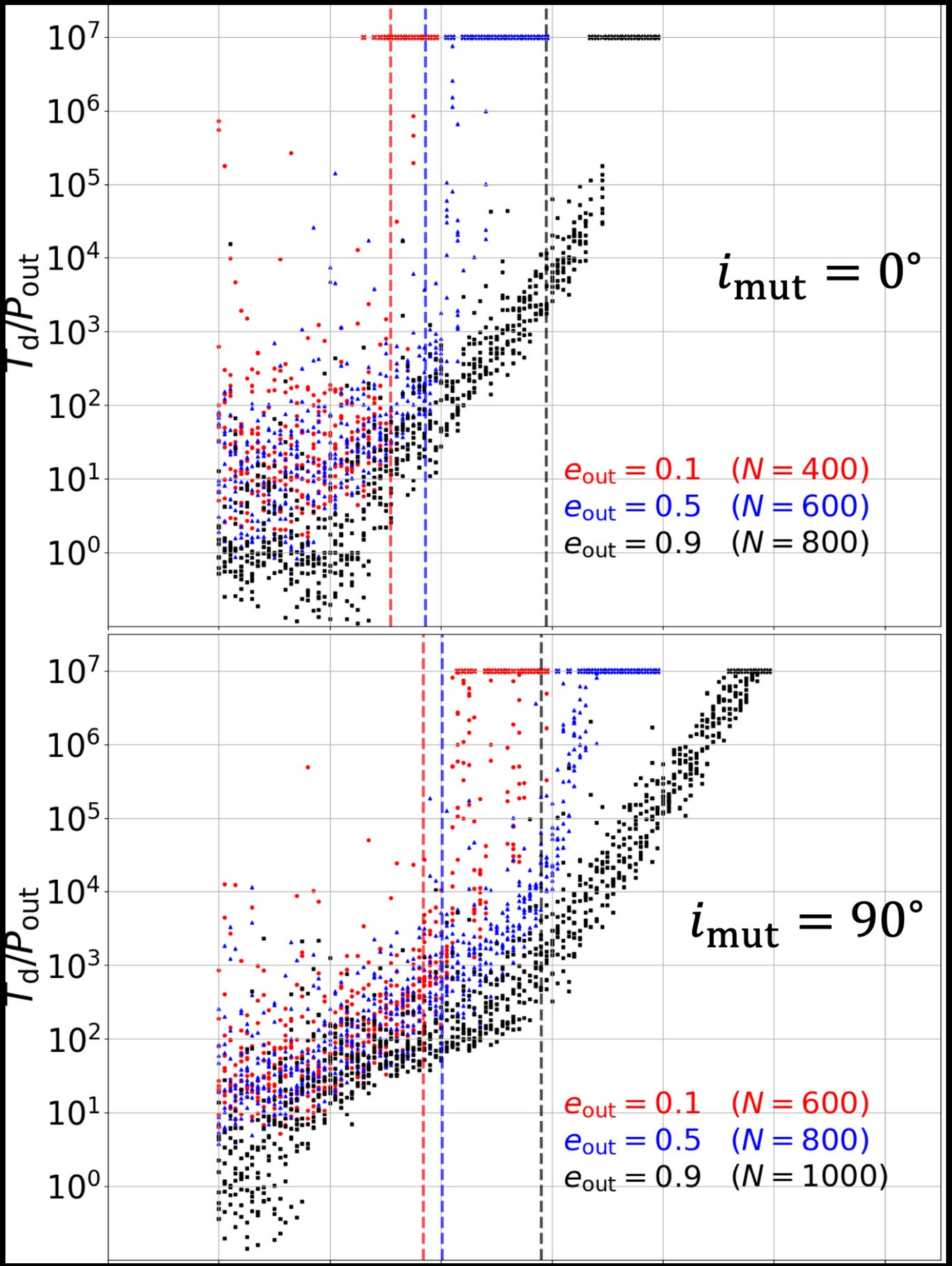


Chaotic nature of disruption timescale distribution

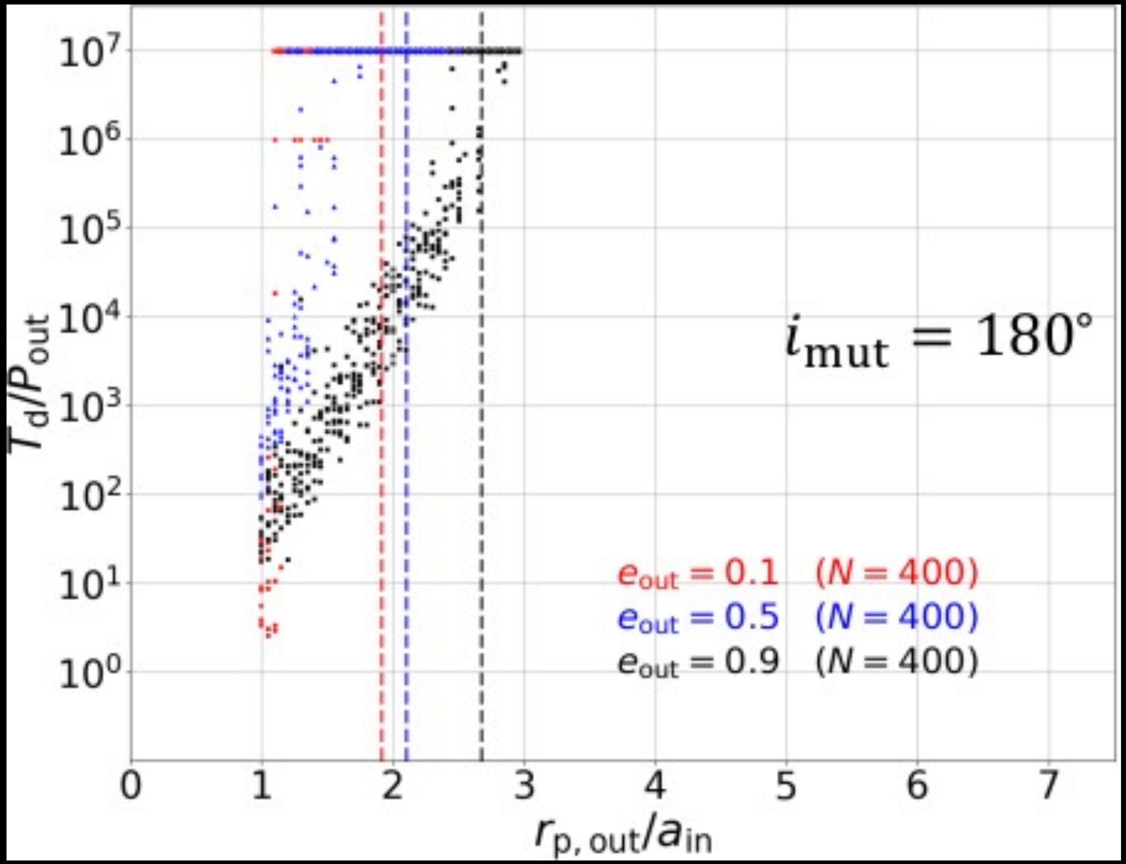


- Tiny difference in the input value of P_{in} leads to one or two order-of-magnitude difference of disruption timescales
- Initial phase difference of the three bodies also leads to one or two order-of-magnitude difference of disruption timescales

T_d/P_{out} ($r_{p,out}/a_{in}$; i_{mut}, e_{out}) with random initial phases



- Liapunov stability boundaries (Vynatheya+2022) are plotted in dashed lines for reference



Conclusions

- Lagrange vs. Liapunov stability for triple systems
  - Conventional criteria correspond to Liapunov stability
  - Lagrange stability is more relevant in considering the fate of astronomical triples, i.e., disruption timescale
- We derive triple disruption timescales as a function of orbital parameters (intrinsic variation by one or two order-of-magnitudes due to the chaotic dynamics of triples)
- Strong dependence on the mutual inclination
  - Strongly misaligned systems ($60<i_{mut}<150$) are destabilized due to the Kozai-Lidov oscillations over longer timescales
  - Coplanar retrograde triples are significantly stabilized