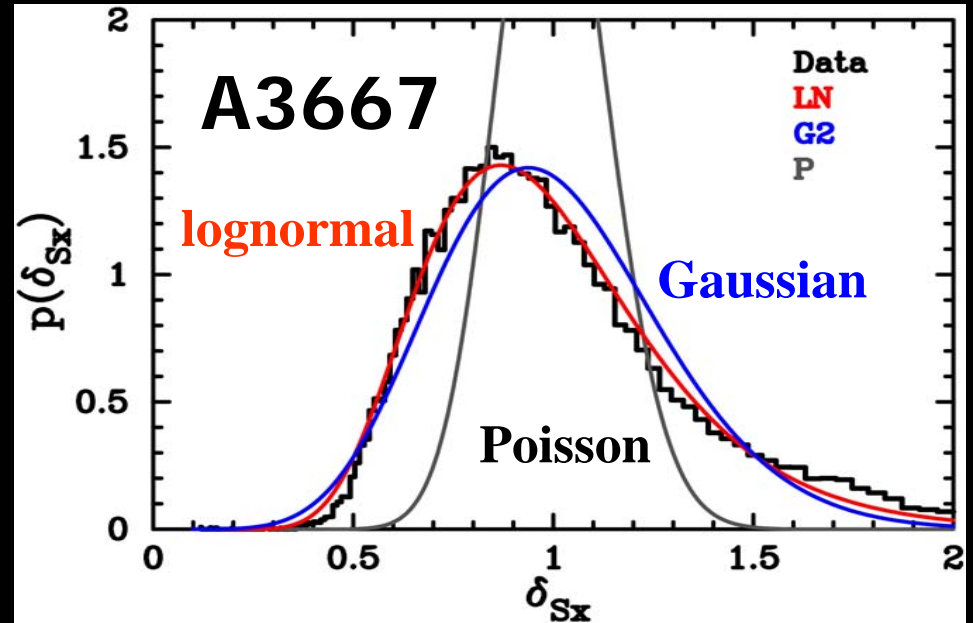
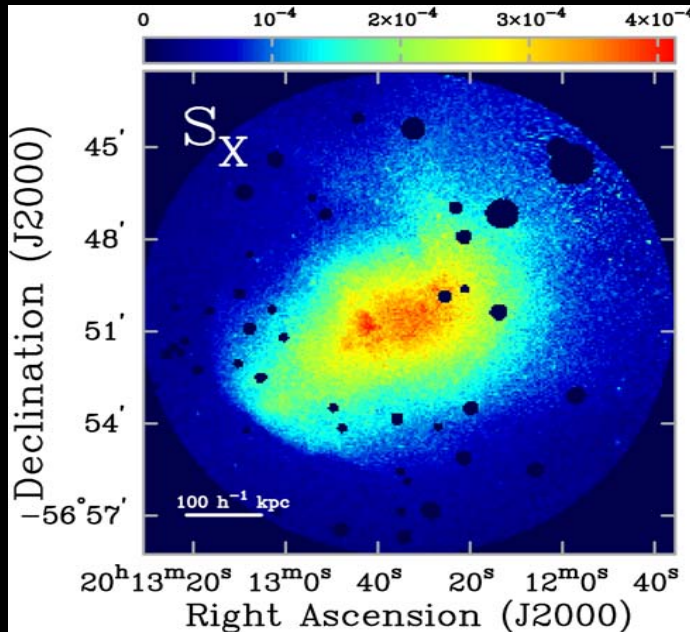


# Inhomogeneities in Galaxy Clusters



**Yasushi Suto** *Department of Physics, The University of Tokyo*

**Large Scale Structure and Galaxy Formation with LAMOST**

**Kavli Institute for Astronomy and Astrophysics at Peking University**

**北京大学科维理天文与天体物理研究所 April 13-17, 2009**

# What are galaxy clusters ?

## Abell (optical) clusters

the Abell radius

$$m_3 < m < m_3 + 2$$

richness class

## Press-Schechter halos

spherical collapse

$$\Delta_{\text{vir}} = 18 \pi^2$$

## *SZ clusters*

$$\Delta I_{\text{SZ}} \propto n_e T_e$$

## Halos in N-body simulations

friend-of-friend

linking length = 0.2

## *X-ray clusters*

$$S_x \propto n_e^2 T_e^{1/2}$$

Definitely they are closely related, but the exact one-to-one correspondence is unrealistic

# Fundamental limitation in cosmology with galaxy clusters

## ■ Cosmological parameters

- Observed cluster abundance as a function of  $T$  and  $z$
- Predicted halo abundance as a function of  $M$  and  $z$

## ■ $M_{\text{halo}} - T_{\text{cluster}}$ relation

- $M_{\text{halo}}$ : size of cluster ? Non-sphericity ?
- $T_{\text{cluster}}$ : non-isothermal, inhomogeneity in intra-cluster medium ?

# Toward a more realistic model of galaxy clusters

- **beyond a spherical homogeneous isothermal  $\beta$ -model**
  - non-sphericity (i.e., triaxial)
  - temperature profile (i.e., polytrope)
- *inhomogeneity in intra-cluster medium*
  - numerical simulations
  - empirical/analytic modeling
  - comparison with observations

# References

- **Hajime Kawahara, YS, Kitayama, Sasaki, Shimizu, Rasia + Dolag (2007)**
  - *Radial Profile and Lognormal Fluctuations of the Intracluster Medium as the Origin of Systematic Bias in Spectroscopic Temperature* **ApJ 659(2007)257**
- **Kawahara, Kitayama, Sasaki + YS (2008)**
  - *Systematic Errors in the Hubble Constant Measurement from the Sunyaev-Zel'dovich effect* **ApJ 674(2008)11**
- **Kawahara, Reese, Kitayama, Sasaki + YS (2008)**
  - *Extracting galaxy cluster gas inhomogeneity from X-ray surface brightness: a statistical approach and application to Abell 3667* **ApJ 687(2008) 936**

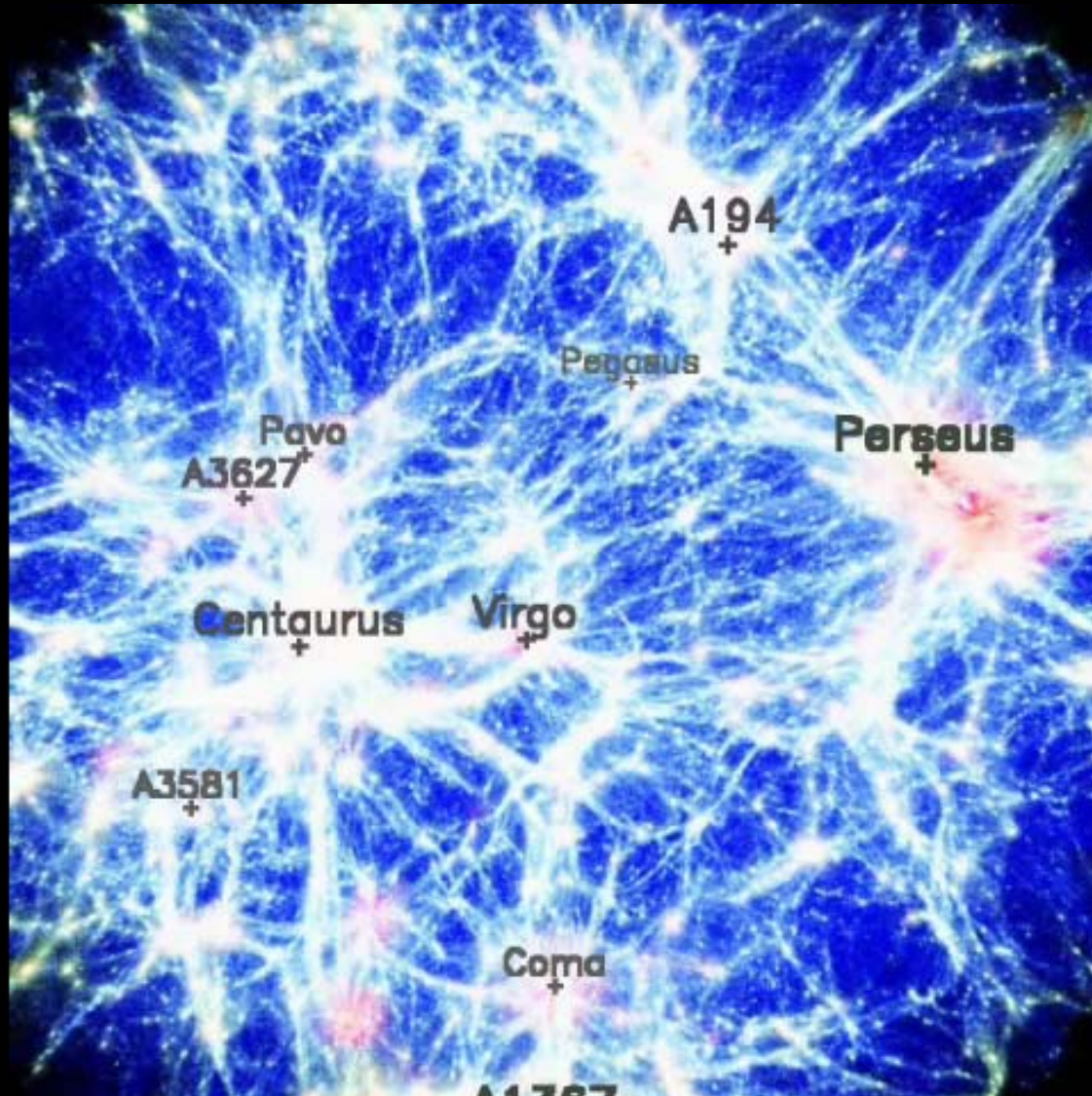
# Simulated clusters in the local universe

- **SPH simulations** (Dolag et al. 2005)
- **Local universe distribution** in a sphere of  $r=110\text{Mpc}$ 
  - Initial condition: the observed galaxy density field of IRAS 1.2 Jy survey (smoothing over  $5h^{-1}\text{Mpc}$ ), linearly evolving back to  $z=50$  + random Gaussian fluctuations on smaller scales
  - + cooling, star formation, SN feedback, and metallicity evolution in  $\Lambda\text{CDM}$

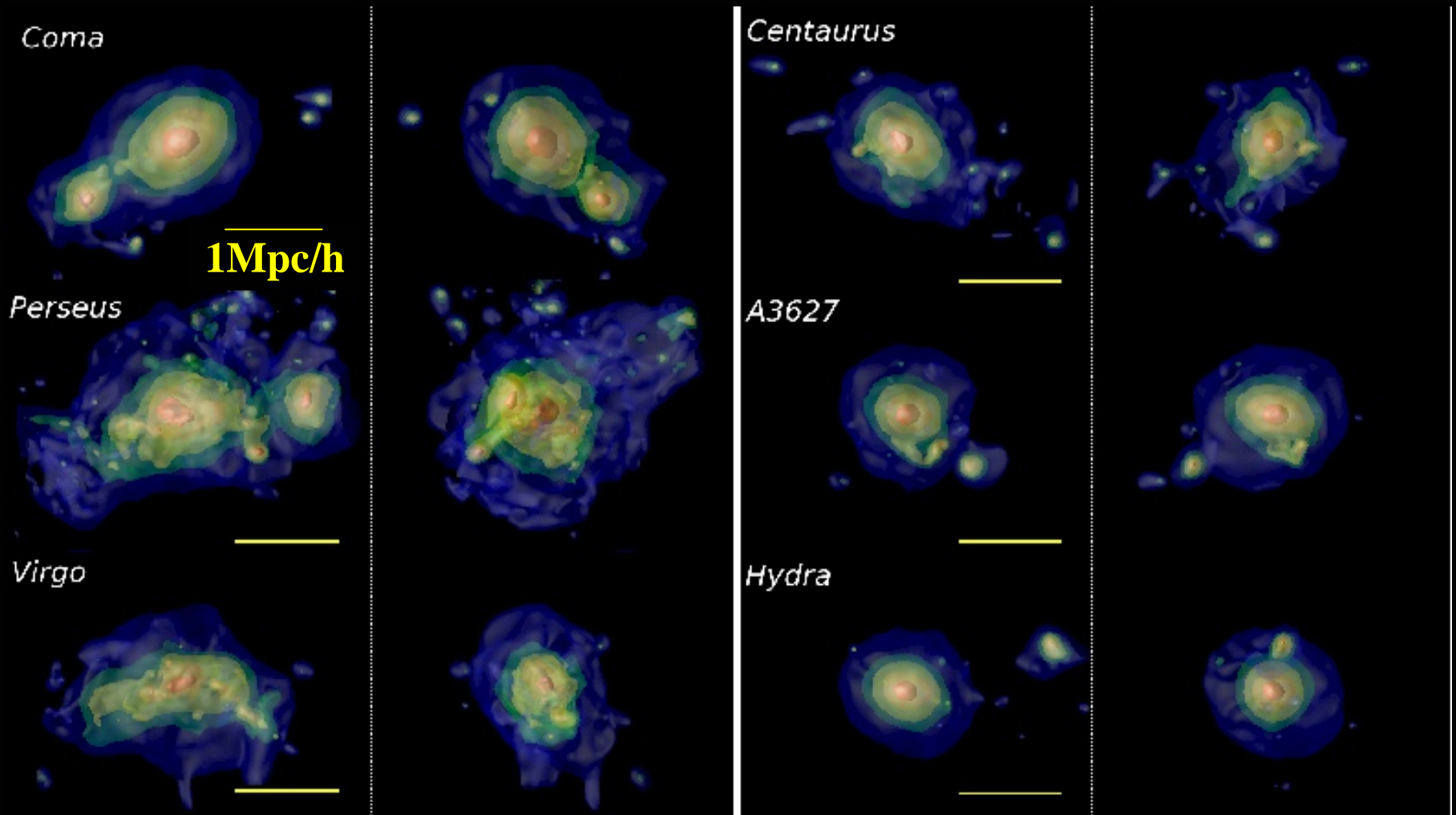


# Simulated local universe

Dolag et al. (2005)



# Projected views of *simulated clusters*





# modeling ICM fluctuations (1)

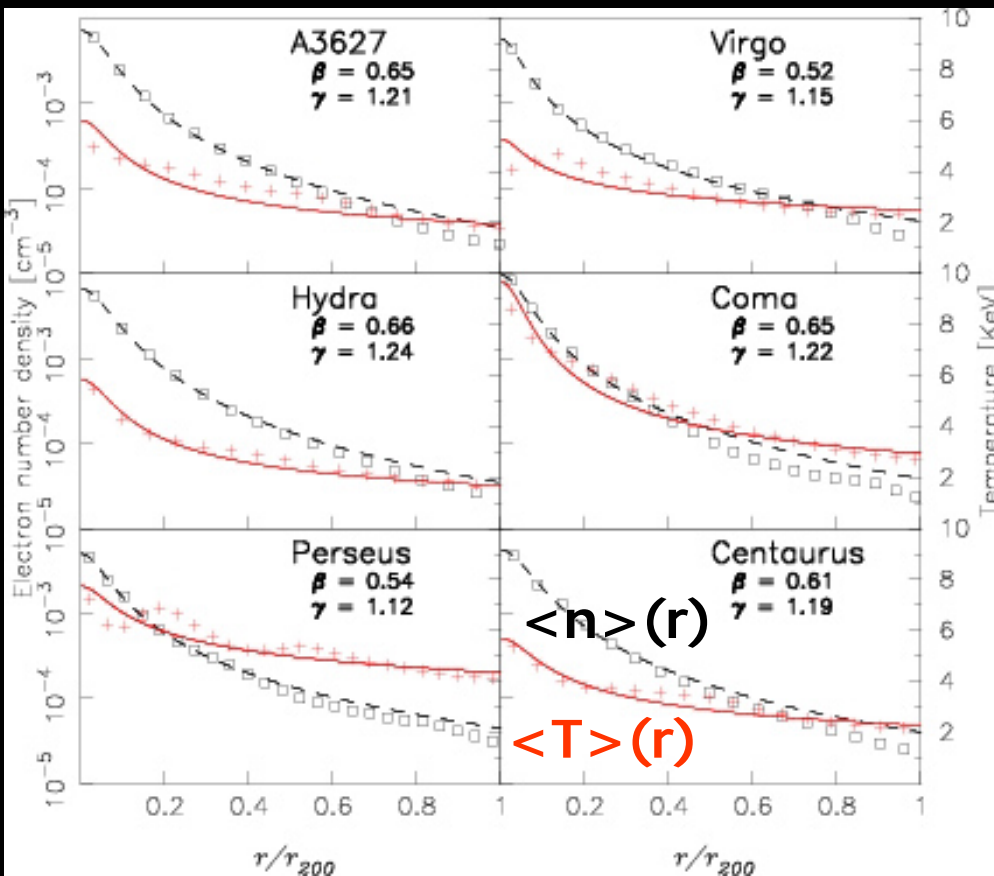
## mean (spherical) radial profile

- Density and temperature radial profiles of simulated clusters

- **Polytropic  $\beta$  model**

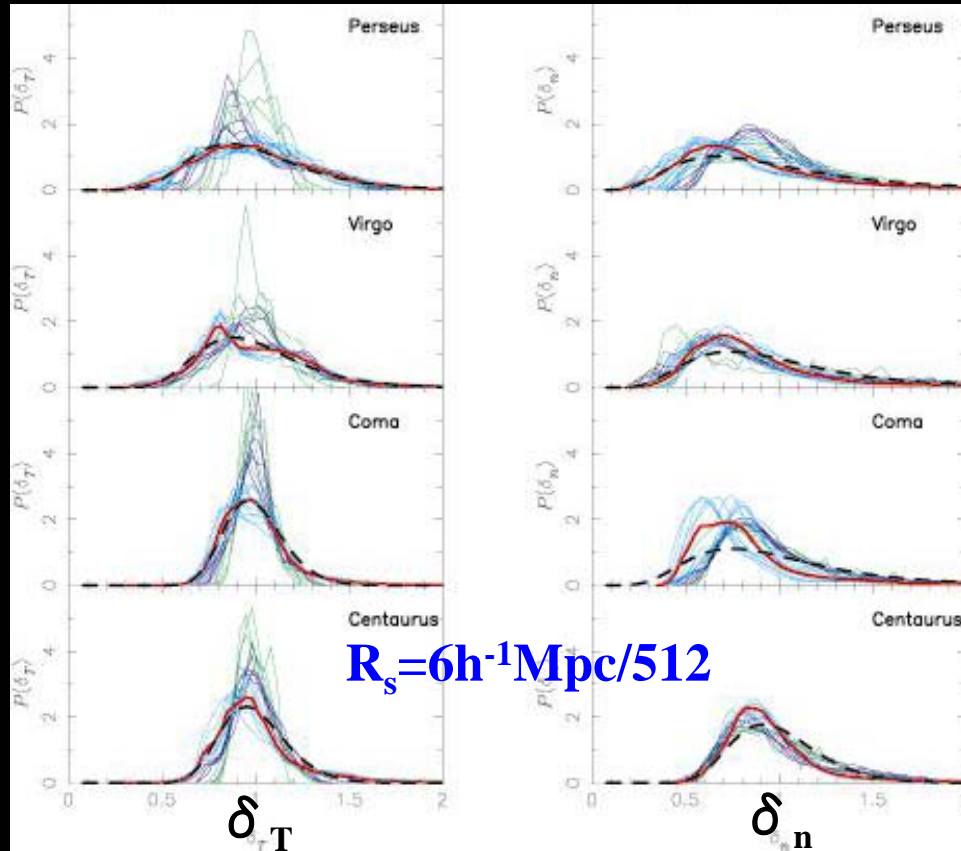
$$\langle n \rangle (r) = n_0 \left[ \frac{1}{1 + (r/r_c)^2} \right]^{3\beta/2}$$

$$\langle T \rangle (r) = T_0 [\langle n \rangle (r) / n_0]^{\gamma-1}$$



# modeling ICM fluctuations (2)

## log-normal PDF for local inhomogeneity



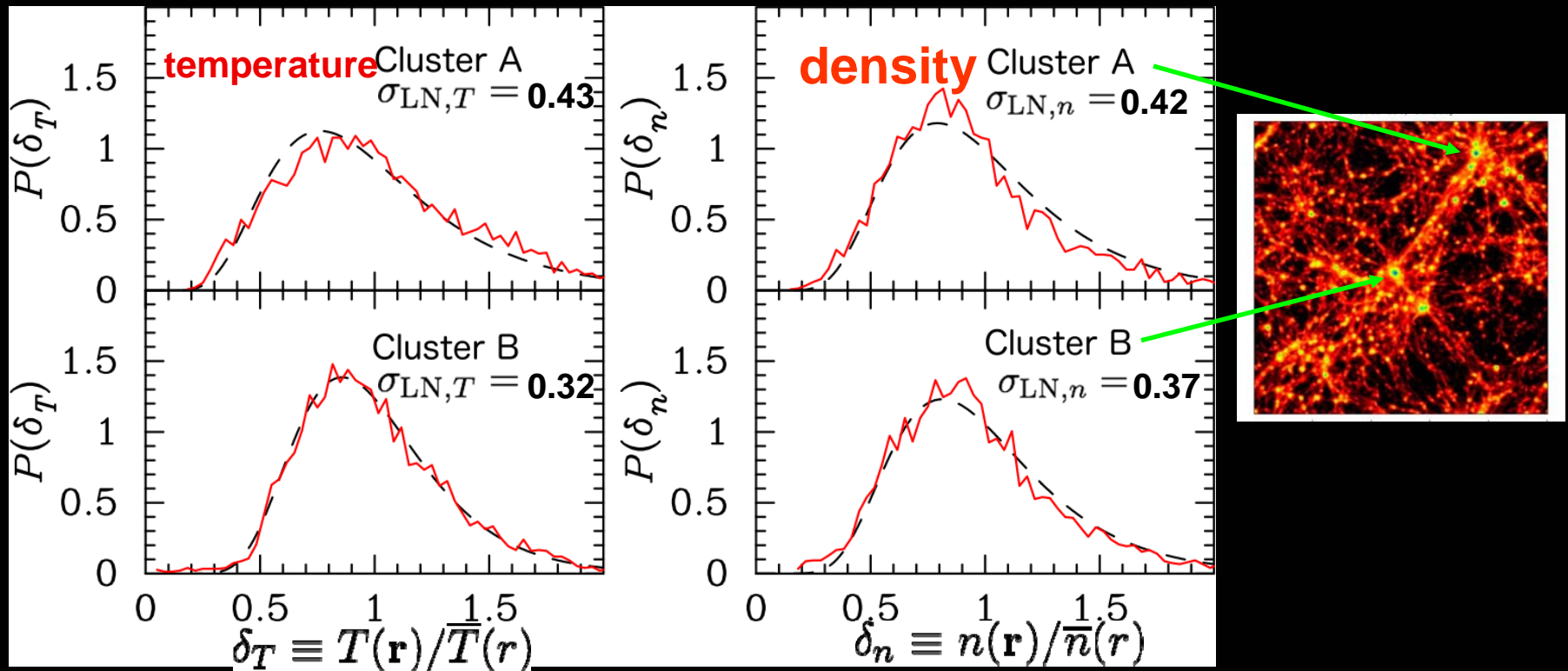
- Local inhomogeneities of density and temperature of simulated clusters

- $\delta_n = n(r, \theta, \phi) / \langle n \rangle(r)$
  - $\delta_T = T(r, \theta, \phi) / \langle T \rangle(r)$

- Log-normal PDF provides reasonable approximations

$$P_{LN}(\delta)d\delta = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(\log \delta + \sigma^2/2)^2}{2\sigma^2}\right] \frac{d\delta}{\delta}$$

# Both SPH and mesh simulations show log-normal distribution



Mesh hydro simulation data  
(D.Ryu private communication)

# Implications

- ① underestimate bias for **X-ray spectroscopic temperature of clusters**
- ② underestimate bias for  $H_0$  estimate from SZ observations
- ③ reconstruction of the 3D ICM fluctuation distribution from X-ray surface brightness (2D) observational data

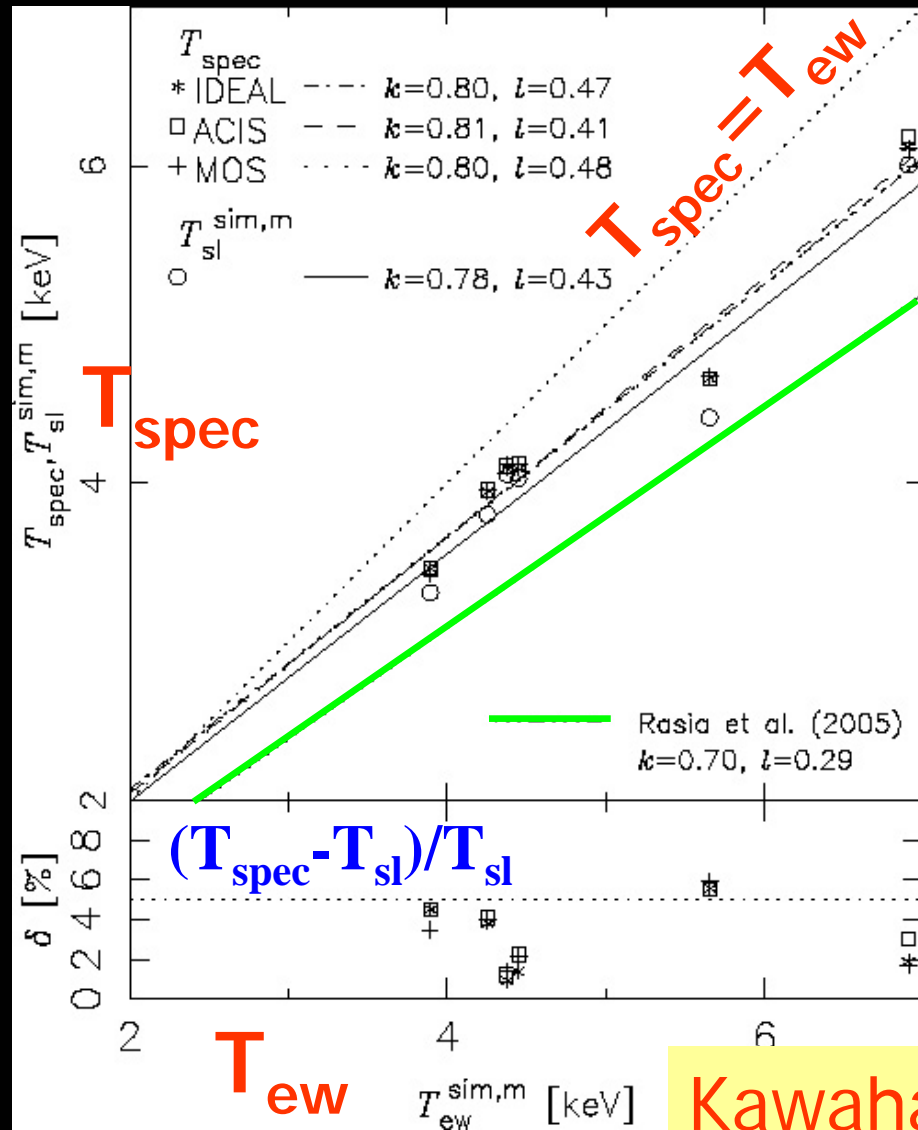
# mass-weighted, emission-weighted, and spectroscopic temperatures of clusters

$$\langle T \rangle_w = \frac{\int T W dV}{\int W dV}$$

Except for idealistic isothermal case, cluster temperature is ill-defined

	definition	W (weight)	
$T_m$	mass-weighted	$n$	
$T_{ew}$	emission-weighted	$n^2 \Lambda(T)$	simulation
$T_{spec}$	spectroscopic	spectral fit	observation
$T_{sl}$	spectroscopic-like	$n^2 T^{-0.75}$	Mazzotta et al. (2004)

# $T_{\text{spec}}$ is systematically smaller than $T_{\text{ew}}$



- Mazzotta et al. (2004) & Rasia et al. (2005) found  $T_{\text{spec}} \sim 0.7 T_{\text{ew}}$  from simulations

- Simulated clusters of Dolag et al. (2005)

$$T_{\text{spec}} \sim 0.85 T_{\text{ew}}$$

(see also Mathiesen & Evrard 2001)

Kawahara et al. ApJ 659 (2007)257



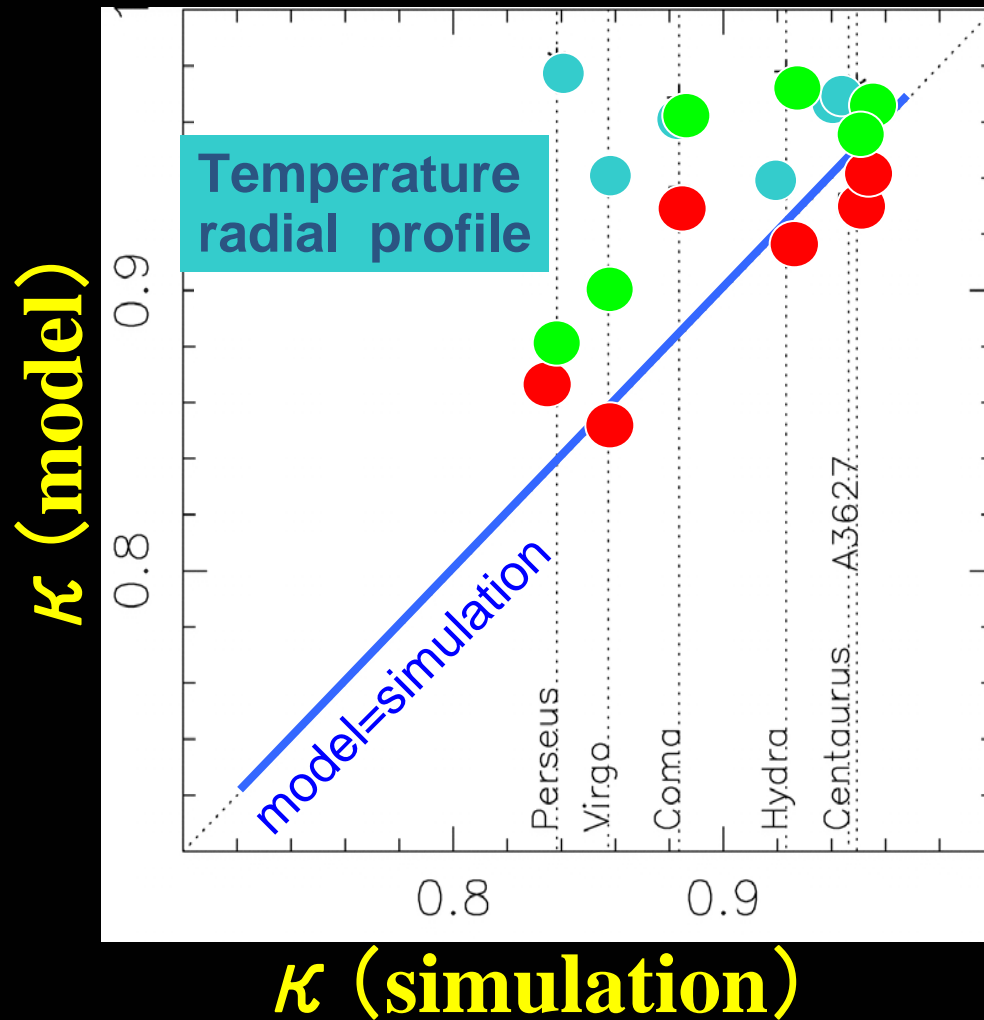
# An analytic model for $T_{\text{spec}}/T_{\text{ew}}$

- Spherical polytropic  $\beta$ -model as global mean radial profiles
- Log-normal density and temperature fluctuations
  - Density and temperature correlations may be ignored
  - Radius independent  $\sigma$  assumed

$$\frac{T_{\text{sl}}}{T_{\text{ew}}} = \frac{T_{\text{sl}}^{\text{RP}}}{T_{\text{ew}}^{\text{RP}}} \exp( -1.25 \sigma_{\text{LN},T}^2 )$$

- Explain numerical simulations well

# Comparison between analytic models and simulations



$$\kappa_{\text{model}} \equiv \frac{T_{\text{sl}}^{\text{model}}}{T_{\text{ew}}^{\text{model}}} \equiv \kappa^{\text{RP}} \kappa^{\text{LI}}$$

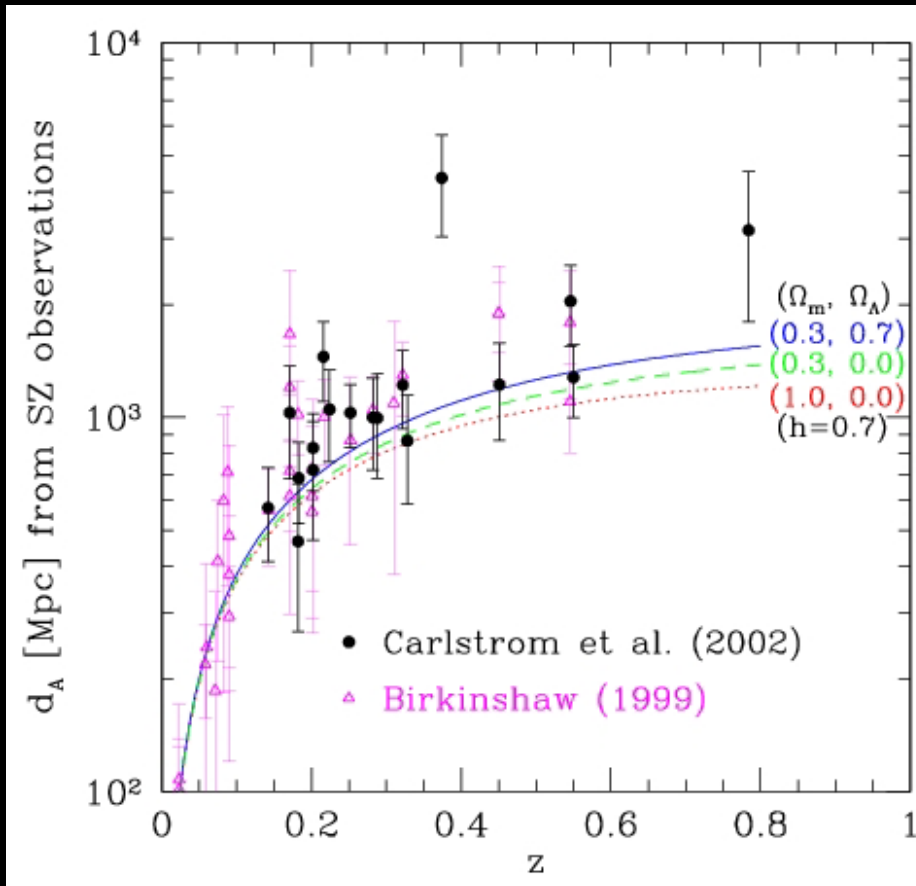
$$= \frac{T_{\text{sl}}^{\text{RP}}}{T_{\text{ew}}^{\text{RP}}} \exp(-1.25\sigma_{\text{LN},T}^2)$$

Kawahara et al.  
ApJ 659 (2007)257

# Implications

- ① underestimate bias for X-ray spectroscopic temperature of clusters
- ② **underestimate bias for  $H_0$   
estimate from SZ observations**
- ③ reconstruction of the 3D ICM fluctuation distribution from X-ray surface brightness (2D) observational data

# $H_0$ estimated from the SZ effect



- WMAP:
  - $73 \pm 3$  km/s/Mpc  
(Spergel et al. 07)
- ROSAT+SZ:
  - $60 \pm 3$  km/s/Mpc  
(Reese et al. 2002)

■ *Overlooked systematic effects ?*

# H<sub>0</sub> from isothermal $\beta$ -model fit

- Polytropic density and temperature profiles

$$\langle n \rangle (r) = n_0 \left[ \frac{1}{1 + (r/r_c)^2} \right]^{3\beta/2}$$

$$\langle T \rangle (r) = T_0 [\langle n \rangle (r) / n_0]^{\gamma-1}$$

- core radius estimated from X-ray + SZ

$$r_{c,iso\beta}(T_{spec}) = \frac{y(0)^2}{S_X(0)} \frac{m_e^2 c^4 \Lambda(T_{spec})}{4\pi(\sigma_T k T_{spec})^2 (1+z)^4} \frac{G(\beta_{fit})}{G(\beta_{fit}/2)^2}$$

$$\beta_{fit} = \beta \frac{\gamma + 3}{4}$$

# Analytic modeling of SZ $H_0$ measurement

- Spherical polytropic  $\beta$  -model as mean radial profiles
- Log-normal density and temperature fluctuations
- **Still force to fit to the isothermal  $\beta$  -model**, and the estimated  $H_0$  is biased as Kawahara et al. ApJ 674(2008)11

$$f_{H, polyLN|iso\beta} \equiv \frac{H_{0,est}}{H_{0,true}} = \chi_\sigma \chi_T(T_{ew}) \frac{\chi_T(T_{spec})}{\chi_T(T_{ew})}$$

**inhomogeneity**  $\chi_\sigma = \exp(\sigma_{LN,n}^2 - \sigma_{LN,T}^2 / 8) \approx (1.1 - 1.3)$

**non-isothermality**

$$\chi_T(T_{ew}) = J(\beta, \gamma, r_c / r_{vir})^{1.5} \left[ \frac{G(\beta(\gamma + 3) / 8)}{G(\beta\gamma / 2)} \right]^2 \approx (0.7 - 0.9)$$

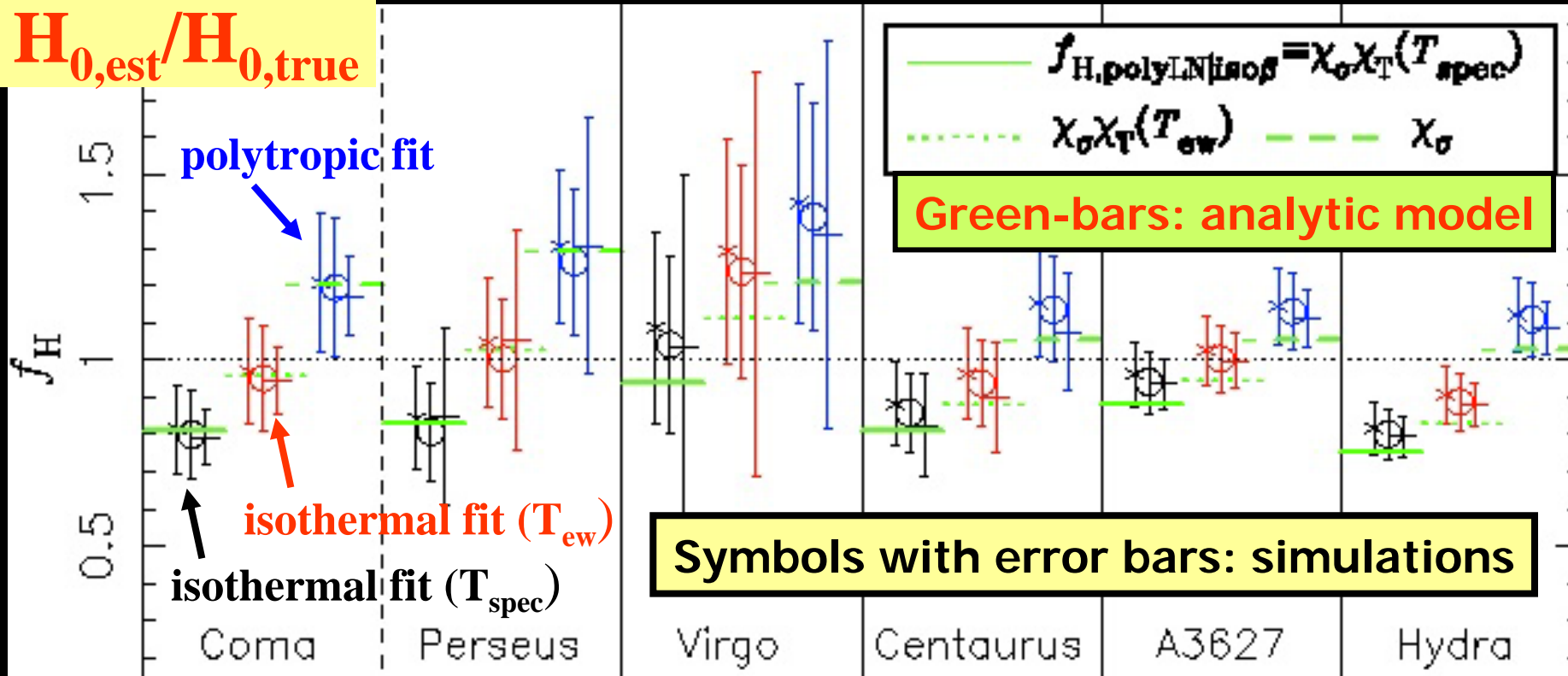
**temperature bias**

$$\frac{\chi_T(T_{spec})}{\chi_T(T_{ew})} \approx \left( \frac{T_{spec}}{T_{ew}} \right)^{1.5} \approx (0.8 - 0.9)$$



# Analytic model vs simulated clusters

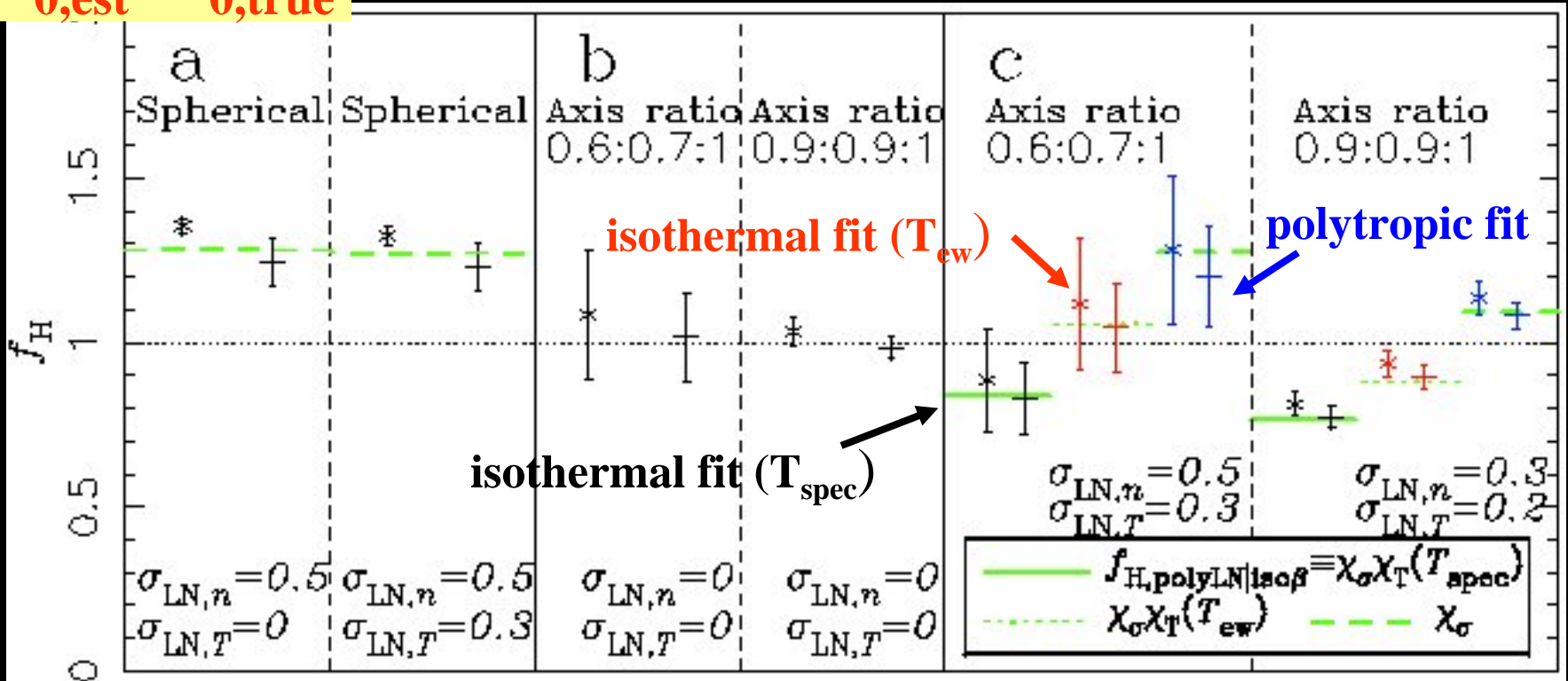
$$H_{0,est}/H_{0,true}$$



- Mean values are in good agreement with the analytic model
- *Additional small bias expected due to non-sphericity of clusters even after averaging over l.o.s. angles*

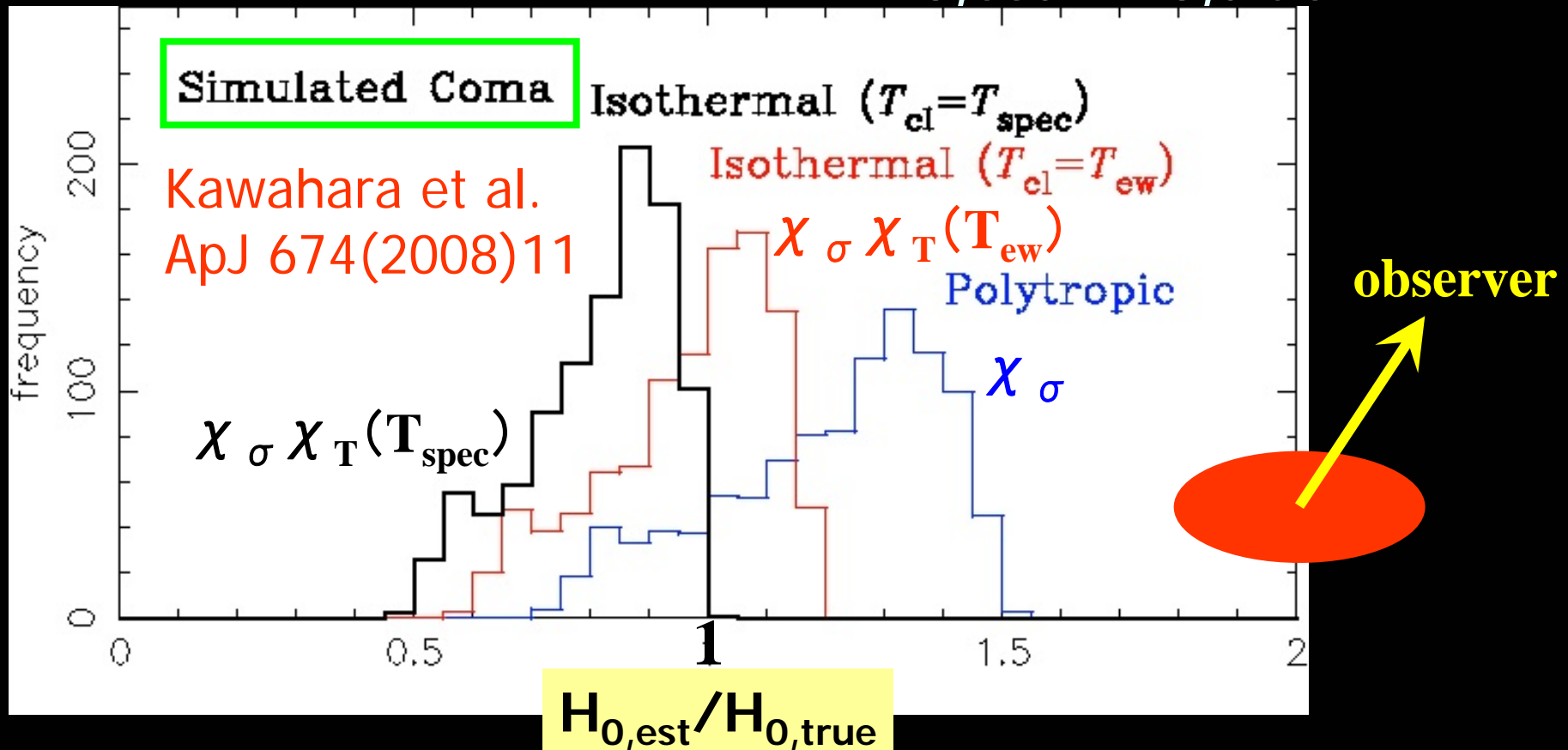
# Non-spherical effect : triaxial clusters

$H_{0,est}/H_{0,true}$



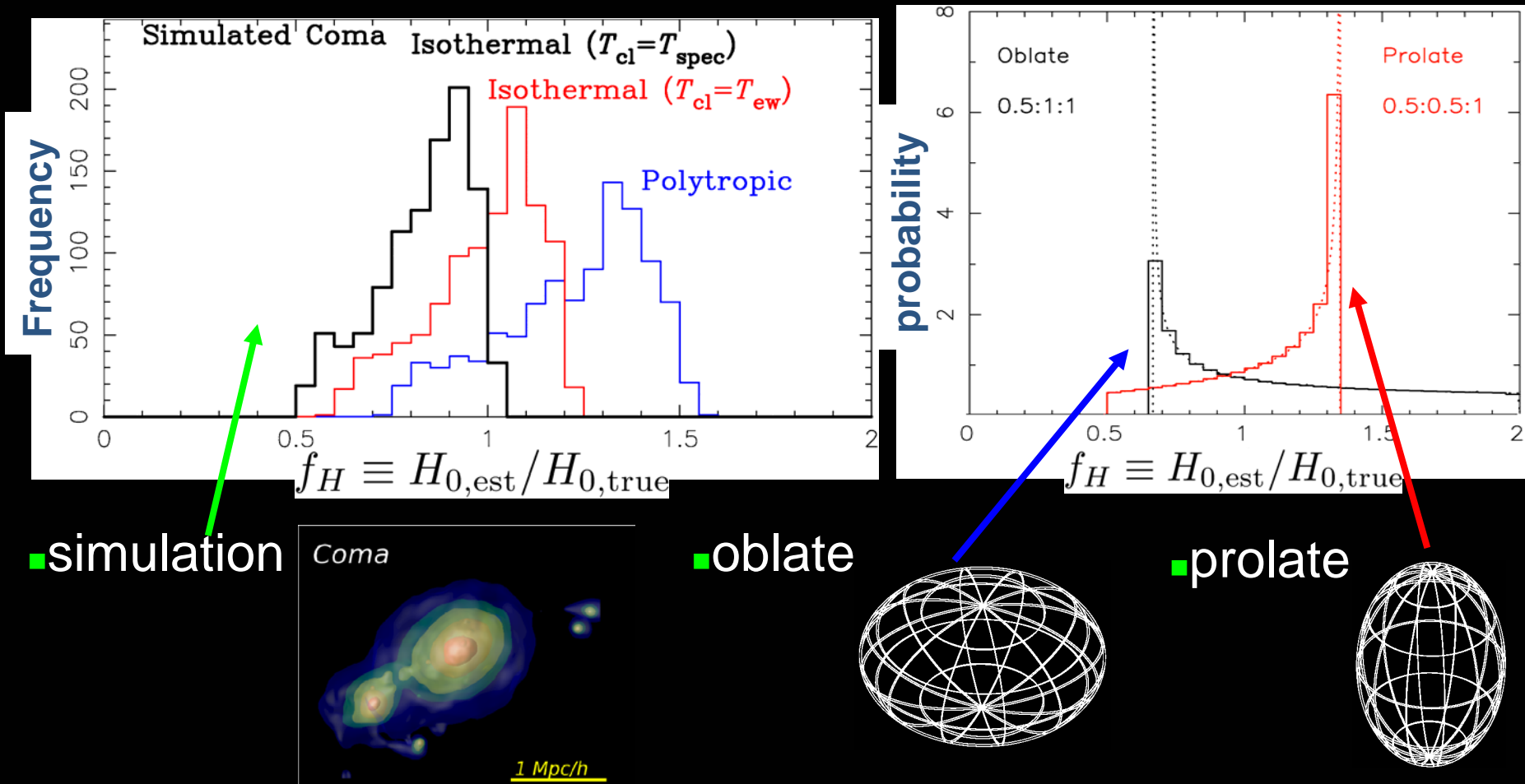
- Synthetic triaxial clusters (Jing & YS 2002)  
+ polytropic  $\beta$  + log-normal fluctuations

# Distribution of $H_{0,est}/H_{0,true}$



- Skewed distribution due to the prolateness
- Previous studies did not find the large bias because they set  $T_{cl}=T_{ew}$  instead of  $T_{spec}$  (Inagaki, Suginozawa & YS 1995, Yoshikawa, Itoh & YS 1998), consistent with our results of the isothermal fit with  $T_{ew}$

# Asymmetry in the estimated $H_0$



- distribution function of  $H_{0,est}$  from many SZ clusters
- ↔ overall average shape of clusters (oblate or prolate)

# Implications

- ① underestimate bias for X-ray spectroscopic temperature of clusters
- ② underestimate bias for  $H_0$  estimate from SZ observations
- ③ reconstruction of the 3D ICM fluctuation distribution from X-ray surface brightness (2D) observational data**

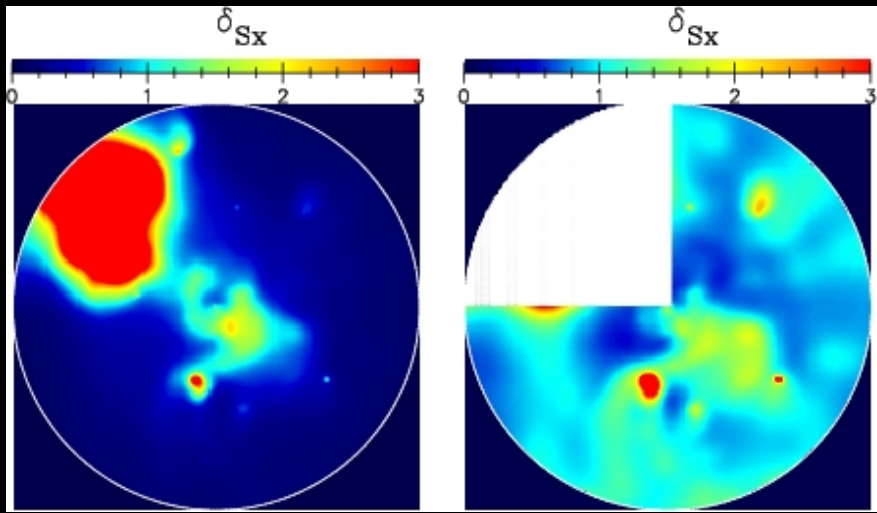
# Do observed ( $\neq$ simulated) clusters show log-normal signature ?

- So far we did not directly confirm the presence of log-normal fluctuations in real ICM
- projection effect: 2D surface brightness PDF from the 3D log-normal PDF in density and temperature ?
- *how to reconstruct the 3D log-normal nature from the observed 2D X-ray data ?*

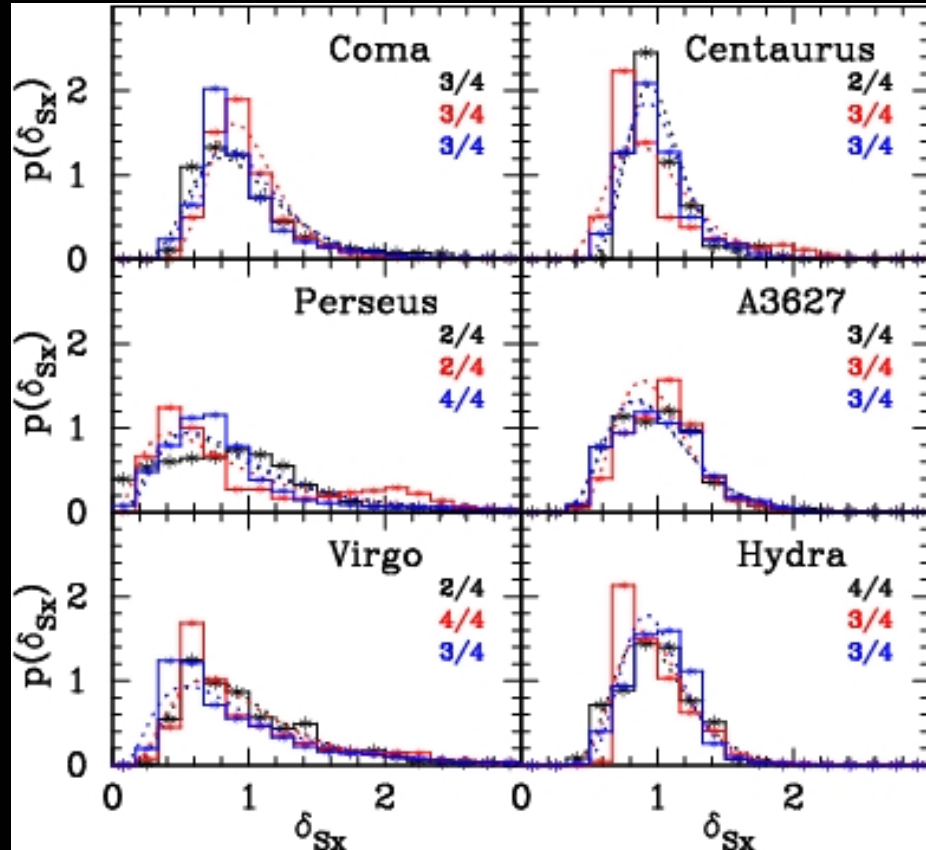
Kawahara, Reese et al. ApJ 687 (2008) 936



# 2D projection effect: test with simulated clusters

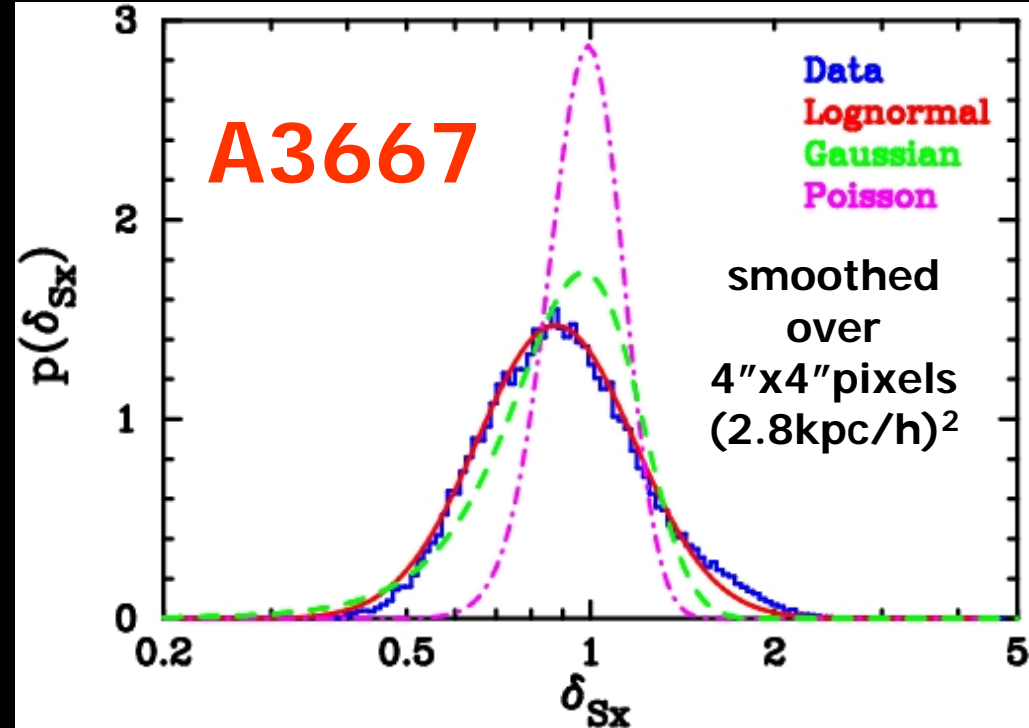
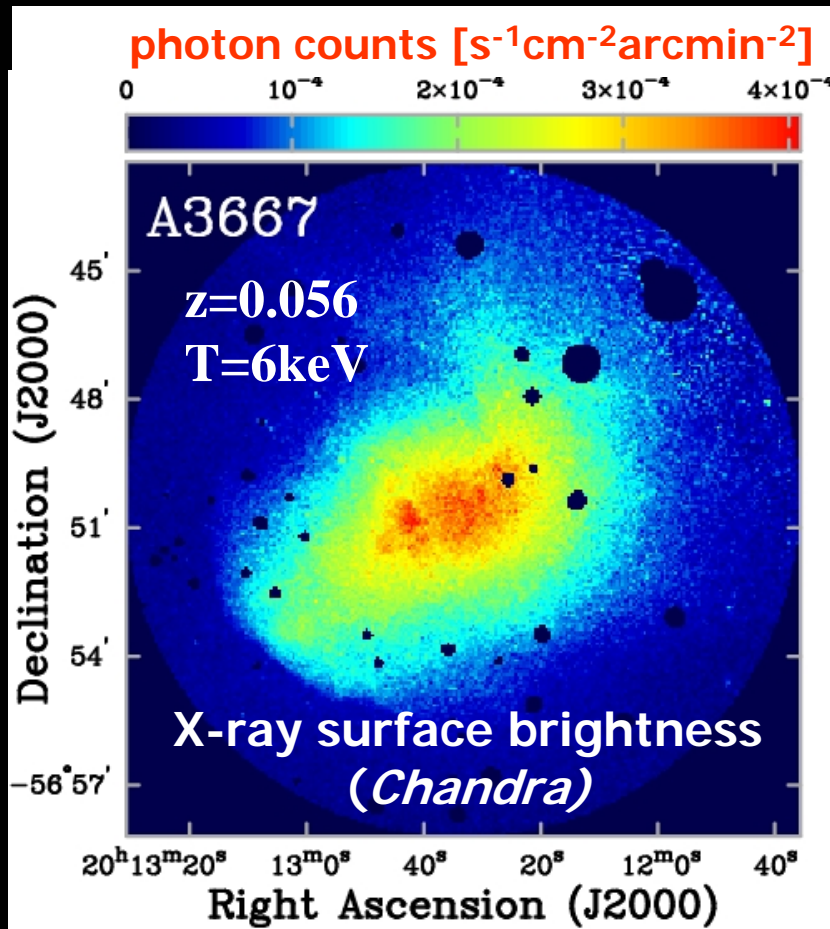


if a prominent substructure resides in a quarter, the quarter is removed in the analysis



- 3D log-normal PDF of gas and temperature results in 2D log-normal PDF of X-ray surface brightness

# Application to Abell 3667



first observational detection of log-normal fluctuation in real ICM !

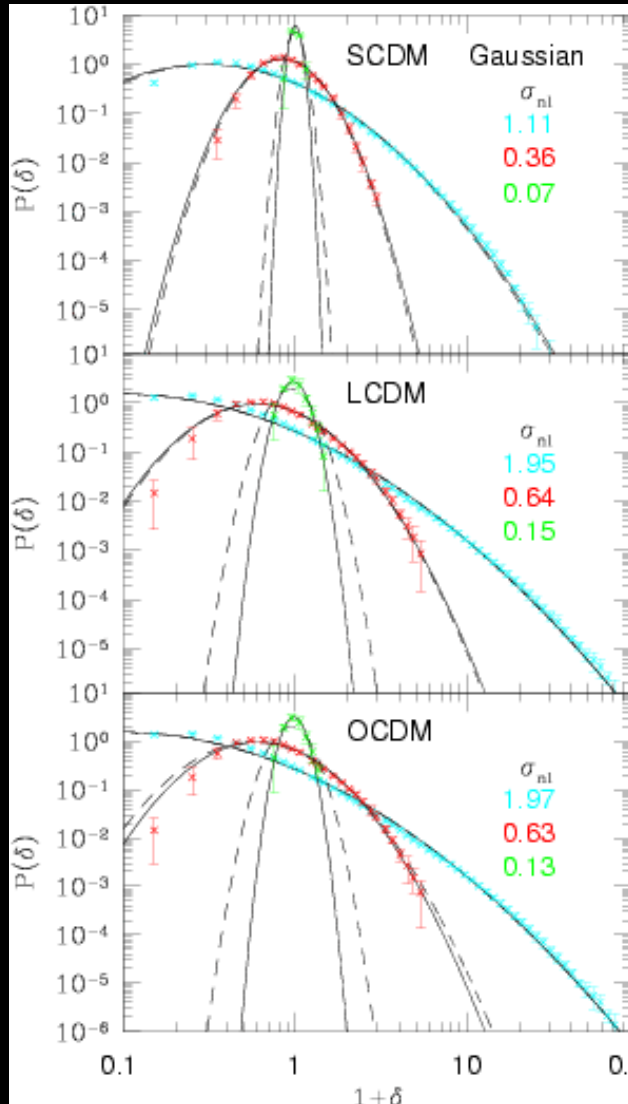
$$\sigma_{LN,Sx,A3667} = 0.3 \rightarrow \sigma_{LN,n} = 0.4$$

consistent with average values of simulated clusters

# a remaining question: origin of log-normal fluctuations ?

- generic statement
  - addition of independent random fields  
⇒ Gaussian
  - multiplication of independent random fields  
⇒ log-normal
- Indeed log-normal distribution is very common in a variety of phenomena
  - cosmological density field
  - surface density of interstellar medium
  - self-gravity ? turbulence ?

# log-normal PDF in cosmological density field



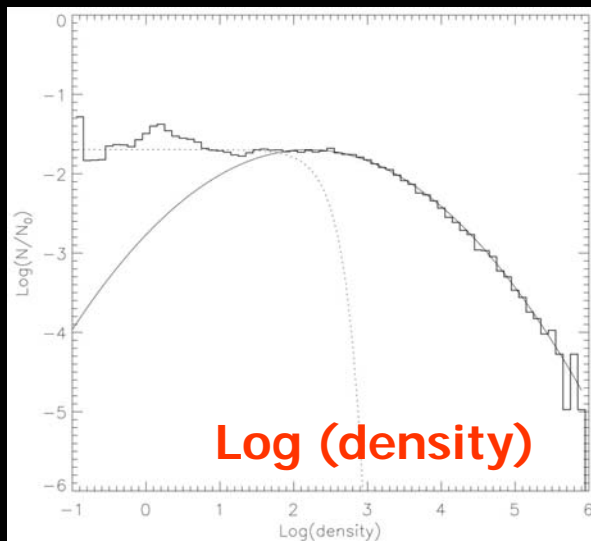
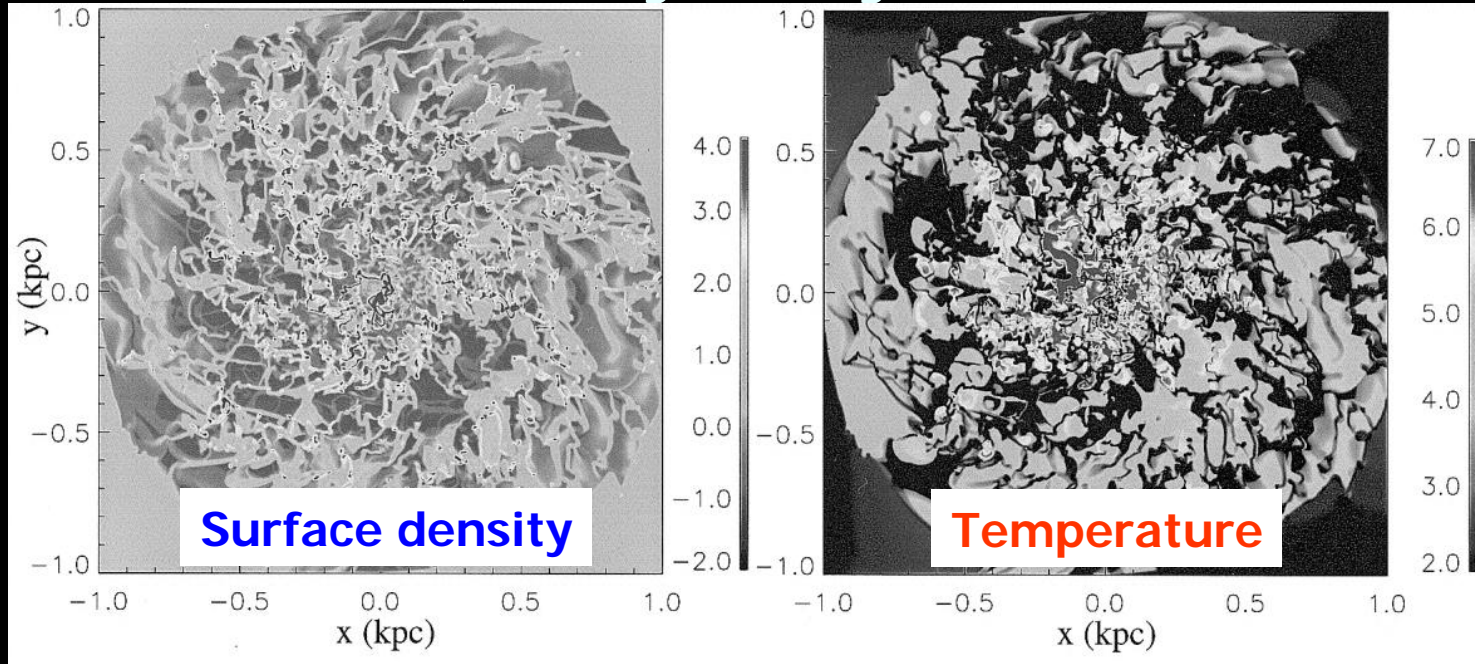
- initial condition: random-Gaussian  
↓ **collisionless N-body simulation**
- approximately log-normal PDF due to nonlinear gravity (>a few Mpc)

$$P_{LN}(\delta) = \frac{1}{(1+\delta)\sqrt{2\pi\ln(1+\sigma^2)}} \exp\left[-\frac{[\ln\sqrt{1+\sigma^2}(1+\delta)]^2}{2\ln(1+\sigma^2)}\right]$$

- empirical model for non-Gaussianity generated by cosmological nonlinear gravitational evolution for  $0.1 < \rho / \langle \rho \rangle < 1000$

Kayo, Taruya & Suto, ApJ 561(2001)22

# log-normal PDF for interstellar medium in Galactic disk (2D hydrodynamic simulation)



log-normal behavior in  
high-density regions for  
 $10^2 < \rho / \langle \rho \rangle < 10^6$

Wada & Norman  
ApJ 547(2001)172

# Summary

- **Analytic modeling of ICM inhomogeneity for the first time: log-normal PDF**
  - Previous claim of the cluster temperature underestimate bias is analytically understood
  - Consistent with Chandra data of A3667
  - $H_{0,est}/H_{0,true} = 0.8-0.9$  is expected for SZ due to inhomogeneity + non-isothermality of ICM
    - Consistent with Reese et al. (2002)
    - Direct comparison between ROSAT and Chandra analyses is now in progress (Reese et al. 2009)