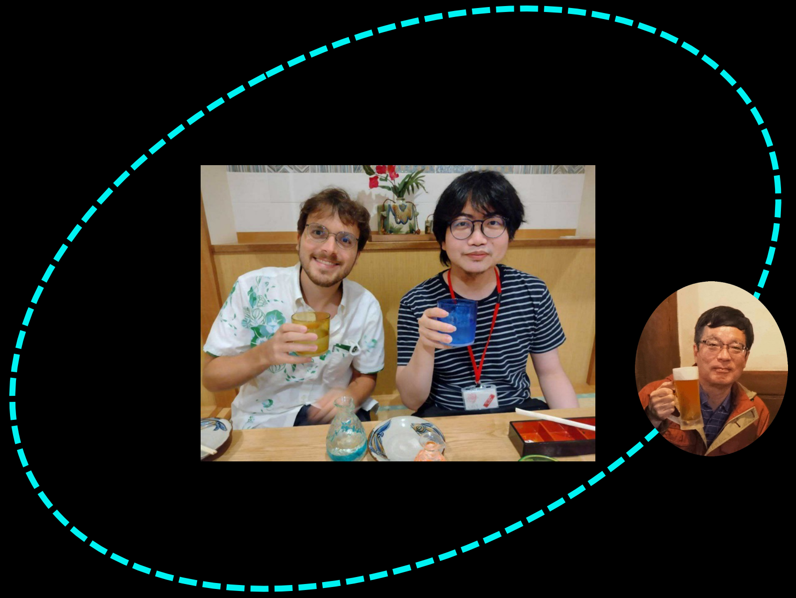
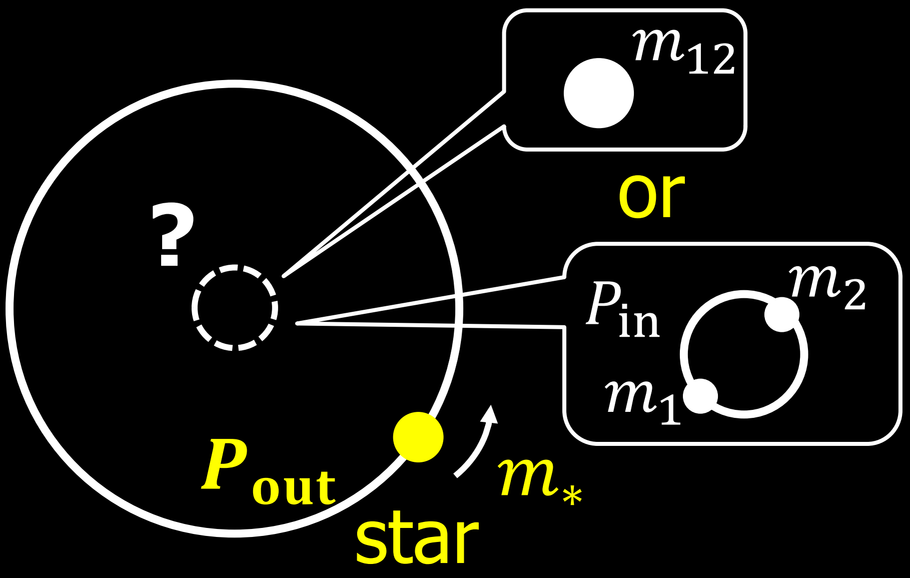
Dynamics of a tertiary body orbiting an inner black-hole binary





Hayashi, Trani & YS: ApJ 939(2022)81
ApJ 943(2023)58

Hayashi, Wang & YS: ApJ 890(2020)112
Hayashi & YS: ApJ 897(2020)29
ApJ 907(2021)48

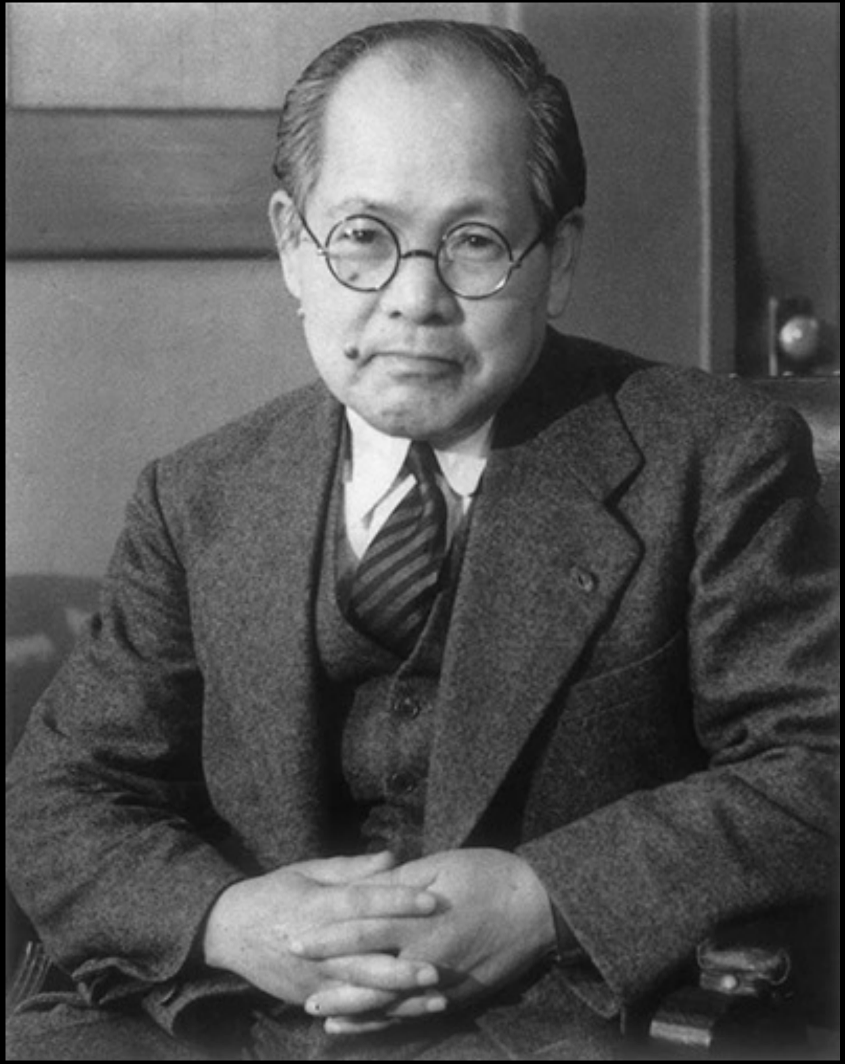
Yasushi Suto Department of Physics and Research Center for the Early Universe
The University of Tokyo

9:00 December 13, 2023 @ RESCEU-NBIA workshop

Niels Bohr Institute
and
theoretical astrophysicicists in Japan

February 13, 2023

Yoshio Nishina

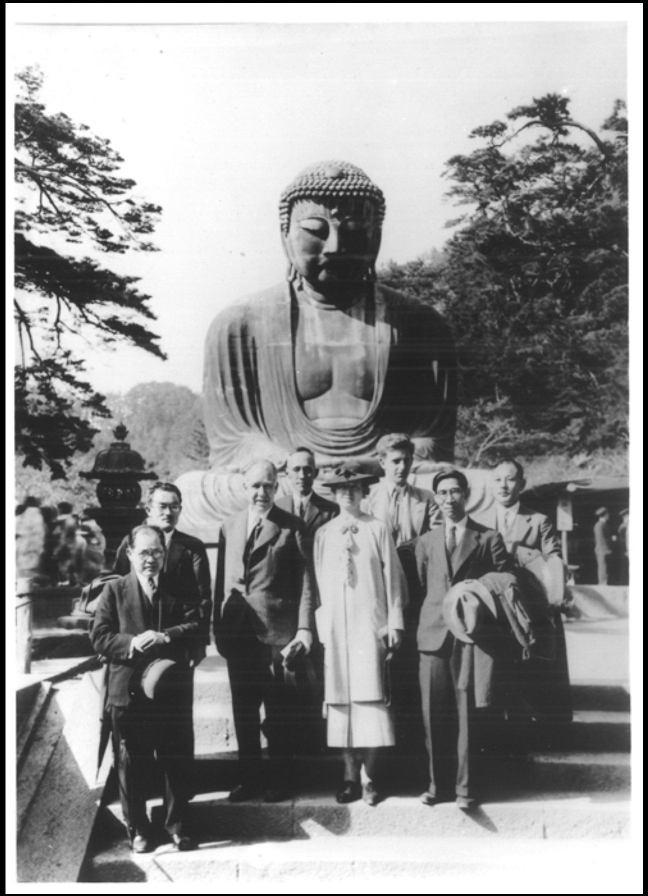


After joining the institute, he studied at the Cavendish Laboratory in the UK, directed by Ernest Rutherford. There, he had a fateful encounter with Niels Bohr, who proposed a model of the atom based on a completely new "principle." Bohr's laboratory in Copenhagen, Denmark, was the center of the birth of "quantum mechanics", and he moved there at Bohr's invitation, where he studied from 1923 to 1928 amongst the world's young geniuses, achieving outstanding experimental results in X-ray spectroscopy.

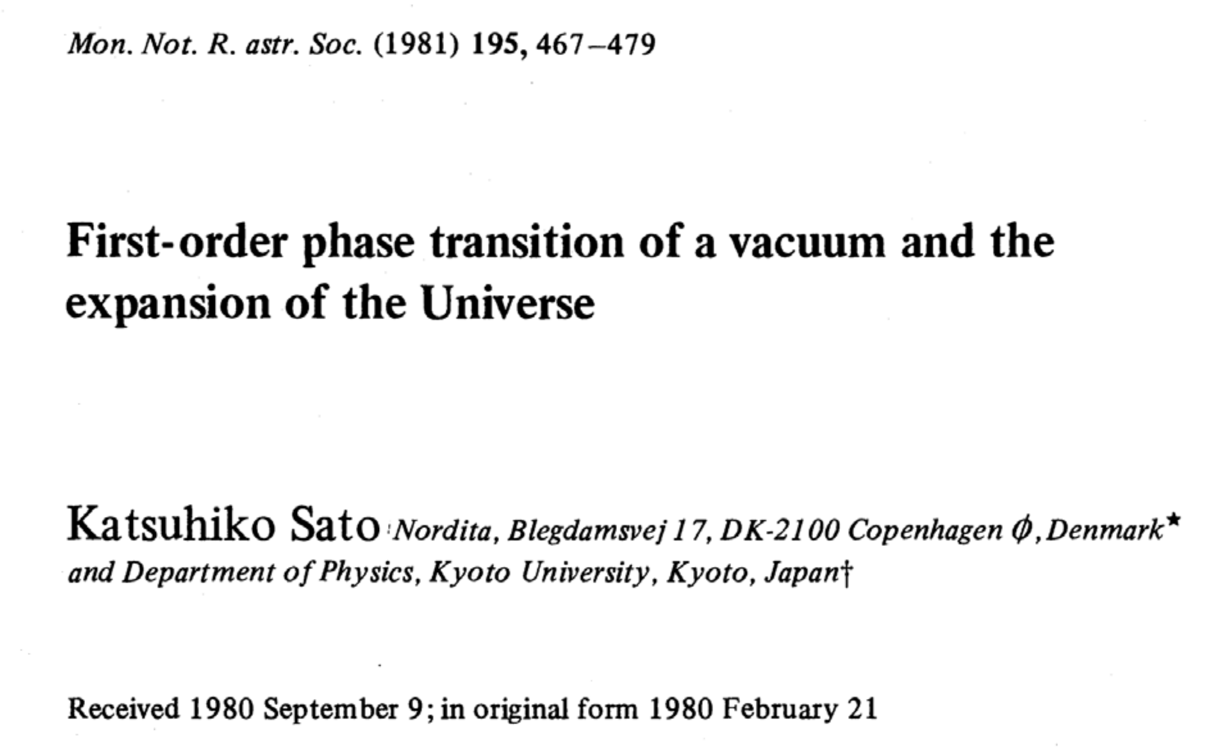
In 1928, his talents as a theorist blossomed and, together with his close friend the theorist Oskar Klein, he derived the "Klein-Nishina formula" for the scattering of X-rays by electrons, based on the newly published "relativistic quantum mechanics" of Paul Dirac, which earned him international recognition as an atomic physicist.

From webpage of the Nishina Memorial Foundation
https://www.nishina-mf.or.jp/doctor_en

Bohr visited Japan in 1937



Nishina ⇒ Yukawa ⇒ C.Hayashi ⇒ K.Sato



Mon. Not. R. astr. Soc. (1981) **195**, 467–479

First-order phase transition of a vacuum and the expansion of the Universe

Katsuhiko Sato *Nordita, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark**
and Department of Physics, Kyoto University, Kyoto, Japan†

Received 1980 September 9; in original form 1980 February 21

Prof. C. Pethick invited Katsuhiko Sato to NBI (Nordita?) for one year (1980)

Nuclear Physics B180 [FS2] (1981) 385–404
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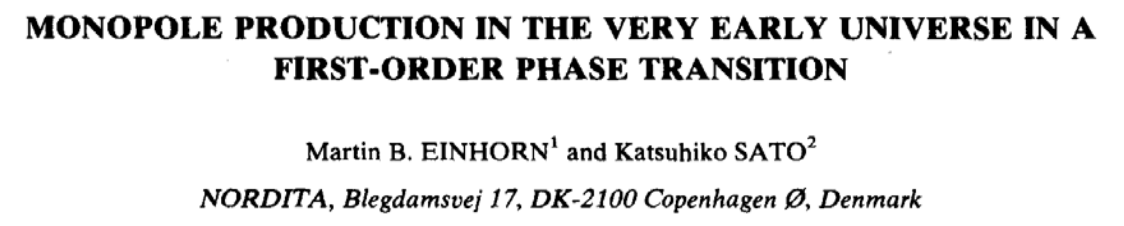
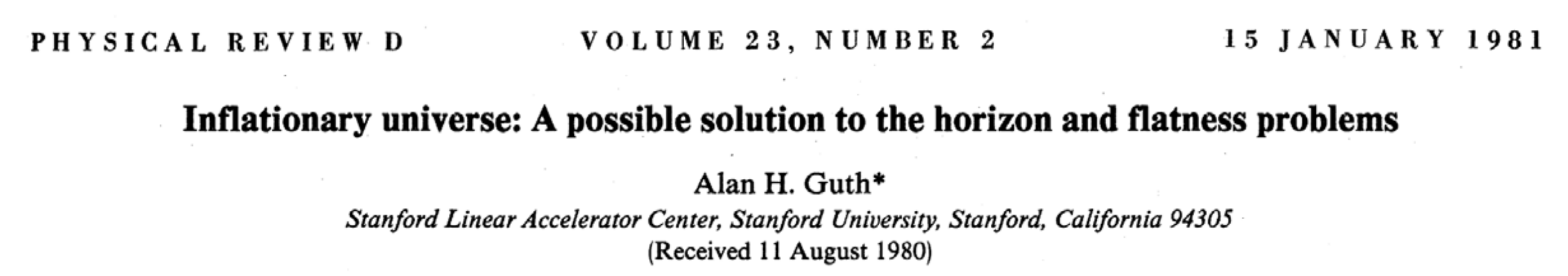


MONOPOLE PRODUCTION IN THE VERY EARLY UNIVERSE IN A FIRST-ORDER PHASE TRANSITION

Martin B. EINHORN[1] and Katsuhiko SATO[2]

NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

Received 30 July 1980
(Revised 10 November 1980)

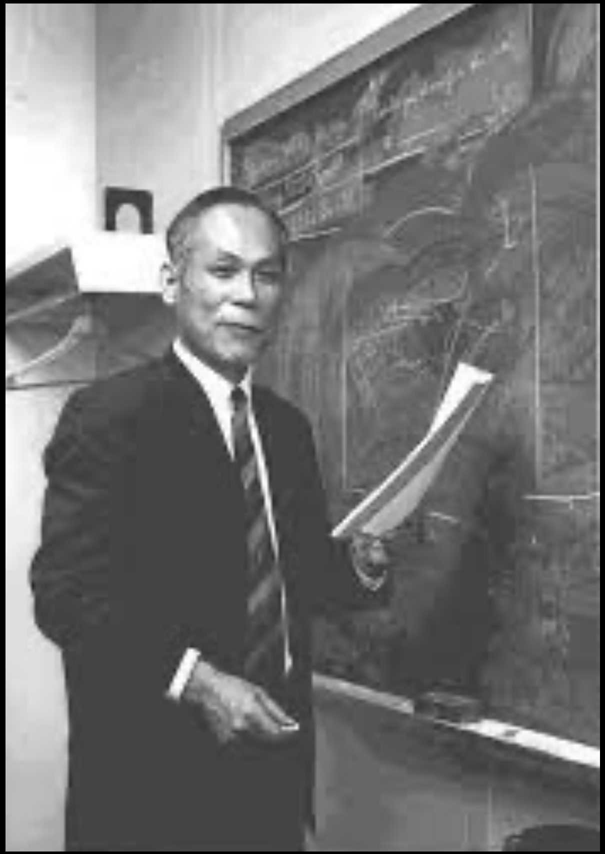


PHYSICAL REVIEW D VOLUME 23, NUMBER 2 15 JANUARY 1981

Inflationary universe: A possible solution to the horizon and flatness problems

Alan H. Guth*
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 11 August 1980)

Chushiro Hayashi Group @Kyoto University



Undergrad at University of Tokyo
Classmate of Yoichiro Nambu

Very incomplete list; more than 30 university professors from his 52 graduate students

- Daiichiro Sugimoto
  - Junichiro Makino
    - Michiko Fujii
  - Toshiyuki Fukushige
    - Ataru Tanikawa
- Humitaka Sato
  - Masaru Shibata
    - Kenta Hotokezaka
- Katsuhiko Sato
  - Yasushi Suto
    - Toshinori Hayashi
  - Jun'ichi Yokoyama
  - Masahiro Kawasaki
  - Shoichi Yamada
    - Ryosuke Hirai
- Takashi Nakamura
  - Tomoya Kinugawa
- Misao Sasaki

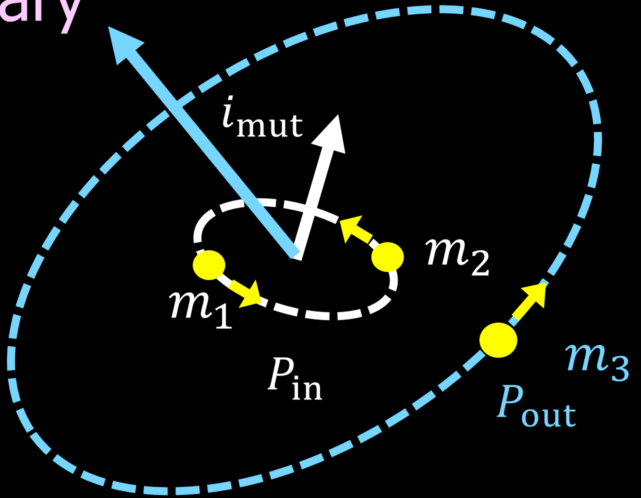
1 Hierarchical three-body systems

Ubiquity of hierarchical triples

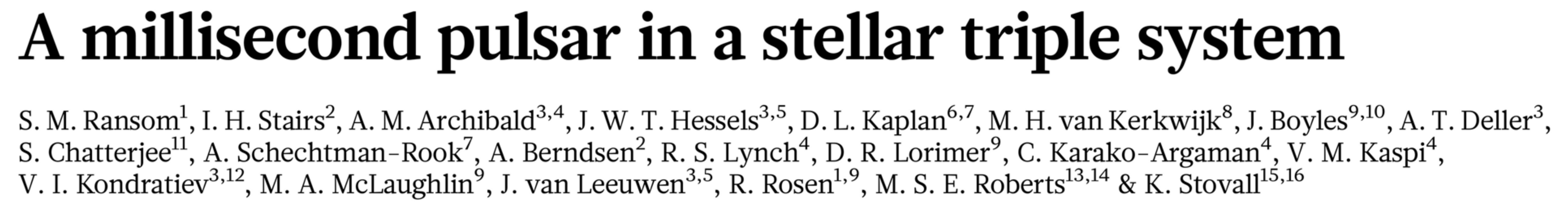
- Stellar systems
  - more than 70% of OBA-type stars and 50% of FGK-type stars a belong to binary/multiple systems (e.g., Alpha Centauri)
- (Exo)Planetary systems
  - planets around binary stars, multi-planets, satellites,,,
- Compact objects
  - Possible pathway towards binary BHs detected by GW
  - Binaries (stars, BHs) around a supermassive BH in galaxies
  - Triples of compact objects, e.g., pulsar-WD binary + tertiary WD (Ransom et al. 2014)

Hierarchical three-body systems

- Gravitational three-body systems are unstable in general
  - stable three-body systems are mostly hierarchical: tight binary + distant tertiary orbiting the center-of-mass of the inner binary
  - observed three-body systems are likely to be hierarchical
- Stable systems are inevitably associated with (undemocratic) hierarchies
  - quite universal in biological, astronomical and social systems
    - quarks and leptons – atoms – molecules – DNAs – cells – organs – animals – villages – cities – nations – planets – stars – star clusters – galaxies – galaxy clusters – universe(s) – multiverse(s)

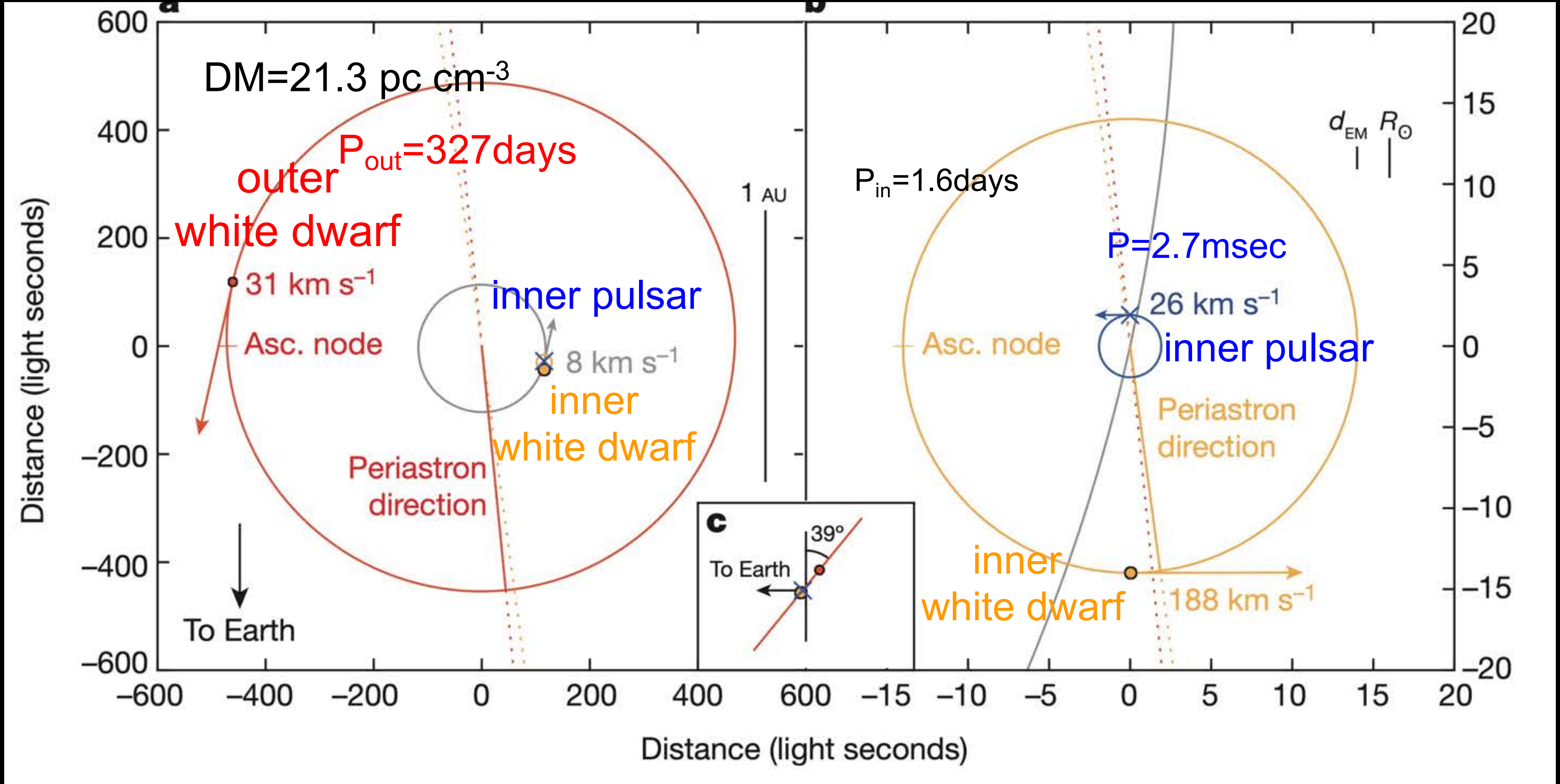


- non-intuitive (counter-intuitive) dynamical behavior of hierarchical triples triggers unexpectedly broad diversities in astronomical phenomena (e.g., ZKL effect)

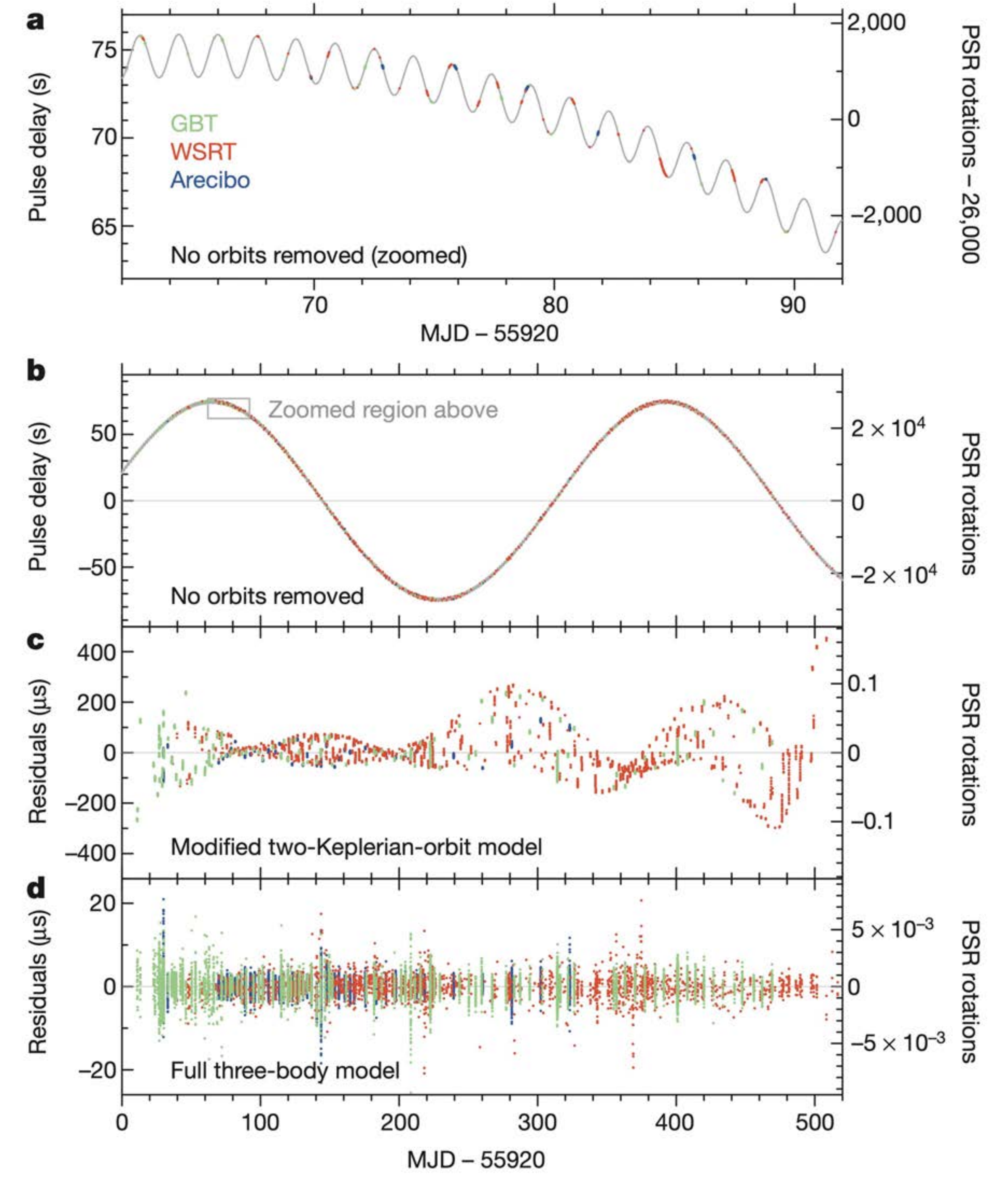


A millisecond pulsar in a stellar triple system

S. M. Ransom[1], I. H. Stairs[2], A. M. Archibald[3,4], J. W. T. Hessels[3,5], D. L. Kaplan[6,7], M. H. van Kerkwijk[8], J. Boyles[9,10], A. T. Deller[3], S. Chatterjee[11], A. Schechtman-Rook[7], A. Berndsen[2], R. S. Lynch[4], D. R. Lorimer[9], C. Karako-Argaman[4], V. M. Kaspi[4], V. I. Kondratiev[3,12], M. A. McLaughlin[9], J. van Leeuwen[3,5], R. Rosen[1,9], M. S. E. Roberts[13,14] & K. Stovall[15,16]



Ransom et al.
Nature
505 (2014)
520



Ransom et al.
Nature 505(2014)520
NS-WD binary + WD

PSR J0337+1715 parameters	
inner orbital period (pulsar+WD)	1.629401788(5) day
outer orbital period (WD)	327.257541(7) day
pulsar spin period	2.73258863244(9) msec
mutual orbital inclination	0.0120(17) deg.
Pulsar mass	1.4378(13) $M_\odot$
Inner WD mass	0.19751(15) $M_\odot$
Outer WD mass	0.4101(3) $M_\odot$

2 Lyapunov vs. Lagrange instabilities of hierarchical triple systems

My cosmology research tree

- Chushiro Hayashi
  - Katsuhiko Sato
    - Yasushi Suto

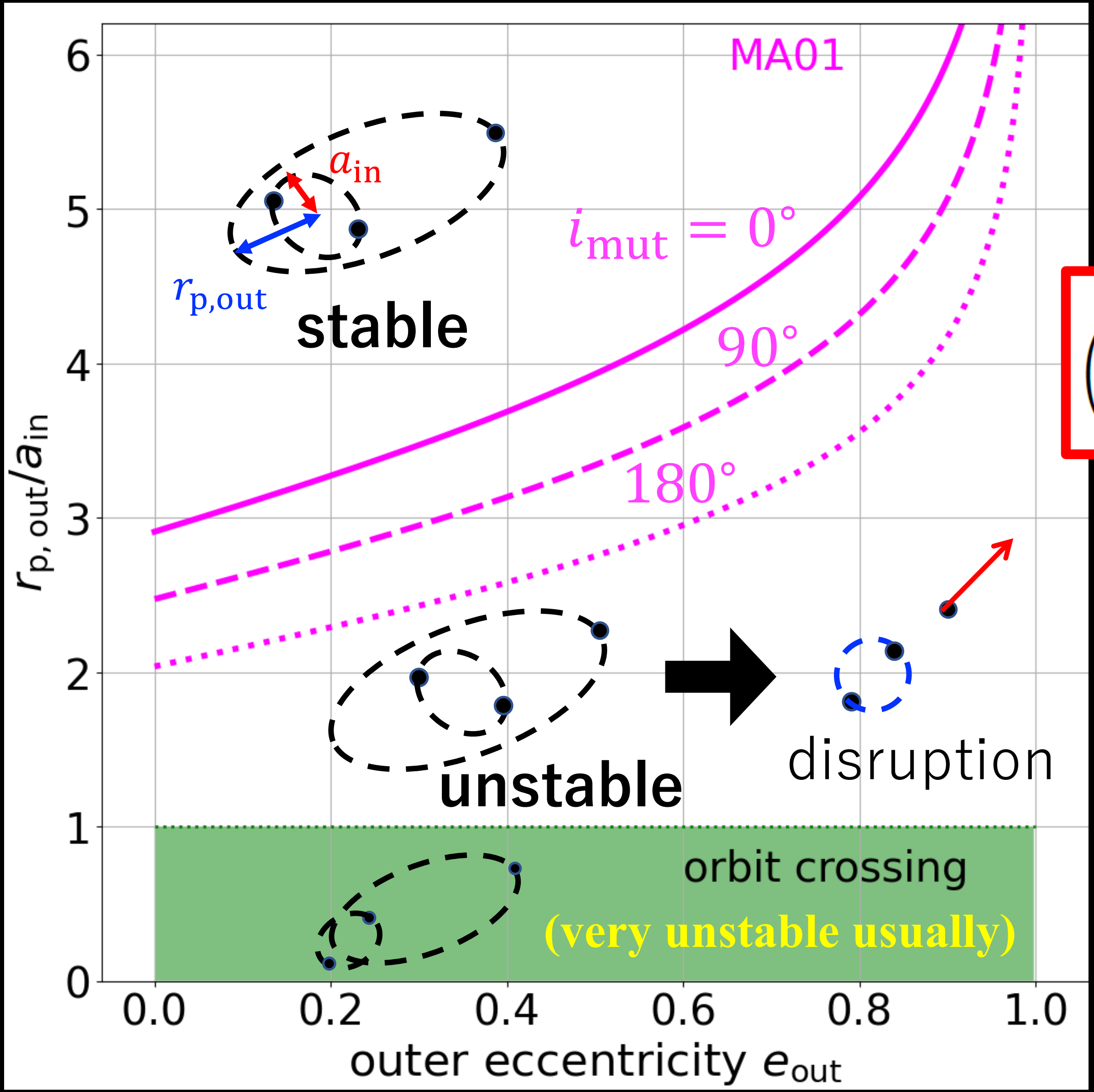
My three-body research tree

- Alessandro Trani
  - Yasushi Suto
    - Toshinori Hayashi

Diversities triggered by triple dynamics

> *Alpha Centauri was a triple system, two suns tightly orbiting one another, and a third, more remote, circling them both. What would it be like to live on a world with three suns in the sky?*
>
> *— Carl Sagan "Contact"*

Triples are unstable in general ⇔ diversity



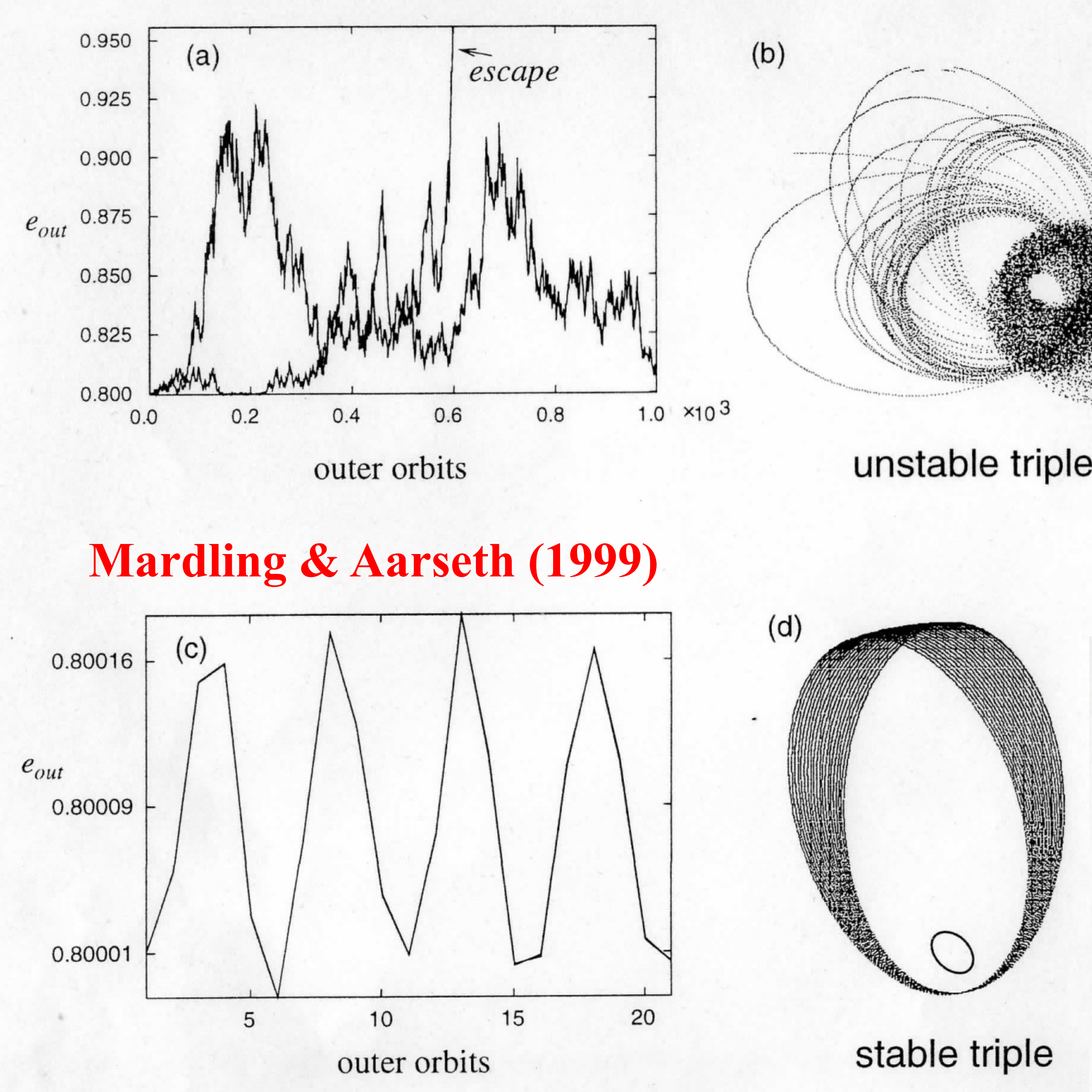
- Stability criterion

  (Mardling & Aarseth 2001)

$$\left(\frac{r_{\rm p,out}}{a_{\rm in}}\right)_{\rm MA} \equiv 2.8\left(1-0.3\frac{i_{\rm mut}}{\pi}\right)\left[\left(1+\frac{m_3}{m_{12}}\right)\frac{(1+e_{\rm out})}{\sqrt{1-e_{\rm out}}}\right]^{2/5}$$

  - Well-known and widely used, but its implication is often misinterpreted…
  - What does it mean?
  - Lyapunov (chaoticity of local trajectory) vs. Lagrange (escape of a body from the system) stability
  - ***Disruption timescale,*** instead of the stable-unstable boundary ?

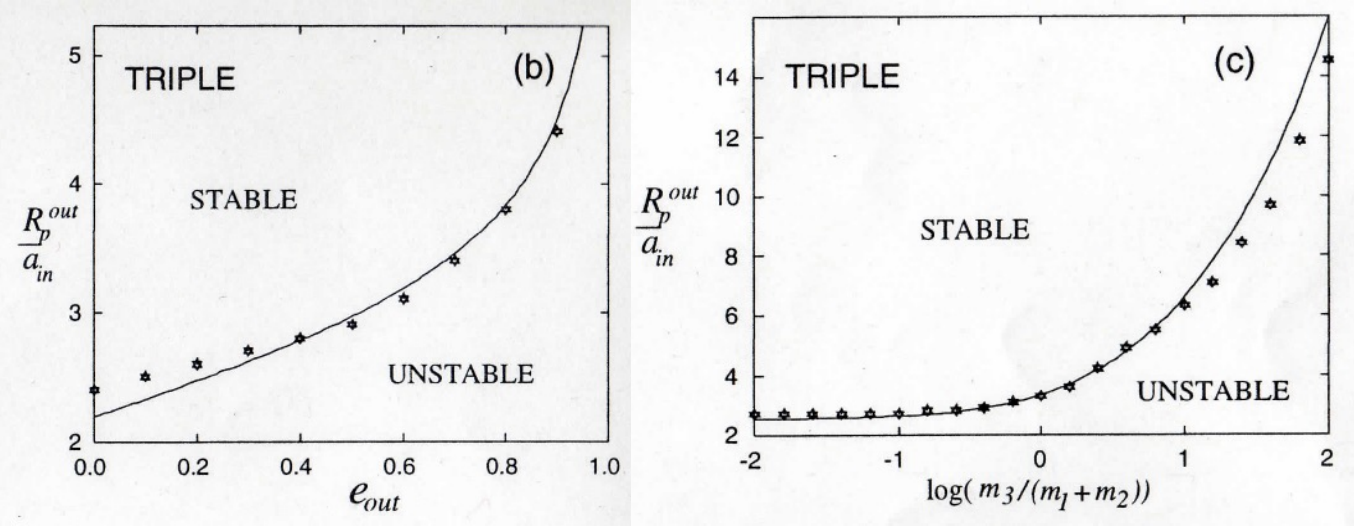
Normal and chaotic evolution of triple systems



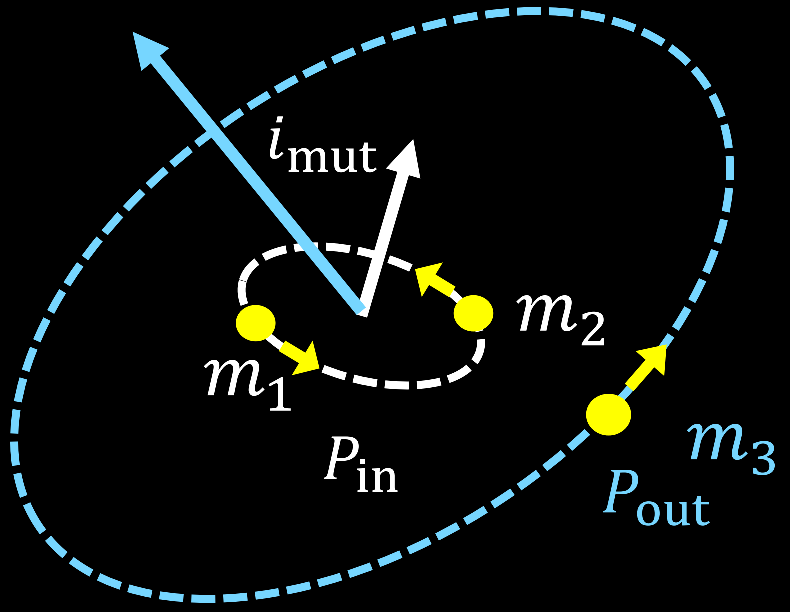
Mardling & Aarseth (1999)

“A system was deemed stable if two orbits, initially differing by 1 part in 10^5 in the eccentricity, remained close after 100 orbits.” (Mardling & Aarseth 1999)

“We deem a triple system stable if it remains bound for 100 outer orbits and if the semimajor axes of both inner and outer orbits do not change by more than 10 percent of the initial value.”
(Vynatheya et al. 2022 MNRAS 516, 4146)

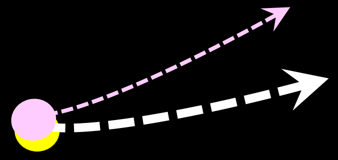


Lyapunov vs. Lagrange instability

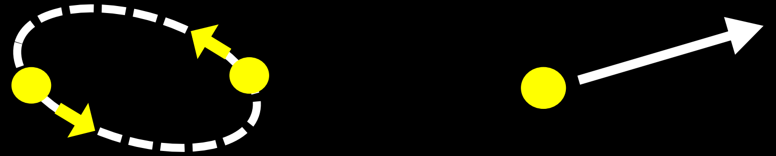


- cannot be studied under secular approximation in which energies of inner and outer orbits are conserved separately.
- Relation between Lyapunov and Lagrange instabilities is not clear

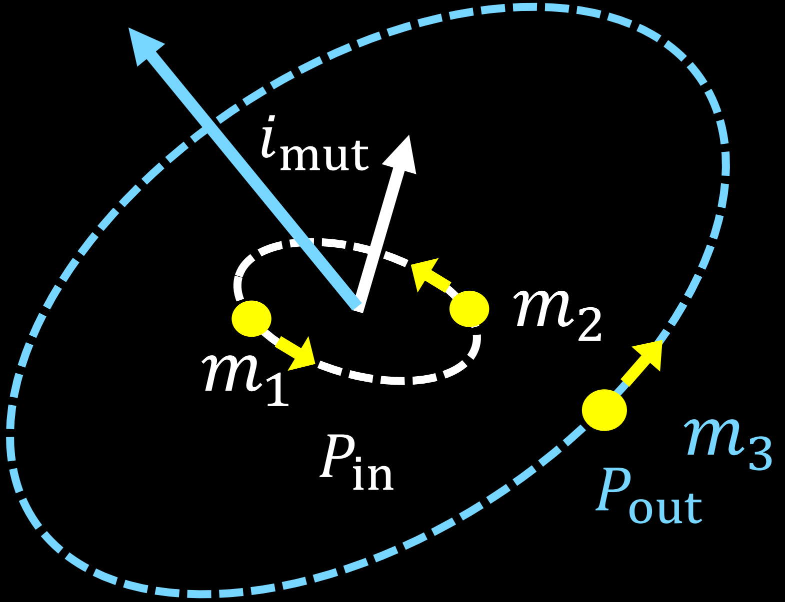
- Lyapunov instability
  - Local divergence of trajectories of bodies (~chaoticity)



- Lagrange instability
  - global escape of a body from the triple system (∼boundedness of an orbit)



Longer-term N-body simulations in Newtonian dynamics ***neglecting GR effects***



$e_{\rm in} = 10^{-5}$(circular)

N-body code
TSUNAMI
(Trani and Spera)

$\frac{r_{\rm p,out}}{a_{\rm in}}$ $e_{\rm out}$

Integration time
up to $10^9 P_{\rm in}$
(roughly $10^{6\text{-}7}$ $P_{\rm out}$)

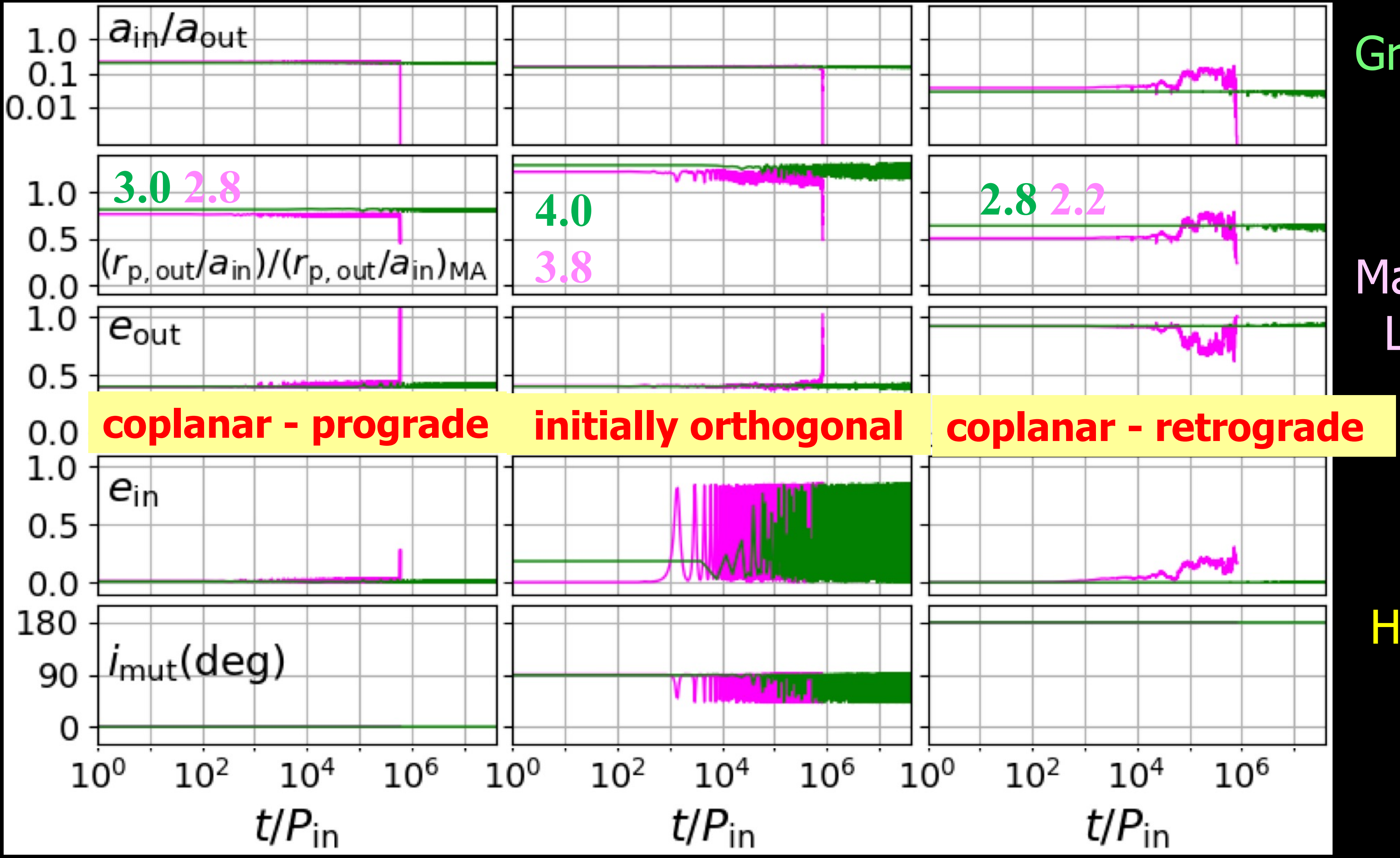
Disruption time
$\frac{T_{\rm d}}{P_{\rm in}}(q_{21}, q_{23}, i_{\rm mut}, \cdots)$

$\boldsymbol{i_{\rm mut}}$**: $\boldsymbol{0^\circ, 90^\circ, 180^\circ}$** Coplanar: prograde, retrograde
Orthogonal

Hayashi, Trani & YS,
arXiv:2207.12672 ApJ 939(2022)81
arXiv:2209.08487 ApJ 943(2023)58

$q_{21} \equiv m_2/m_1 (\leq 1)$ Mass ratio (inner binary)
$q_{23} \equiv m_2/m_3$ Mass ratio (tertiary)

Examples of orbital evolution of triples



Green:
Lagrange stable
up to $t=10^8$ P_{in}

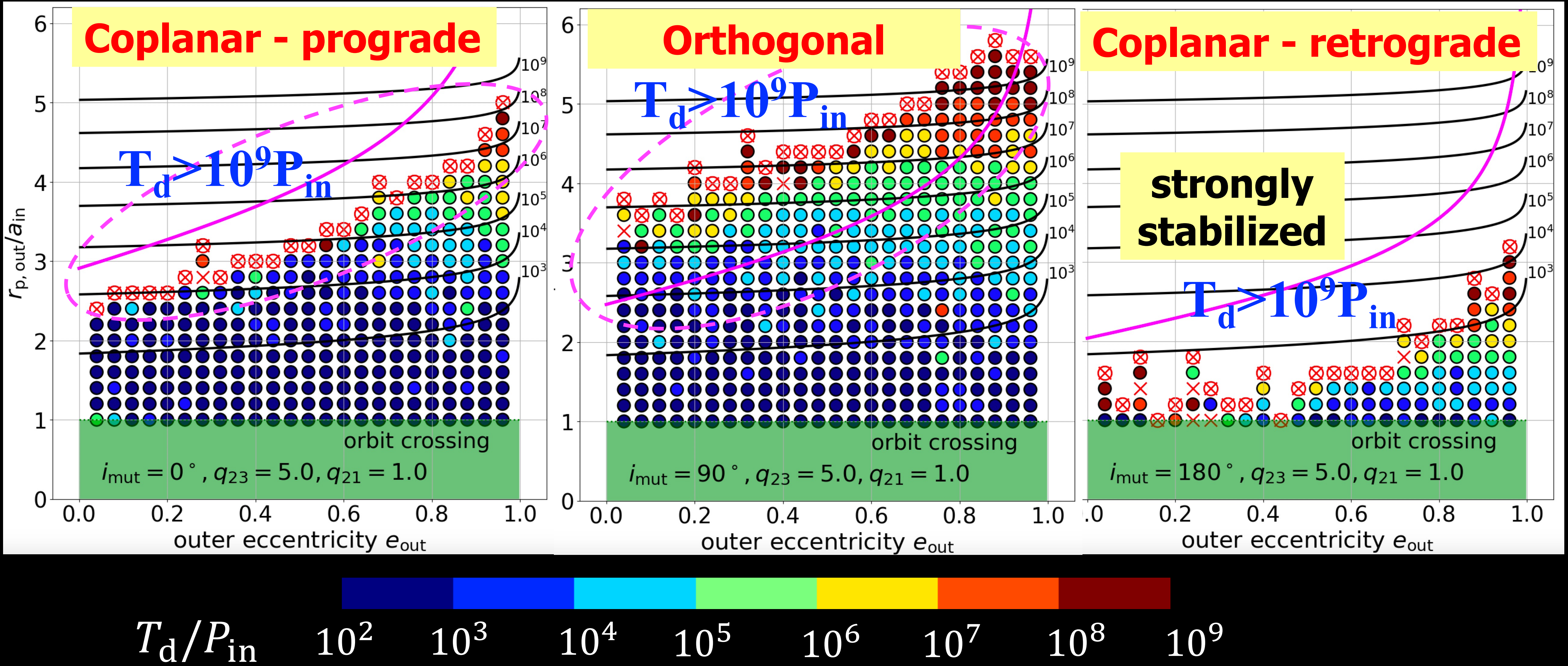
Magenta:
Lagrange unstable
around $t=10^6$ P_{in}

Hayashi, Trani & YS
arXiv:2207.12672
ApJ 939 (2022)81

Disruption timescales on $a_{out}(1-e_{out})/a_{in}$ – e_{out} plane

(Initial phases are fixed)

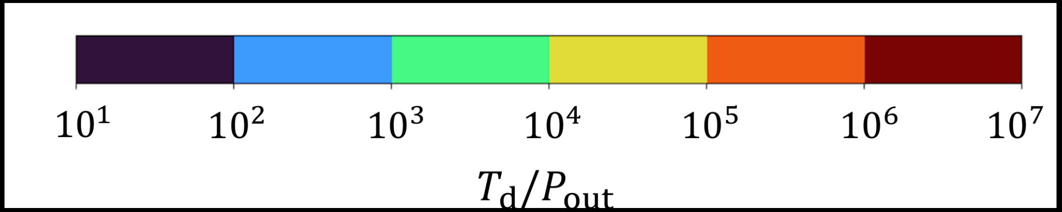
$m_1=m_2=5m_3$ —— Dynamical stability boundary by Mardling & Aarseth (2001)



Inclination dependence

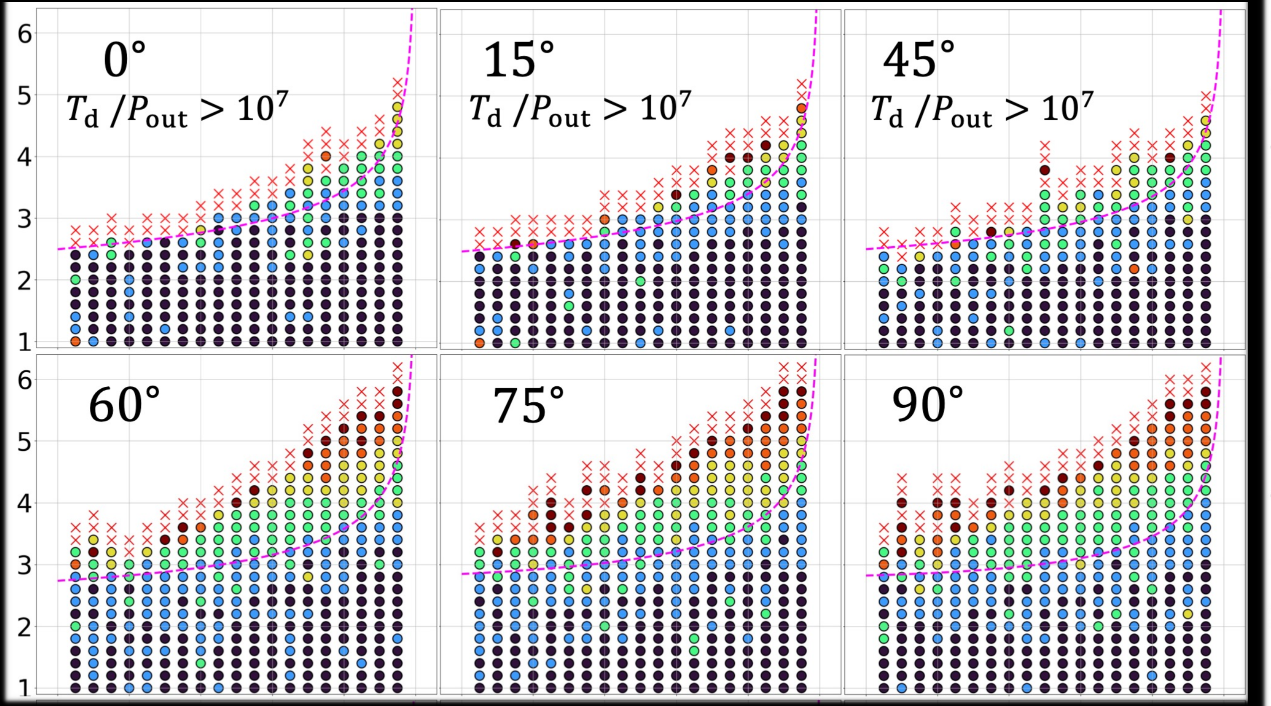
T_d/P_{out} on $a_{out}(1-e_{out})/a_{in}$ – e_{out} plane

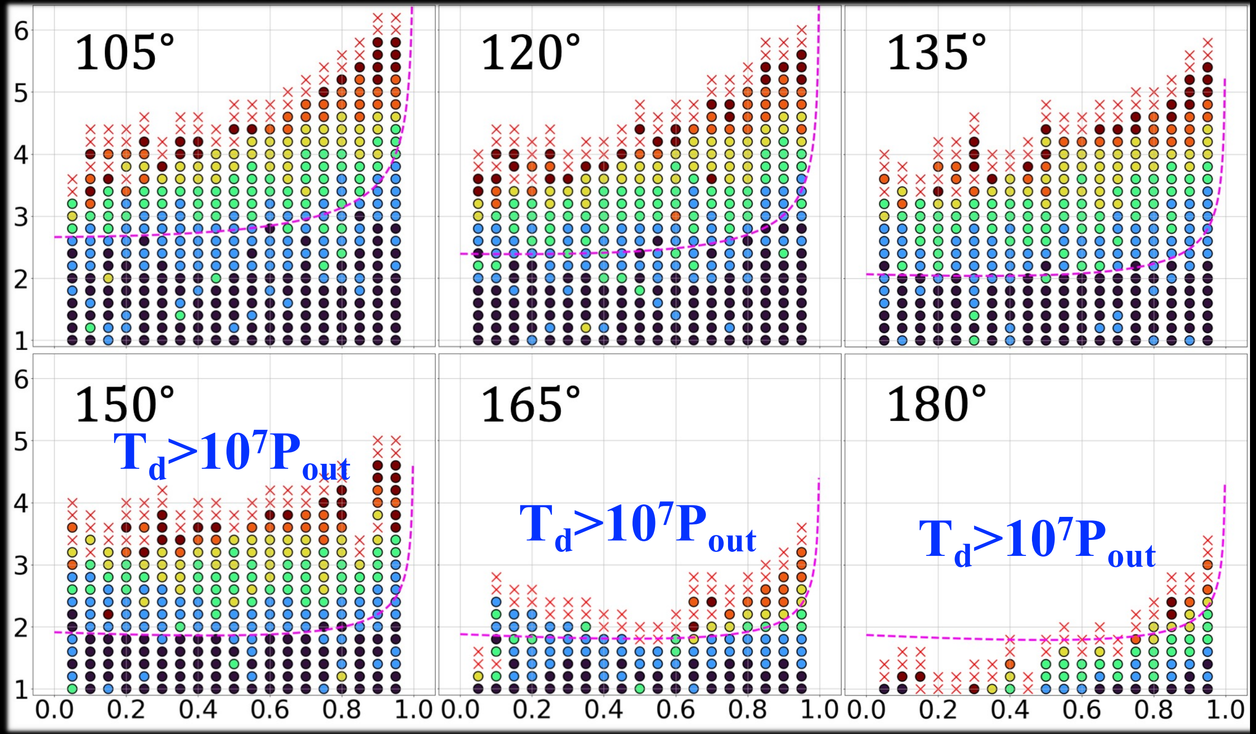
(Initial phases are fixed)



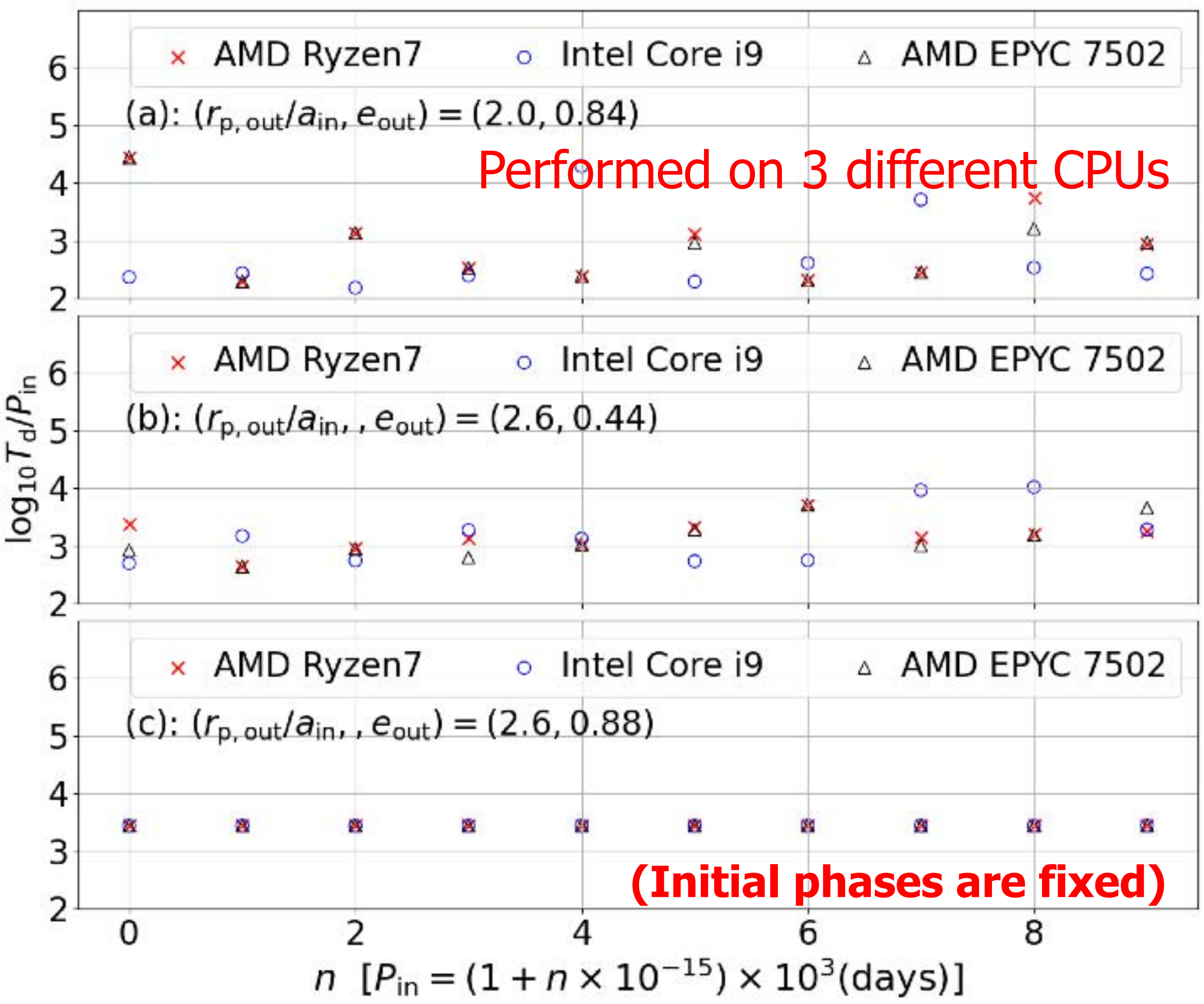
60<i_{mut} (deg)<150
destabilized by the Kozai-Lidov oscillations
i_{mut} (deg)>160
significantly stabilized due to inefficient energy transfer between inner and outer orbits

Very different from the stability boundary by Mardling & Aarseth (2001) ------------





Chaotic nature of disruption timescale distribution



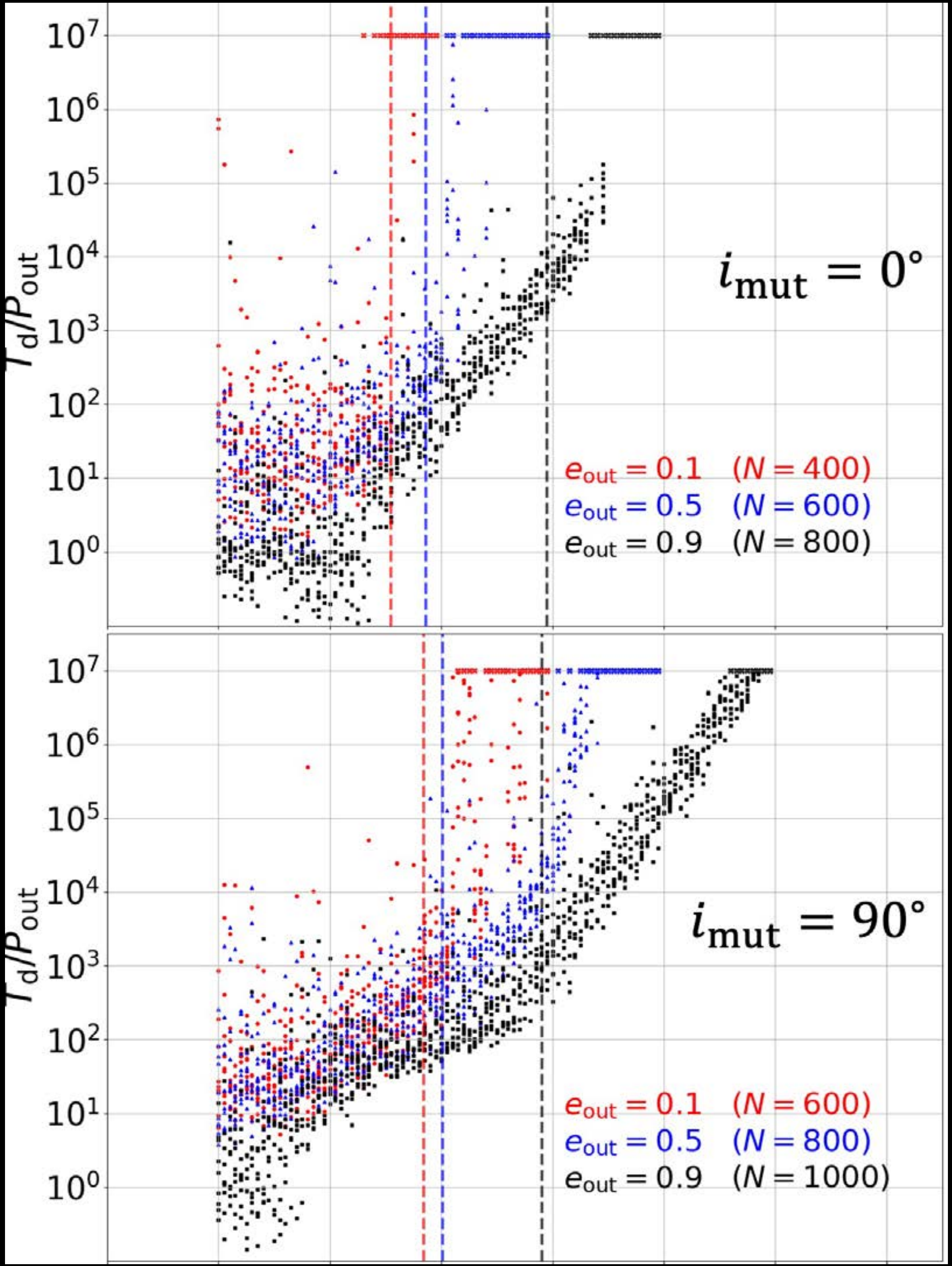
Performed on 3 different CPUs

(Initial phases are fixed)

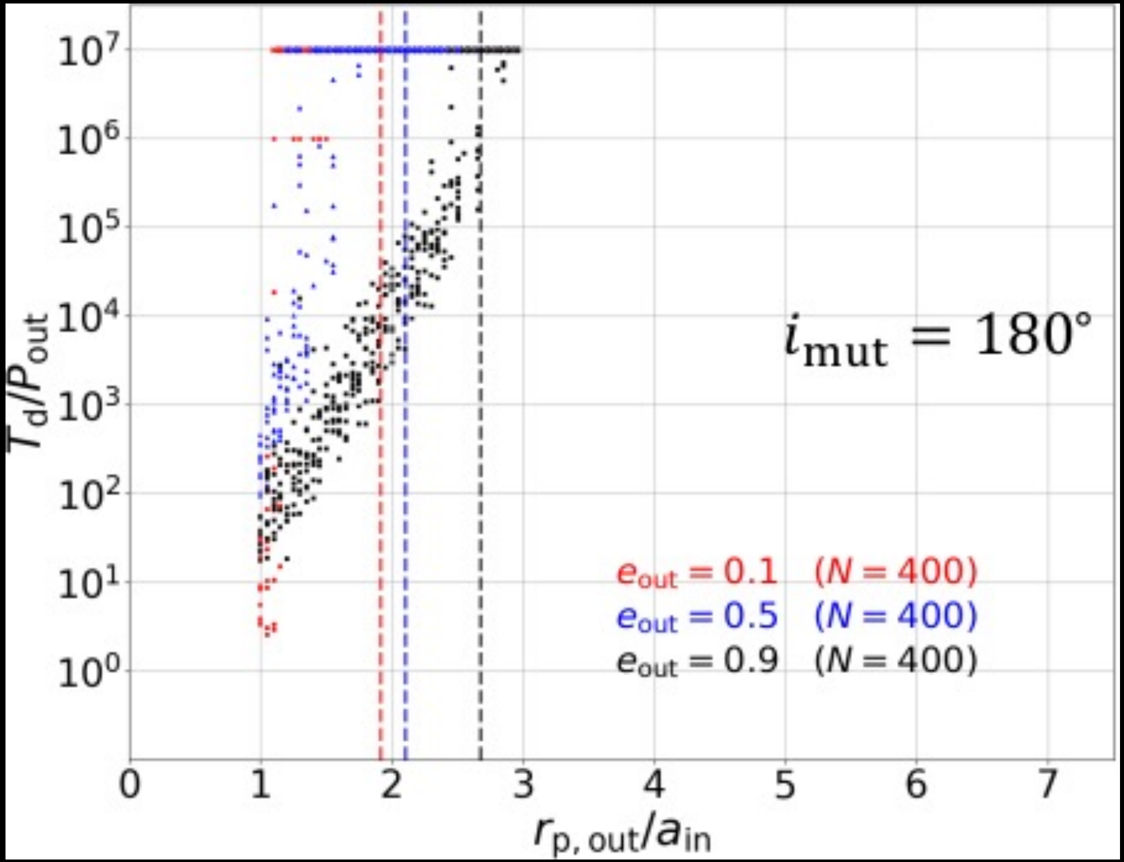
- Tiny difference in the input value of P_{in} leads to one or two order-of-magnitude difference of disruption timescales
- Initial phase difference of the three bodies also leads to one or two order-of-magnitude difference of disruption timescales
- We do not understand why just one or two order-of-magnitudes …

Hayashi, Trani & YS
ApJ 939(2022)81

T_d/P_{out} ($r_{p,out}/a_{in}$; i_{mut}, e_{out}) with random initial phases

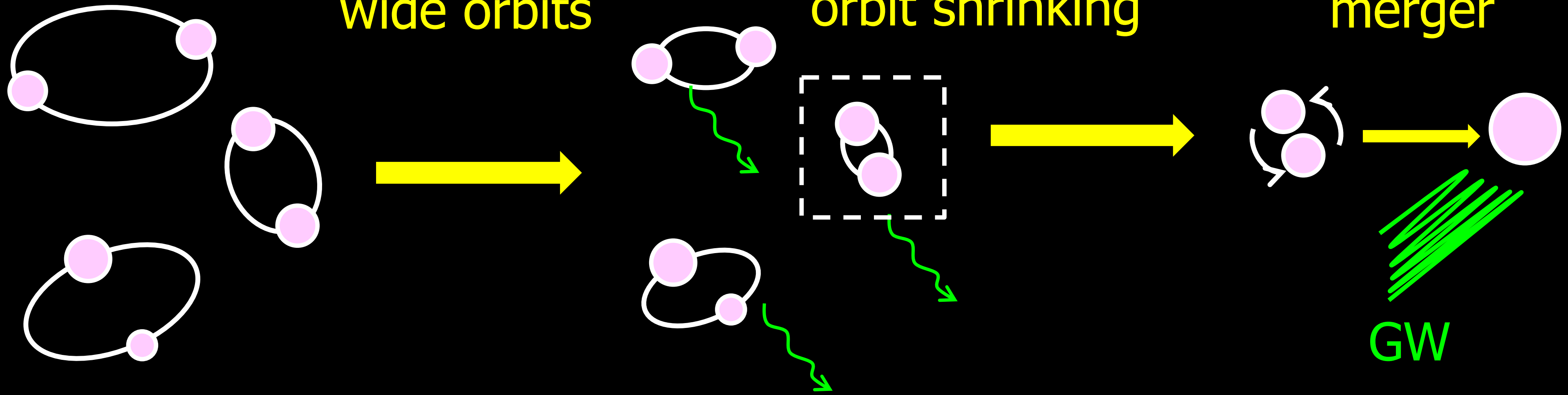


- Lyapunov stability boundaries (Vynatheya et al. 2022) are plotted in dashed lines for reference



3 Dynamical signature of triple systems including inner binary black-holes

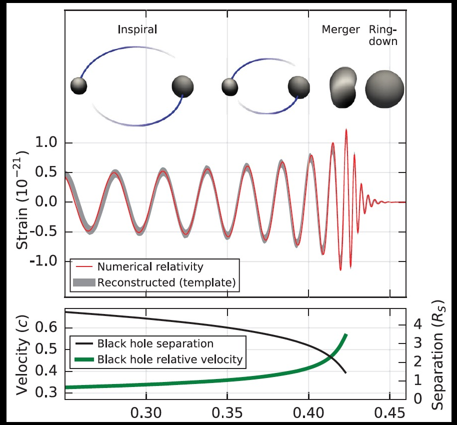
Generic picture of binary BH evolution



binary black holes form in
wide orbits

orbit shrinking

merger



LIGO/Virgo

weak GW(low-frequency)

BBHs would spend longer time in wide orbits before merging

Abundant longer orbital-period BBHs may remain undetected (e.g. ~10day orbital period~$10^{-6}\mathrm{Hz}$).

Detection strategy complementary to GW ?

Proposals to search for star-BH binaries

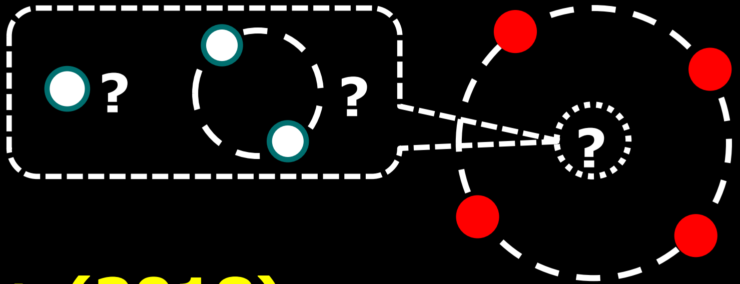
Gaia mission (2013-)

Astrometry of stars in Galaxy

$\sim 10^9$ stars eventually

RV with 200-350m/s precision

for brightest stars (Katz 2018)



Yamaguchi+ (2018)

5-year mission may detect

200-1000 star-BH binaries

TESS mission (2018-)

photometry of nearby stars ($\sim 12\mathrm{mag}$)

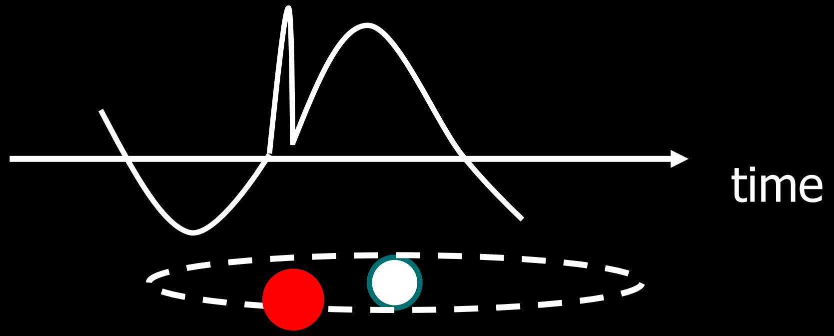
transit planets

Masuda & Hotokezaka (2019)

Light curve modulation

(relativistic effects, tidal deformation)

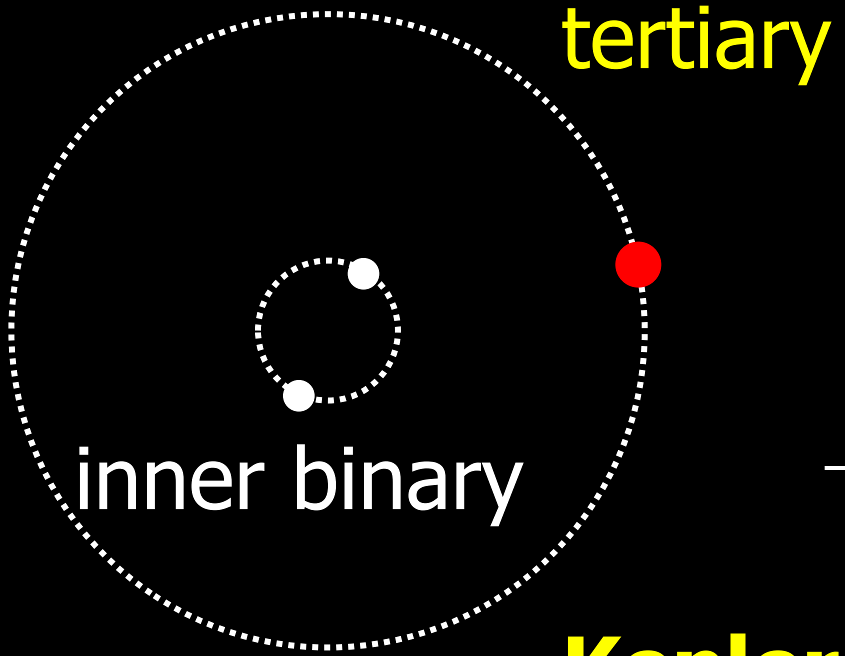
$\Rightarrow (10-100)$ star-BH binaries may be identified

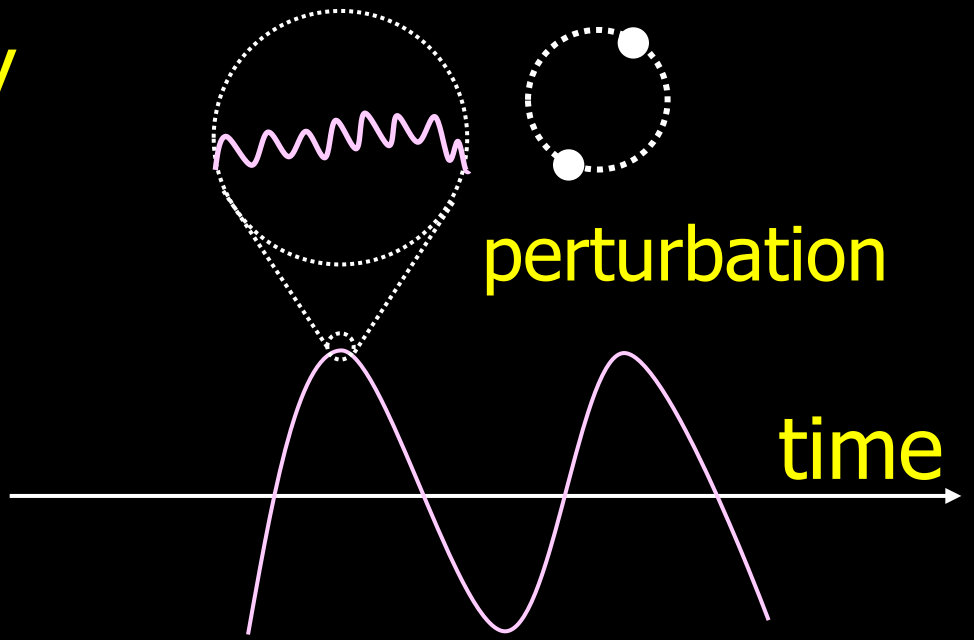


Some of them may be indeed a star-binary BH triple!

Can precise radial velocity follow-up unveil the inner BBH?

Radial velocity modulation of a tertiary star due to an inner binary



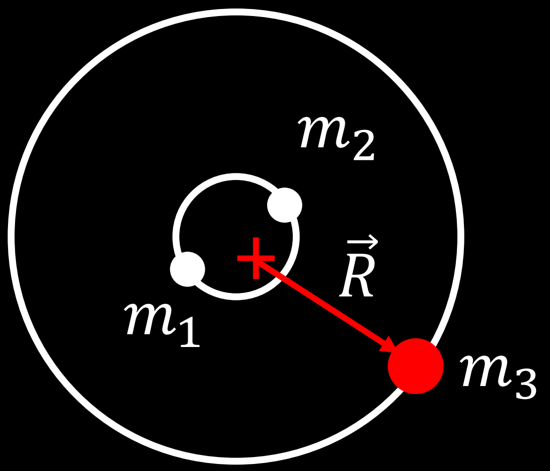


Kepler motion of the tertiary

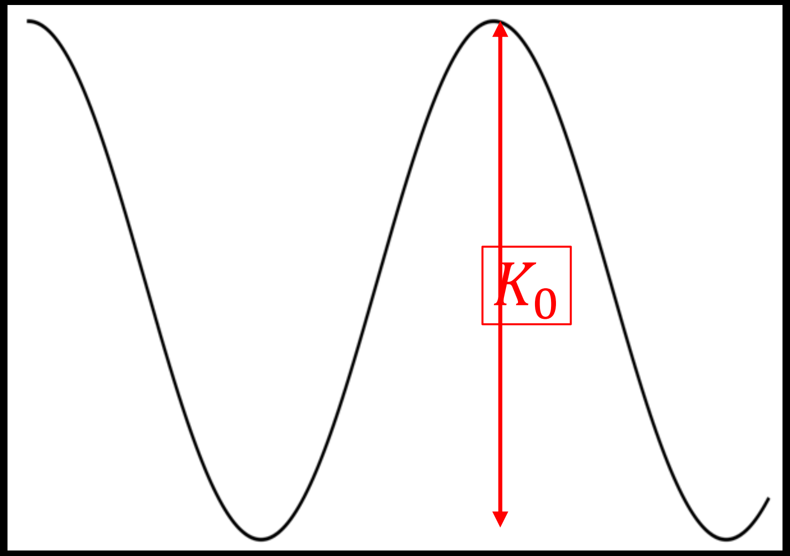
Toshinori Hayashi

Hayashi, Wang + YS: ApJ 890(2020)112
Hayashi + YS: ApJ 897(2020)29

RV modulations for coplanar triples

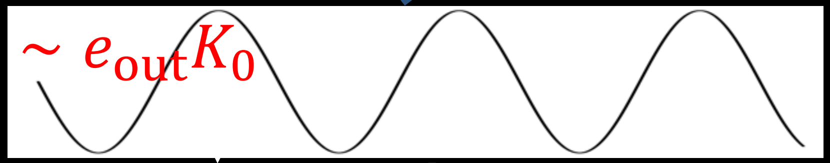


$$\mathrm{RV} =$$



period: $\boldsymbol{P_{out}}$

$$+$$



period: $\boldsymbol{P_{out}/2}$

first order in e_{out}

$$+$$

$$\mathcal{H} = \mathcal{H}_{\text{Kep}} - \mathcal{R}$$

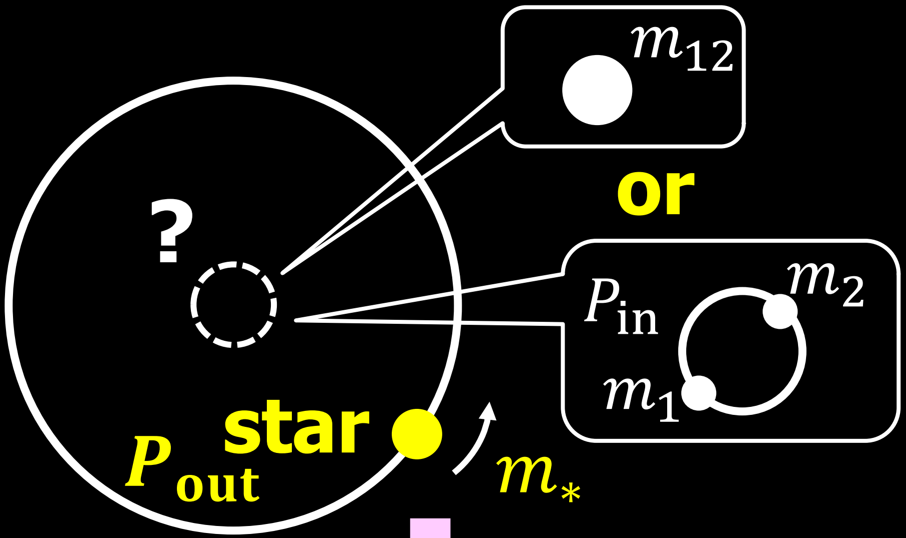
Perturbation theory

$$\left(\sqrt{\frac{m_1}{m_2}} + \sqrt{\frac{m_2}{m_1}}\right)^{-2} \left(\frac{a_{\text{in}}}{a_{\text{out}}}\right)^{3.5} K_0$$

period: $\boldsymbol{P_{in}/2}$

inner binary

RV modulations for non-coplanar triples

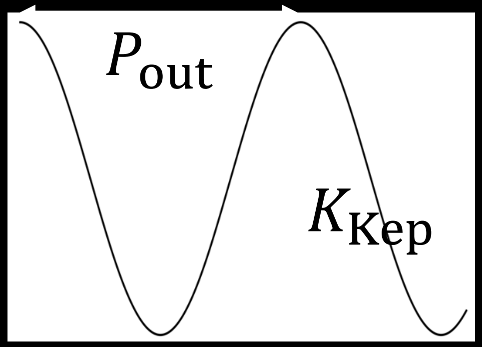


high-precision RV follow-up

Keplerian motion RV
+ RV variations by inner binary

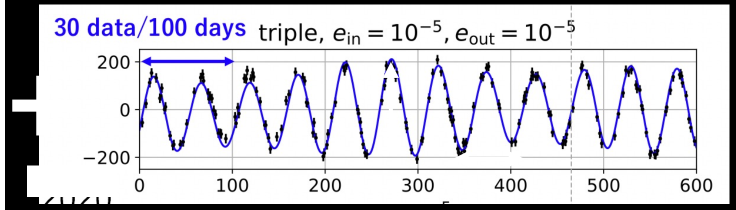
(i) Coplanar triple

$$\mathrm{Amp} \sim K_{\mathrm{Kep}} \left(\frac{P_{\mathrm{in}}}{P_{\mathrm{out}}}\right)^{\frac{7}{3}}$$



Kepler motion

+



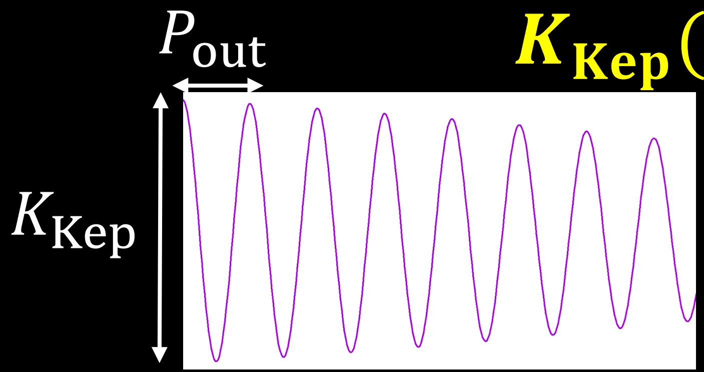
$$\mathrm{period} \sim P_{\mathrm{in}}/2$$

+ Short-term RV variations (inner-binary perturbation)

(ii) Non-coplanar triple

Inclination $I_{\mathrm{out}}(t)$ modulated in the Kozai-Lidov timescale

$$K_{\mathrm{Kep}}(t) = K_0 \sin I_{\mathrm{out}}(t)$$



Amplitude of Kepler RV varies with the timescale

Coplanar circular triples

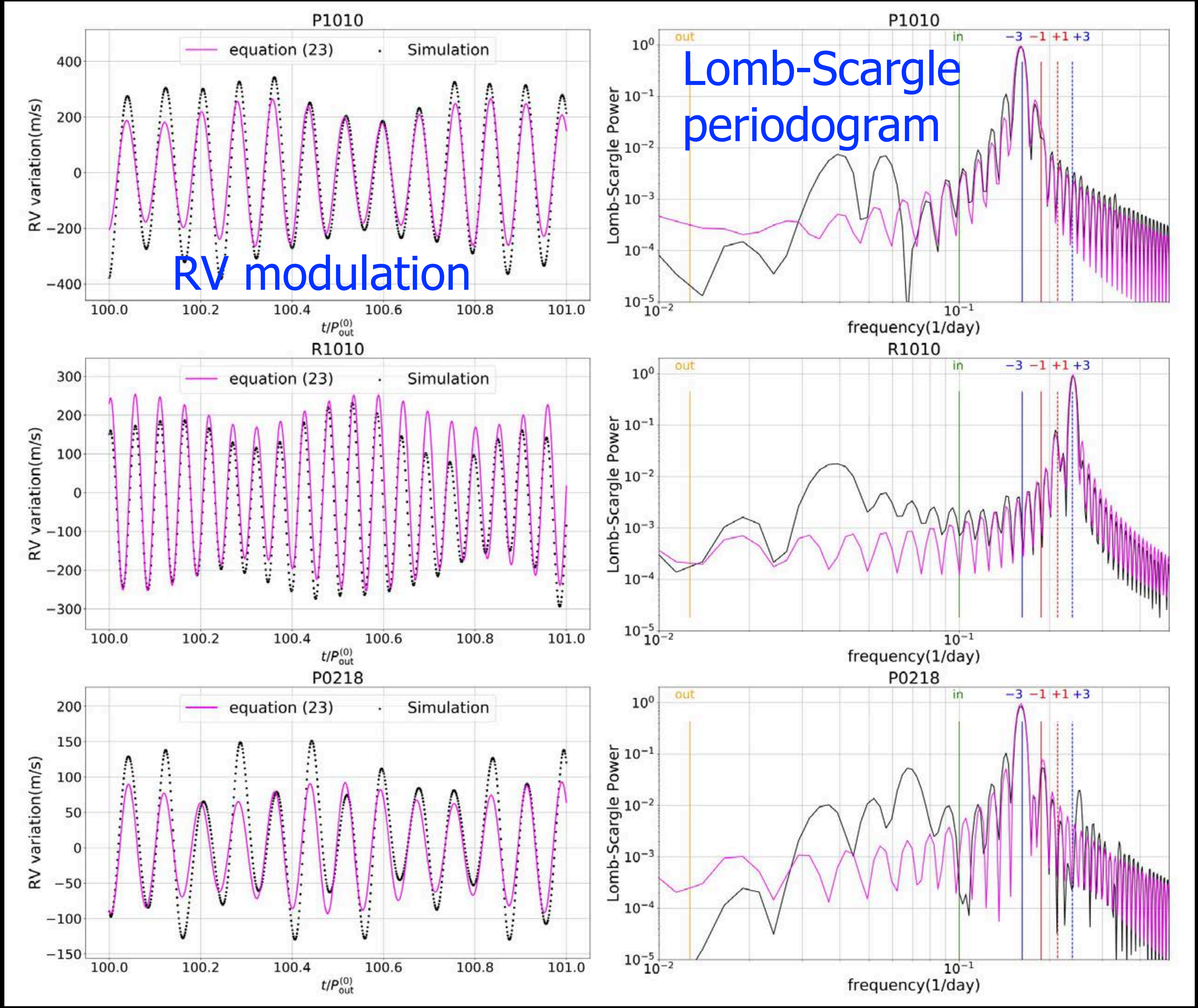
Simulation against
Perturbative model
(Morais & Correia 2008, 2012)

$$\nu_{-3} \equiv 2\nu_{\text{in}} - 3\nu_{\text{out}},$$
$$\nu_{-1} \equiv 2\nu_{\text{in}} - \nu_{\text{out}}.$$

Prograde
equal-mass

Retrograde
equal-mass

Prograde
unequal-mass



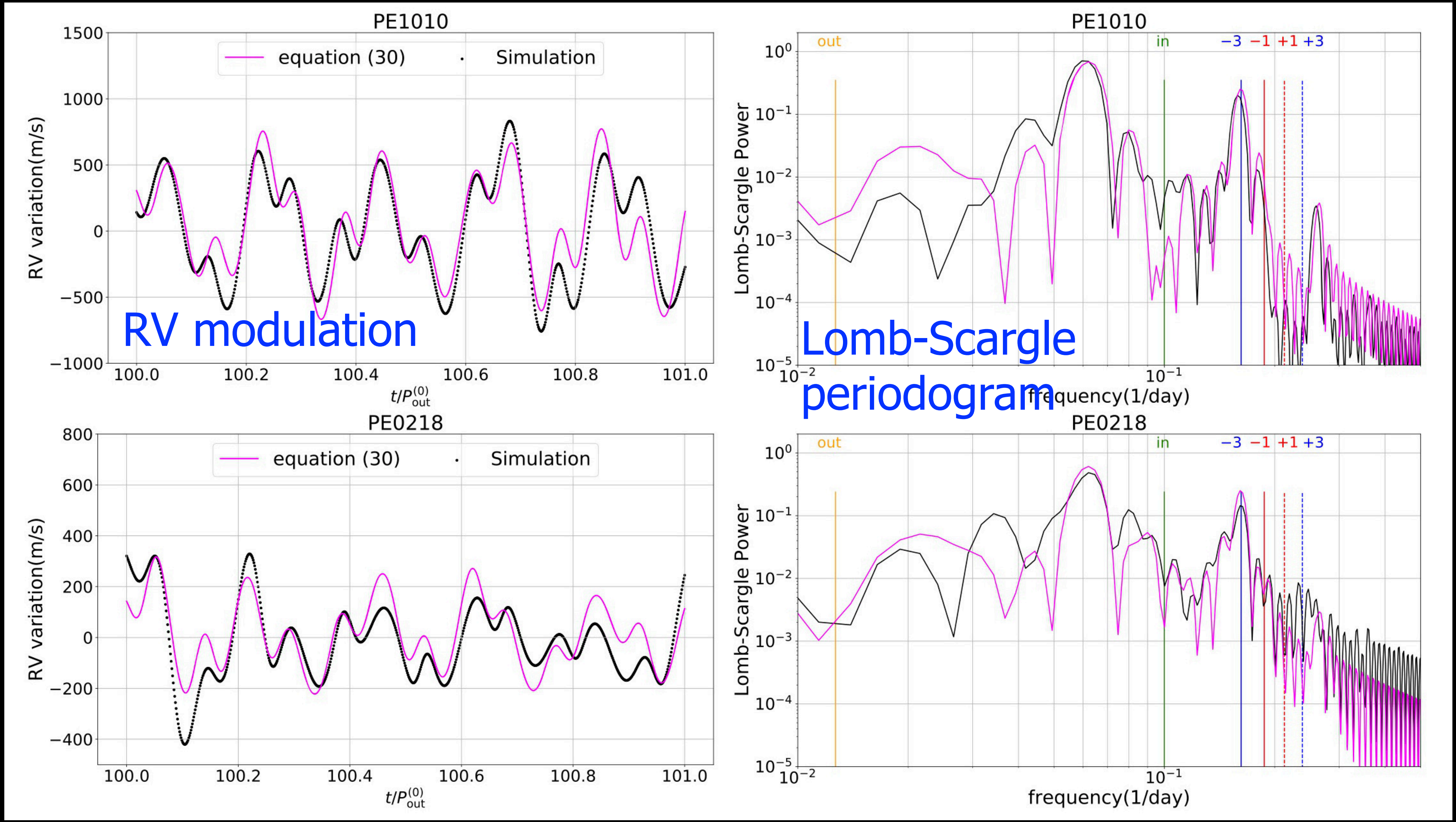
Hayashi & YS (2020)

Coplanar eccentric triples

Simulation against Perturbative model (Morais & Correia 2008, 2012)

Prograde equal-mass

Prograde unequal-mass



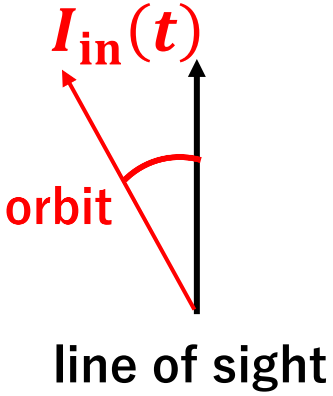
RV modulation

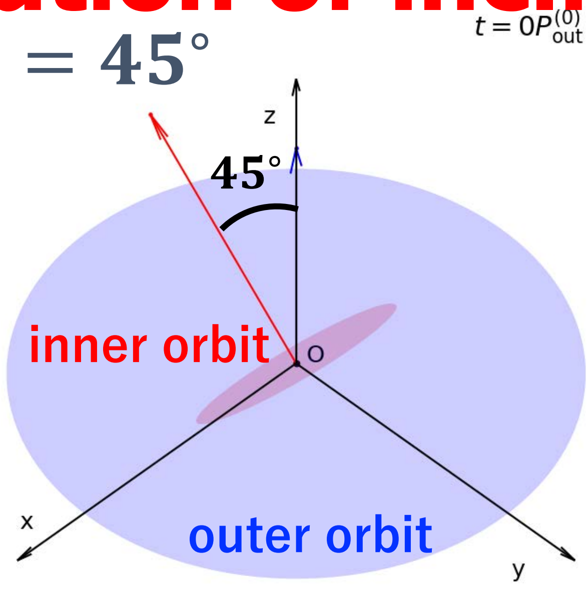
Lomb-Scargle periodogram

Hayashi & YS (2020)

Evolution of inclination for non-coplanar triples

$i_{\mathrm{mut}} = 45°$



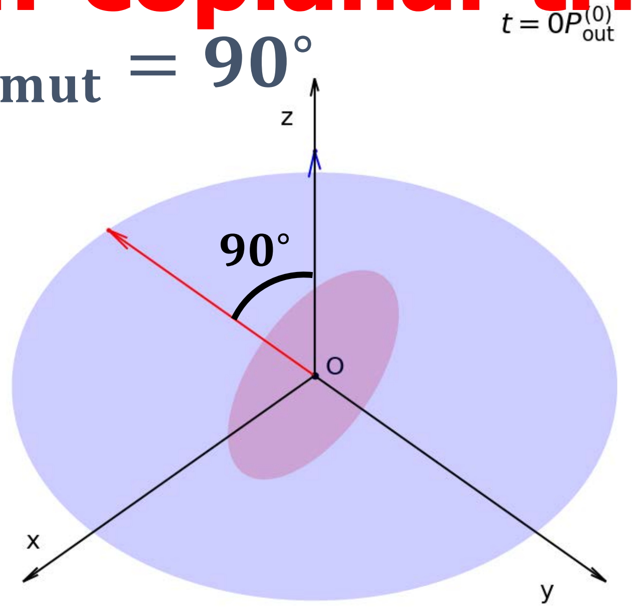


$P_{\mathrm{out}} = 78.9$ days
$P_{\mathrm{in}} = 10$ days

$m_1 = m_2 = 10M_{\odot}$
$m_* = 3M_{\odot}$

$e_{\mathrm{out}} = 0.03$
$e_{\mathrm{in}} = 10^{-5}$

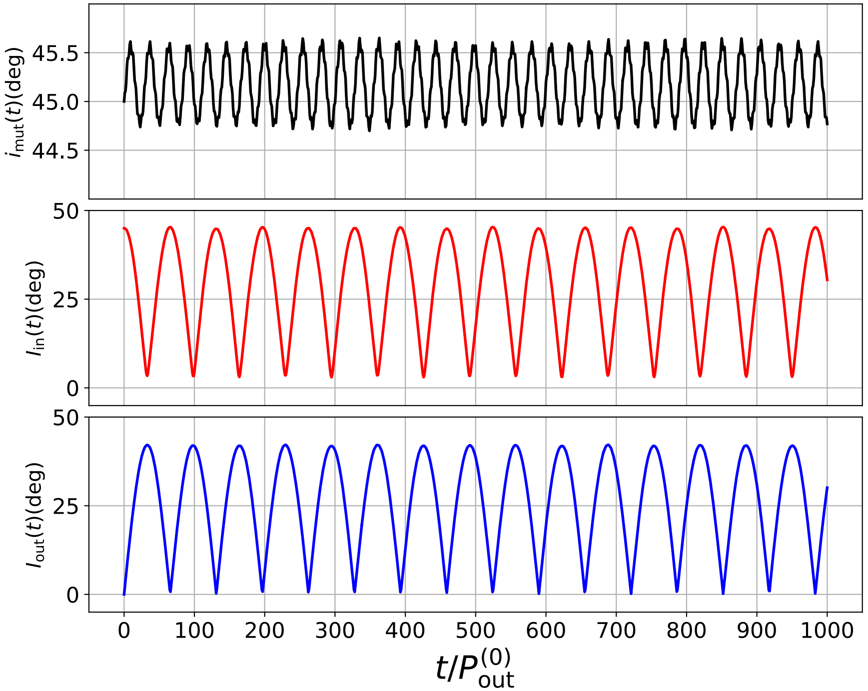
$i_{\mathrm{mut}} = 90°$



$i_{\mathrm{mut}}(t)$

$I_{\mathrm{in}}(t)$

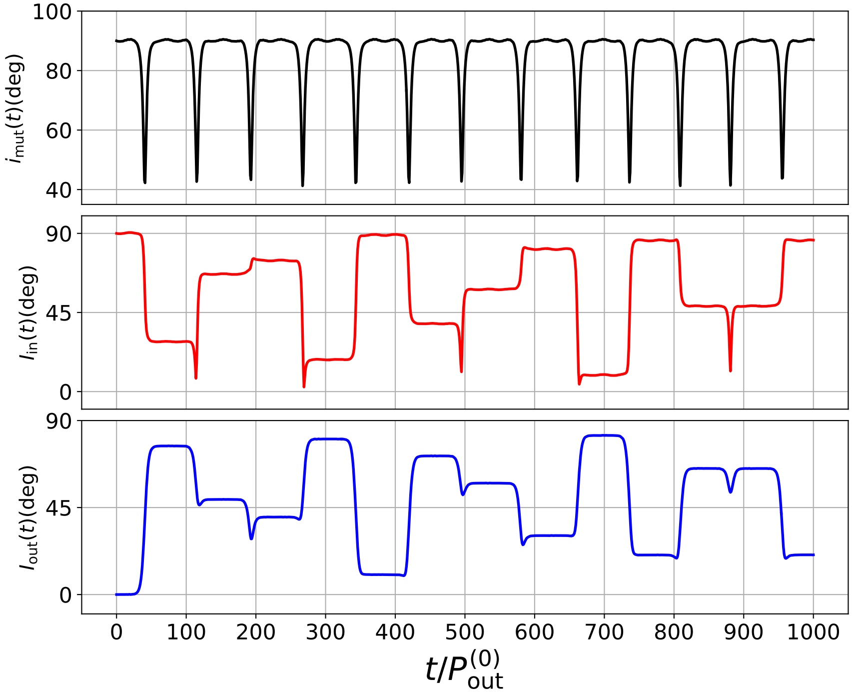
$I_{\mathrm{out}}(t)$



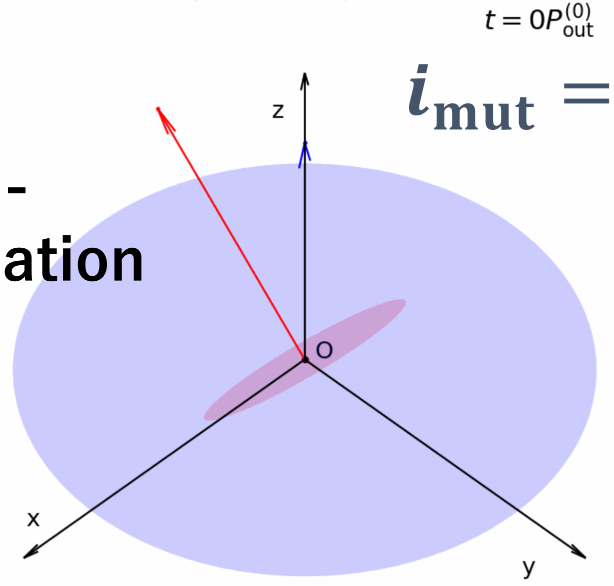
$i_{\mathrm{mut}}(t)$

$I_{\mathrm{in}}(t)$

$I_{\mathrm{out}}(t)$



Evolution of inclination for non-coplanar triples

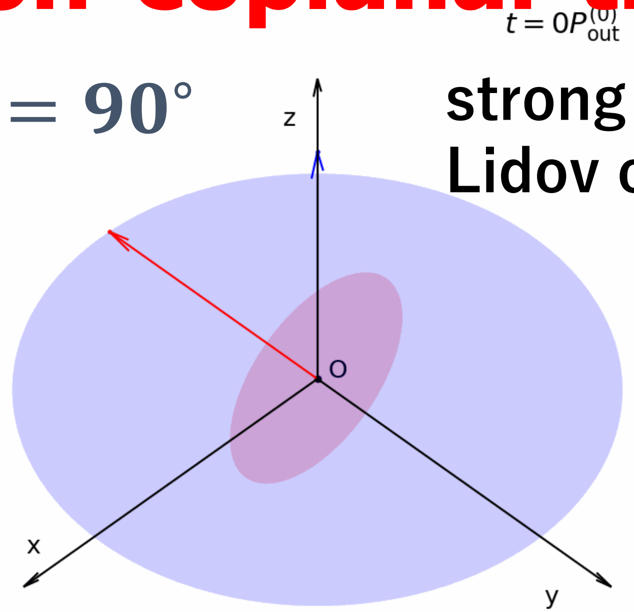


$$i_{\rm mut} = 45^\circ$$

weak Kozai-Lidov oscillation

⇒ small-amplitude regular precession

$$i_{\rm mut} = 90^\circ$$



strong Kozai-Lidov oscillation

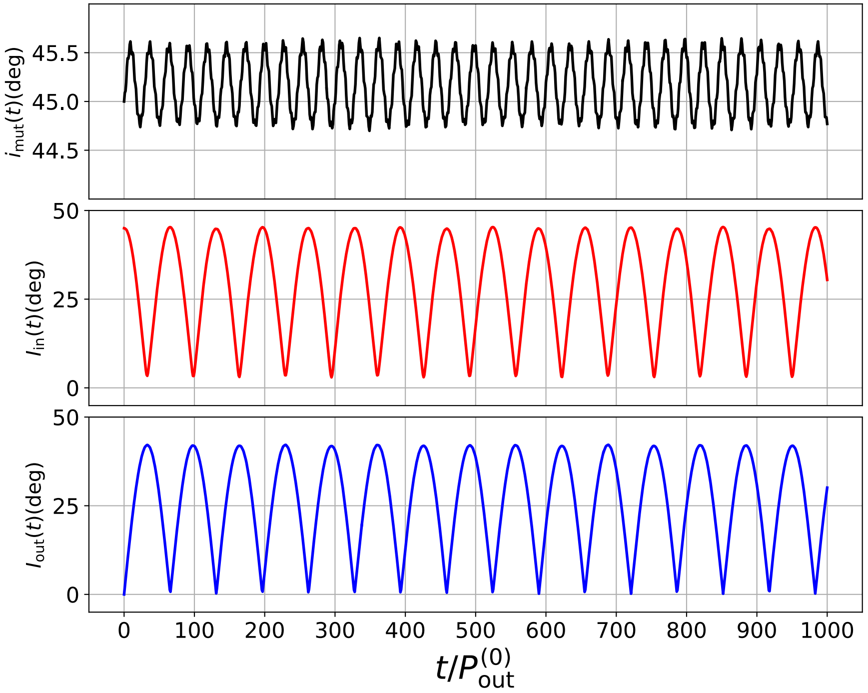
⇒ large-amplitude sporadic precession

$$K_{\rm Kep} = K_0 \sin I_{\rm out}(t)$$

$i_{\rm mut}(t)$

$I_{\rm in}(t)$

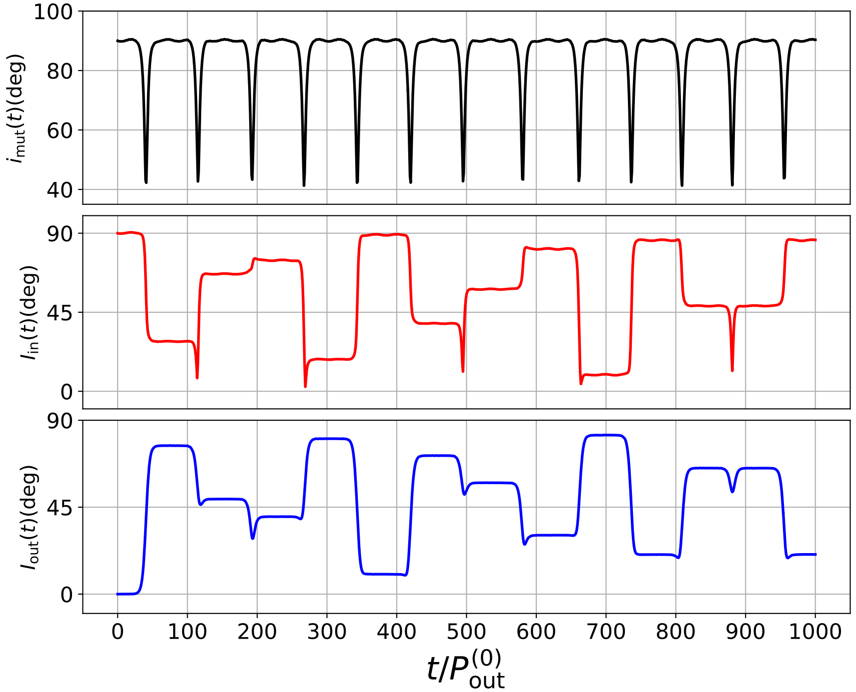
$I_{\rm out}(t)$



$i_{\rm mut}(t)$

$I_{\rm in}(t)$

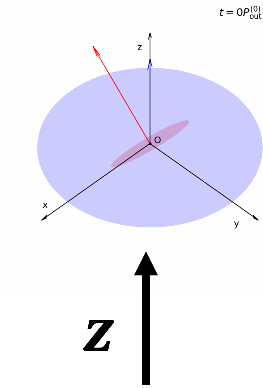
$I_{\rm out}(t)$



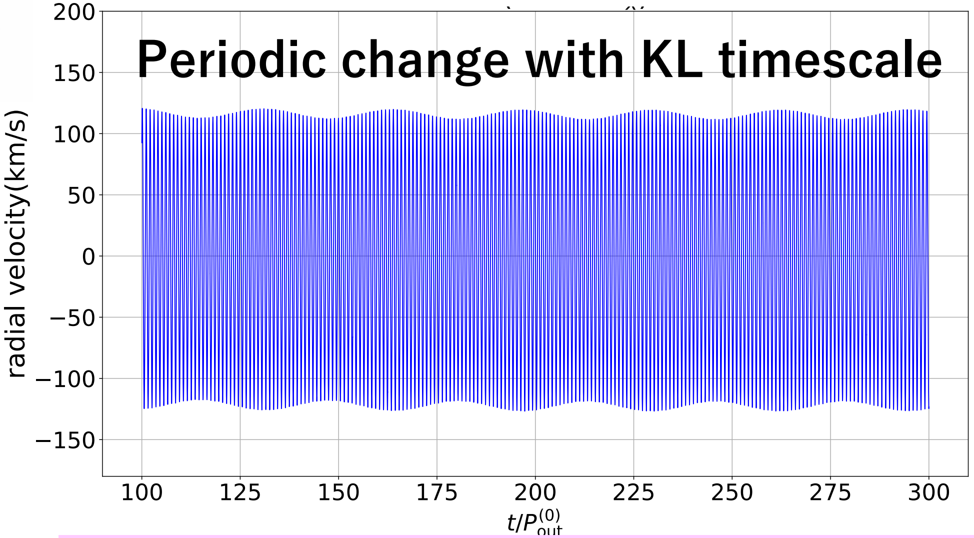
Evolution of radial velocity for non-coplanar triples

$i_{\mathrm{mut}} = 45°$

$K_{\mathrm{Kep}} = K_0 \sin I_{\mathrm{out}}(t)$

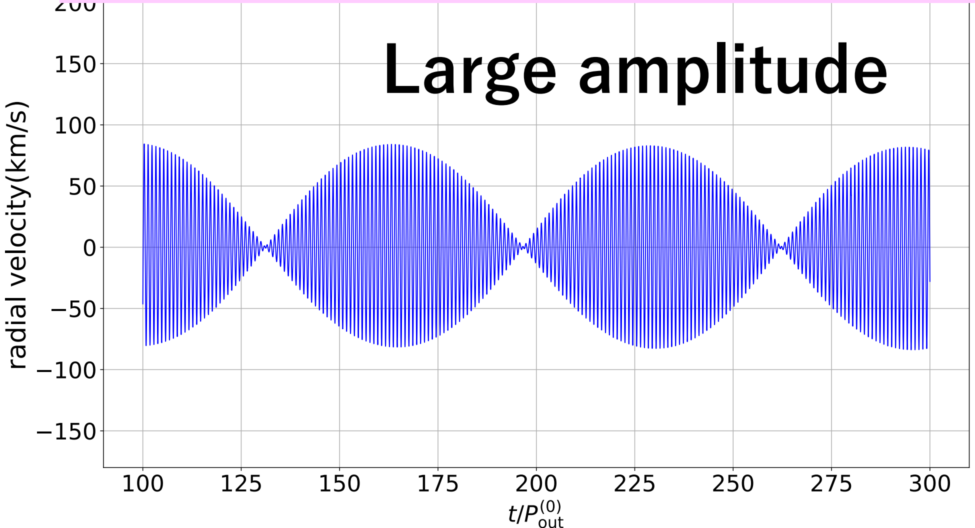


x-direction (near edge-on) total RV



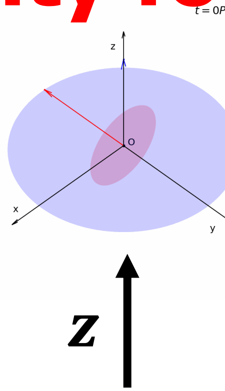
Periodic change with KL timescale

z-direction (near face-on) total RV

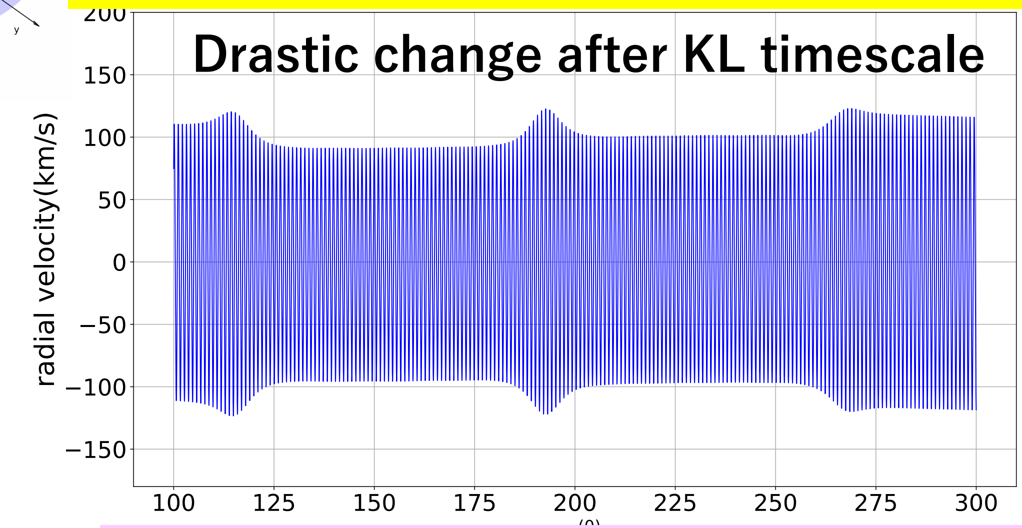


Large amplitude

$i_{\mathrm{mut}} = 90°$

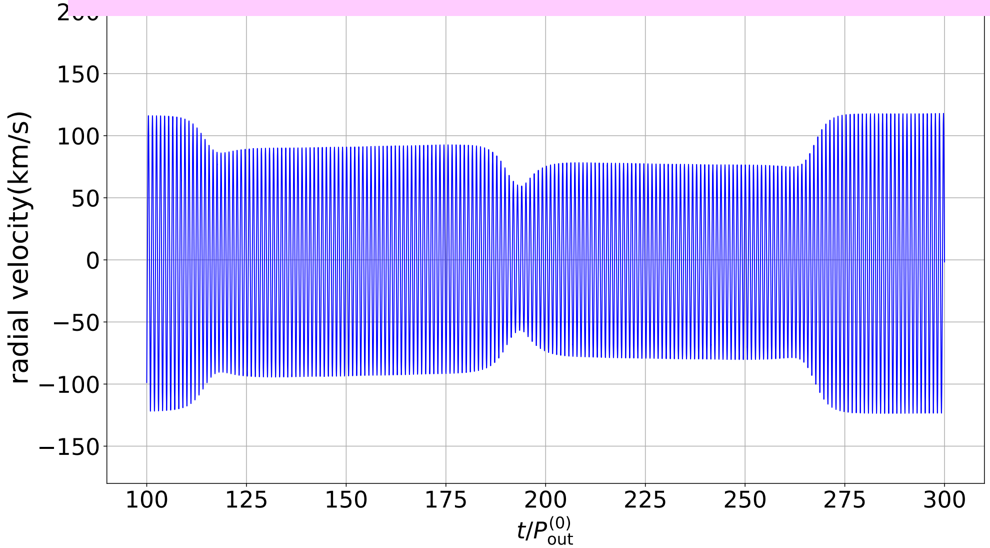


x-direction (near edge-on) total RV



Drastic change after KL timescale

z-direction (near face-on) total RV



4 Summary

Dynamical stability of triple systems

- Lagrange vs. Lyapunov stability for triple systems
  - Conventional criteria correspond to Lyapunov stability
  - Lagrange stability is more relevant in considering the fate of astronomical triples, i.e., disruption timescale
- We derive triple disruption timescales as a function of orbital parameters (within intrinsic variation of one or two order-of-magnitudes due to the chaotic dynamics of triples)
- Strong dependence on the mutual inclination
  - Strongly misaligned systems ($60^\circ < i_{\mathrm{mut}} < 150^\circ$) are destabilized due to the Kozai-Lidov oscillations over longer timescales
  - Coplanar retrograde triples are significantly stabilized

Dynamical signature of inner binary black holes in triple systems

- Radial velocity (RD) monitoring of future star-black hole binary candidates may reveal inner binary black holes (instead of single black holes) in those systems
  - short-term RD variations

    Hayashi, Wang + YS: ApJ 890(2020)112
    - periodic modulations of O(1) percent of the Kepler orbital velocity amplitude with a half inner orbital period
  - long-term RD variations in inclined triples

    Hayashi + YS: ApJ 897(2020)29
    - the semi-amplitude of the Kepler orbital velocity modulated periodically by the precession of the inner and outer orbits over roughly the von Zeipel-Kozai-Lidov oscillation timescale