

# Clustering statistics on a light-cone in the cosmological redshift space

Yasushi Suto

Dept. of Phys. and RESCEU, The University of Tokyo, Japan

November 17, 1999 (The 4th RESCEU Symposium)



based on the collaboration with

Y.P. Jing, Tetsu Kitayama, Hiromitsu Magira,

Takahiko Matsubara, Hiroaki Nishioka, Kazuhiro Yamamoto

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# 1 Cosmological effects in the high- $z$ universe

## linear redshift-space distortion

coherence between density and velocity fields

## finger-of-God

nonlinear velocity field in virialized regions

## cosmological redshift-space distortion

anisotropy due to the geometry of the universe

## light-cone effect

cosmological observations are on a light-cone

## gravitational lensing

distortion due to the mass along the line-of-sight

## bias

(luminous) objects  $\neq$  (dark) mass distribution



## Noise ?

hampers the proper understanding of the clustering  
in the universe

## Signal ?

provides additional cosmological information

**Predictions of the two-point clustering statistics  
on a light-cone in the cosmological redshift space**

## 2 Cosmological redshift-space distortion

### ★ separations in the redshift space

parallel to the line-of-sight:  $x_{s\parallel}(z) = c\delta z/H_0$

perpendicular to the line-of-sight:  $x_{s\perp}(z) = cz\delta\theta/H_0$

$\Downarrow$

### ★ mapping to those in the real (comoving) space

$$x_{\parallel}(z) = c_{\parallel}(z)x_{s\parallel}(z), \quad x_{\perp}(z) = c_{\perp}(z)x_{s\perp}(z),$$

with

$$c_{\parallel}(z) = \frac{1}{\sqrt{\Omega_0(1+z)^3 + (1 - \Omega_0 - \lambda_0)(1+z)^2 + \lambda_0}}$$

$$c_{\perp}(z) = \frac{H_0(1+z)}{cz} D_A(z; \Omega_0, \lambda_0)$$

(Alcock & Paczyński 1979; Ballinger, Peacock & Heavens 1996; Matsubara & YS 1996)

### **Anisotropic power-spectrum in the cosmological redshift space**

$$P^{(\text{CRD})}(k_{s\perp}, k_{s\parallel}; z) = \frac{1}{c_{\perp}(z)^2 c_{\parallel}(z)} P^{(S)}\left(\frac{k_{s\perp}}{c_{\perp}(z)}, \frac{k_{s\parallel}}{c_{\parallel}(z)}; z\right),$$

where  $k_{s\parallel}(z) = c_{\parallel}(z)k_{\parallel}(z)$ ,  $k_{s\perp}(z) = c_{\perp}(z)k_{\perp}(z)$ , and

$$P^{(S)}(k_{\perp}, k_{\parallel}; z) = P_{\text{mass}}^{(R)}(k; z) \times b^2(z) \\ \times \left[ 1 + \beta(z) \left( \frac{k_{\parallel}}{k} \right)^2 \right]^2 \times D[k_{\parallel} \sigma_p(z)]$$

# Angle-averaged $P(k)$ in redshift space at $z = 0$ and 2.2 in SCDM, LCDM and OCDM theories vs. N-body simulations ( $N = 256^3$ )

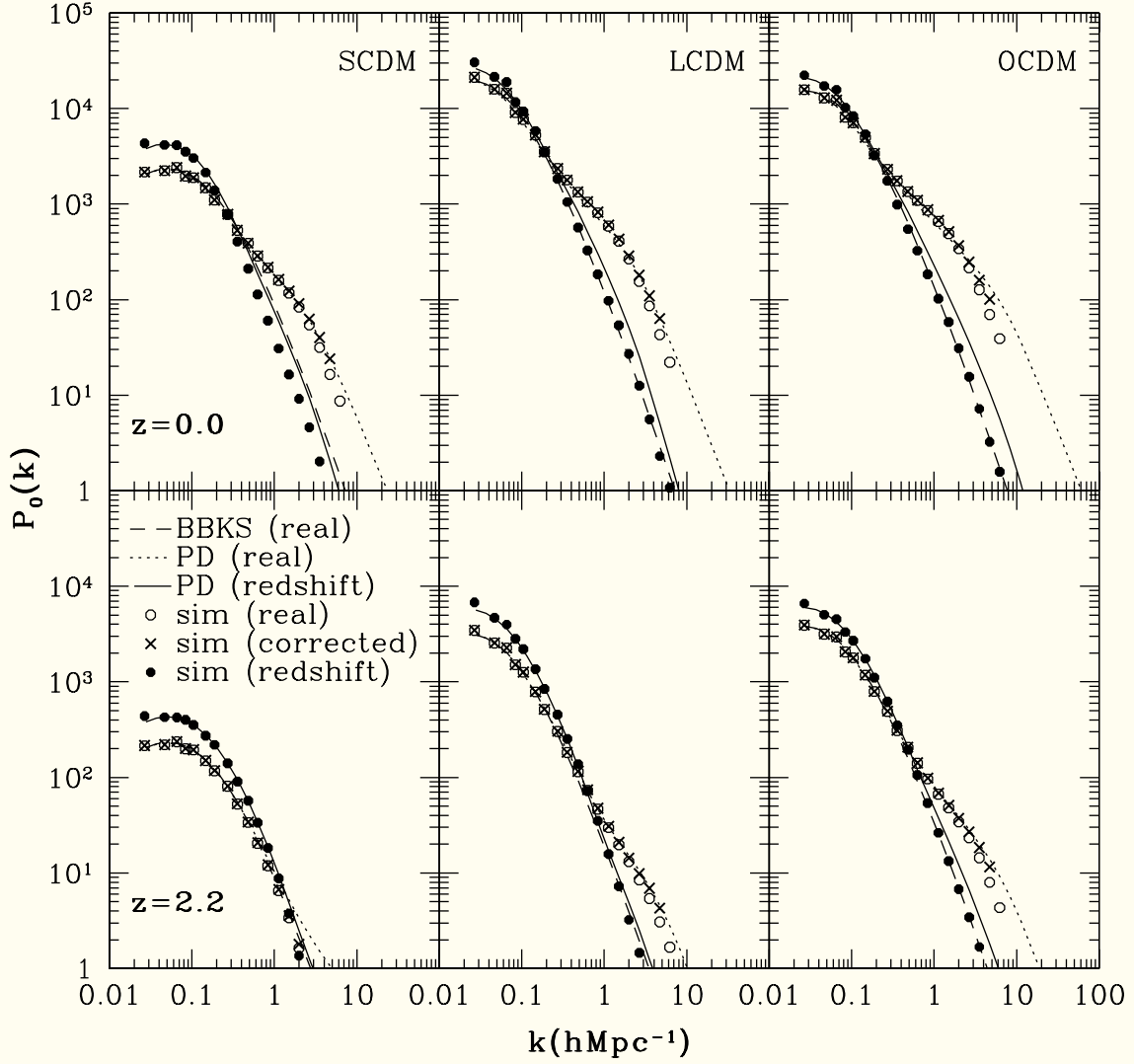


Figure 1: Power spectra for representative CDM models at  $z = 0$  (upper panels) and  $z = 2.2$  (lower panels) neglecting the geometrical effect in the redshift-space distortion. The fluctuation amplitudes are normalized according to the cluster abundances. (Magira, Jing & YS 2000)

2D contours of  $P(k)$  in redshift space  
at  $z = 2.2$  in SCDM, LCDM and OCDM:

linear theory (top panels)

nonlinear model (middle panels)

N-body simulations (bottom panels)

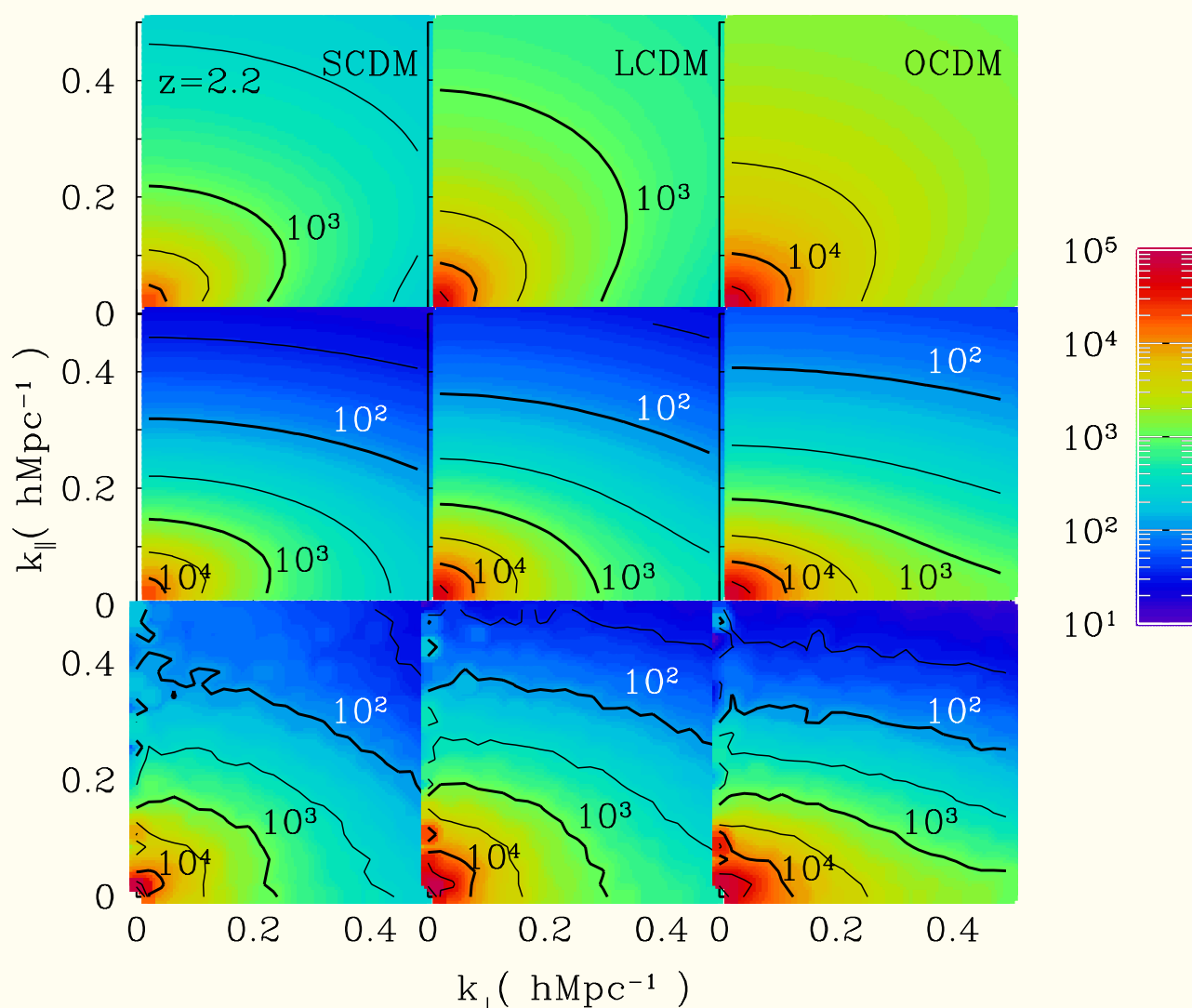


Figure 2: (Magira, Jing & YS 2000)

Expected confidence contours on  $\Omega_0$ - $\lambda_0$  plane  
at  $z = 2.2$  in SCDM, LCDM and OCDM  
from cosmological redshift-space distortion:

$N = 5 \times 10^3$  (upper panels)

$N = 5 \times 10^4$  (middle panels)

$N = 5 \times 10^5$  (bottom panels)

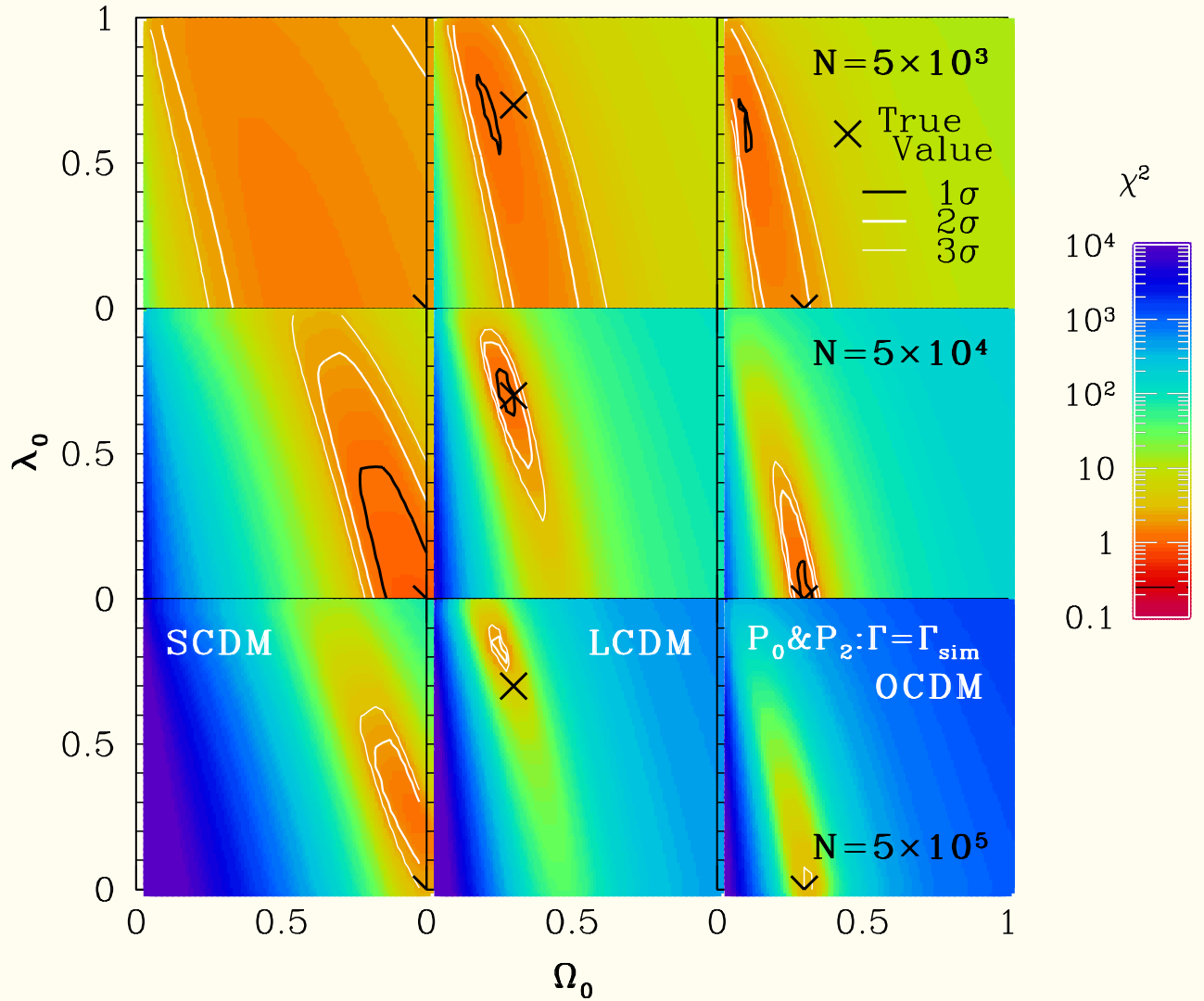


Figure 3: (Magira, Jing, & YS 1999)

### 3 Cosmological light-cone effect

★ All cosmological observations are carried out on a null hypersurface or a light-cone.

A fairly conventional approximation is to replace the light-cone by the constant-time hypersurface at  $z = 0$  (at least for surveys with depth  $z < 0.1$ ).

For  $z > 0.1$ , however, this approximation apparently breaks down.



★ One should simultaneously take account of the intrinsic evolution of the clustering.

- future X-ray selected clusters
- on-going wide-field surveys of galaxies and quasars including 2dF and SDSS

#### count-in-cell statistics for higher-order moments

Matsubara, YS, & Szapudi, ApJ 491(1997)L1

#### perturbation analysis for $\beta$ -parameter

Nakamura, Matsubara & YS, ApJ 494(1998)13

#### general formulation

Mataresse, Coles, Lucchin, Moscardini, MNRAS 286(1997)115

Yamamoto & YS, ApJ 517(1999)1

#### review

YS, Magira, Jing, Matsubara & Yamamoto,  
Prog.Theor.Phys.Suppl., 133(1999)183

**Stay tuned !** the next talk by Yamamoto, and a poster by Nishioka

### 3.1 Two-point correlation functions of X-ray selected clusters

**Express cluster properties in terms of the halo mass**

**gas temperature**  $T_X$  virial equilibrium

**X-ray luminosity**  $L_X$  observed L-T relation + band correction

**X-ray flux**  $S_X$  luminosity distance

**bias parameter**  $b(z, M)$  analytic model (Mo & White 1996) + N-body simulations (Jing 1998)

**selection function** halo mass function from the Press-Schechter theory

**redshift-space distortion** Kaiser (1987), Peacock & Dodds (1996), Magira et al. (2000)

**light-cone effect** Yamamoto & YS (1999), Yamamoto, Nishioka & YS (1999)

Two-point correlation functions of clusters on the light-cone brighter than the X-ray flux-limit  $S_{\text{lim}}$

$$\xi_{X-\text{cl}}^{\text{LC}}(R; > S_{\text{lim}}) = \frac{\int_{z_{\text{max}}}^{z_{\text{min}}} dz \frac{dV_c}{dz} n_0^2(z) \xi_{\text{cl}}^{\text{S}}(R, z(r); > S_{\text{lim}})}{\int_{z_{\text{max}}}^{z_{\text{min}}} dz \frac{dV_c}{dz} n_0^2(z)}$$

(YS, Yamamoto, Kitayama & Jing 1999)



# Model predictions in SCDM, LCDM and OCDM of redshift-space distortion and light-cone effect for X-ray cluster correlation functions

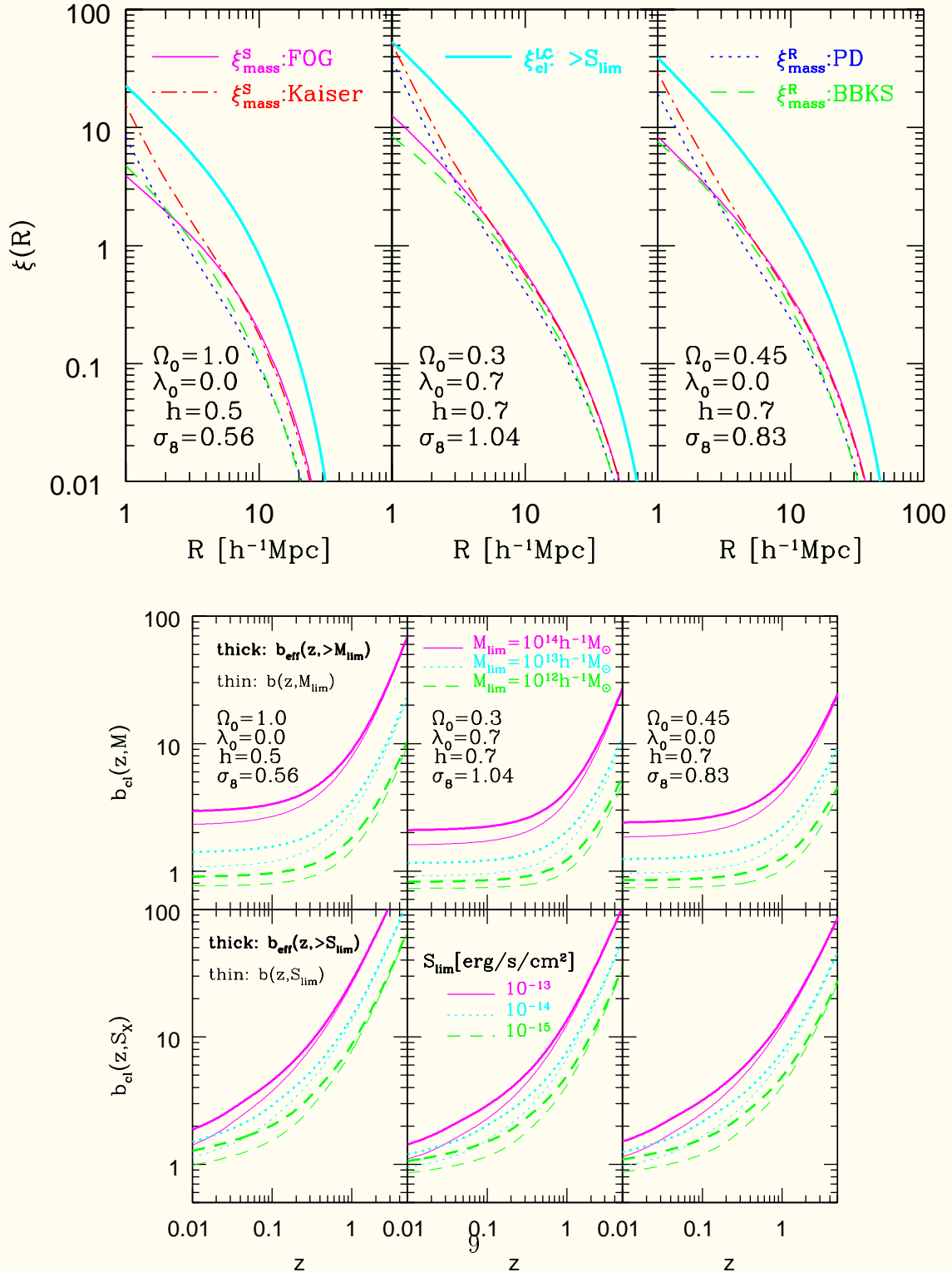


Figure 4: (YS, Yamamoto, Kitayama & Jing 1999)

## Difference due to various selection functions X-ray flux-, temperature-, luminosity-limit

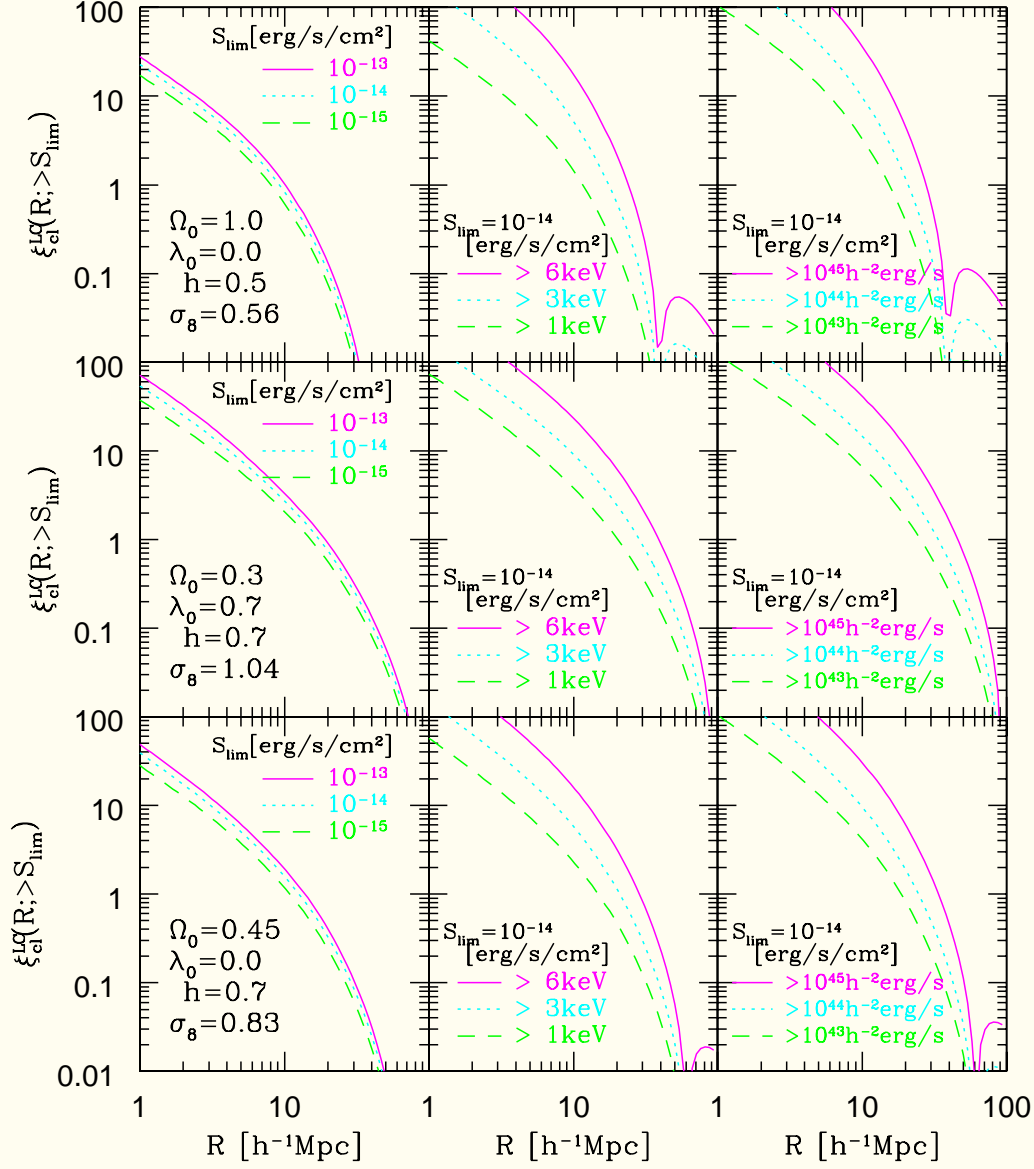


Figure 5: *Left:*  $S_{lim} = 10^{-13}$  (solid lines),  $10^{-14}$  (dotted) and  $10^{-15}\text{erg/s/cm}^2$  (dashed). *Center:*  $T_{lim} = 1$  (solid),  $3$  (dotted) and  $6\text{keV}$  (dashed) in the X-ray flux-limited sample ( $S_{lim} = 10^{-14}\text{erg/s/cm}^2$ ). *Right:*  $L > 10^{45}$  (solid),  $10^{44}$  (dotted) and  $10^{43}h^{-2}\text{erg/s/cm}^2$  (dashed) in the X-ray flux-limited sample ( $S_{lim} = 10^{-14}\text{erg/s/cm}^2$ ).

$\Omega_0$ -dependence of X-ray cluster correlation  
lengths defined by  $\xi_{\text{cl}}^{\text{LC}}(r_{\text{c0}}; S_{\text{lim}}) = 1$

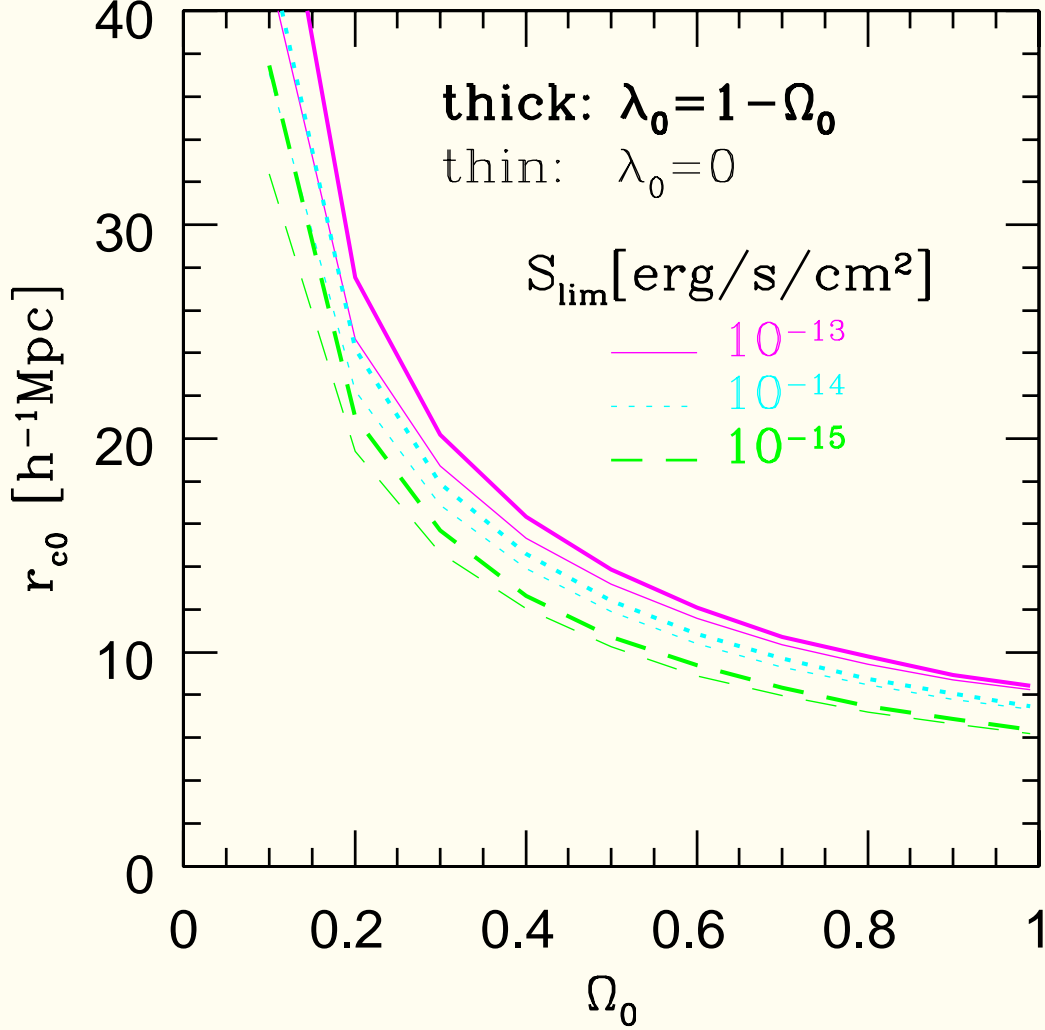


Figure 6: The X-ray flux-limit  $S_{\text{lim}}$  is  $10^{-13}$  (solid lines),  $10^{-14}$  (dotted) and  $10^{-15} \text{erg/s/cm}^2$  (dashed). For each  $S_{\text{lim}}$ , we plot the case of  $\lambda_0 = 1 - \Omega_0$  in thick lines, and  $\lambda_0 = 0$  in thin lines. Fluctuation amplitudes are normalized by the cluster abundance.

## 3.2 Two-point correlation functions and power spectra of SDSS galaxy and QSO samples

Theoretical predictions which take into account the light-cone and cosmological distortion effects simultaneously.

**Decompose  $P^{(\text{CRD})}(k, \mu_k; z)$  into multipoles**

$$P_l^{(\text{CRD})}(k; z) \equiv \frac{2l+1}{2} \int_{-1}^1 d\mu_k P^{(\text{CRD})}(k, \mu_k; z) L_l(\mu_k)$$

**Compute  $\xi_l^{(\text{CRD})}(x; z)$  from  $P_l^{(\text{CRD})}(k; z)$**

$$\xi_l^{(\text{CRD})}(x; z) = \frac{1}{2\pi^2 i^l} \int_0^\infty P_l^{(\text{CRD})}(k; z) j_l(kx) k^2 dk$$

**Average over the light-cone**

$$P_l^{(\text{LC,CRD})}(k_s) = \frac{\int_{z_{\text{max}}}^{z_{\text{min}}} dz \frac{dV_c}{dz} [\phi(z) n_0^{\text{com}}(z)]^2 c_\perp(z)^2 c_\parallel(z) P_l^{(\text{CRD})}(k_s; z)}{\int_{z_{\text{max}}}^{z_{\text{min}}} dz \frac{dV_c}{dz} [\phi(z) n_0^{\text{com}}(z)]^2 c_\perp(z)^2 c_\parallel(z)}$$

$$\xi_l^{(\text{LC,CRD})}(x_s) = \frac{\int_{z_{\text{max}}}^{z_{\text{min}}} dz \frac{dV_c}{dz} [\phi(z) n_0^{\text{com}}(z)]^2 c_\perp(z)^2 c_\parallel(z) \xi_l^{\text{CRD}}(x_s; z)}{\int_{z_{\text{max}}}^{z_{\text{min}}} dz \frac{dV_c}{dz} [\phi(z) n_0^{\text{com}}(z)]^2 c_\perp(z)^2 c_\parallel(z)}$$

$\Downarrow$

★ Evaluate the effects for SDSS galaxy ( $B < 19$ ) and QSO ( $B < 20$ ) samples using the luminosity functions of Loveday et al. (1992) and Boyle, Shanks & Peterson (1988)  $\rightarrow \phi(z)$ .

(YS, Magira & Yamamoto 1999)

**Angle-averaged  $P(k)$  on the light-cone  
in the cosmological redshift space  
divided by the linear theory prediction  
in real space at  $z = 0$ :  
in SCDM (Left) and LCDM (Right)  
for Galaxies (Upper) and QSOs (Lower)**

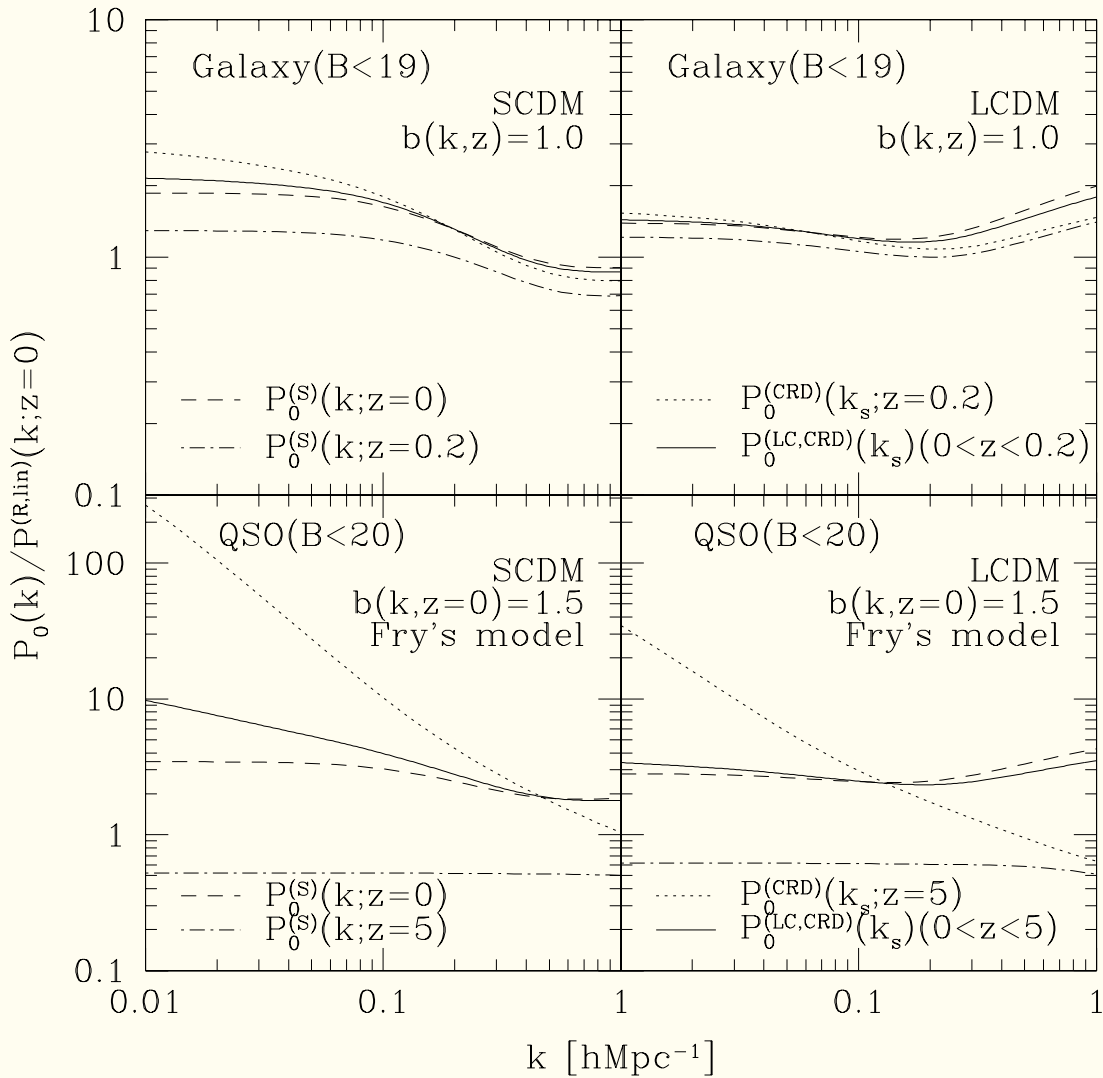


Figure 7: (YS, Magira & Yamamoto 1999)

**Angle-averaged  $\xi(x)$  on the light-cone  
in the cosmological redshift space  
divided by the linear theory prediction  
in real space at  $z = 0$ :  
in SCDM (Left) and LCDM (Right)  
for Galaxies (Upper) and QSOs (Lower)**

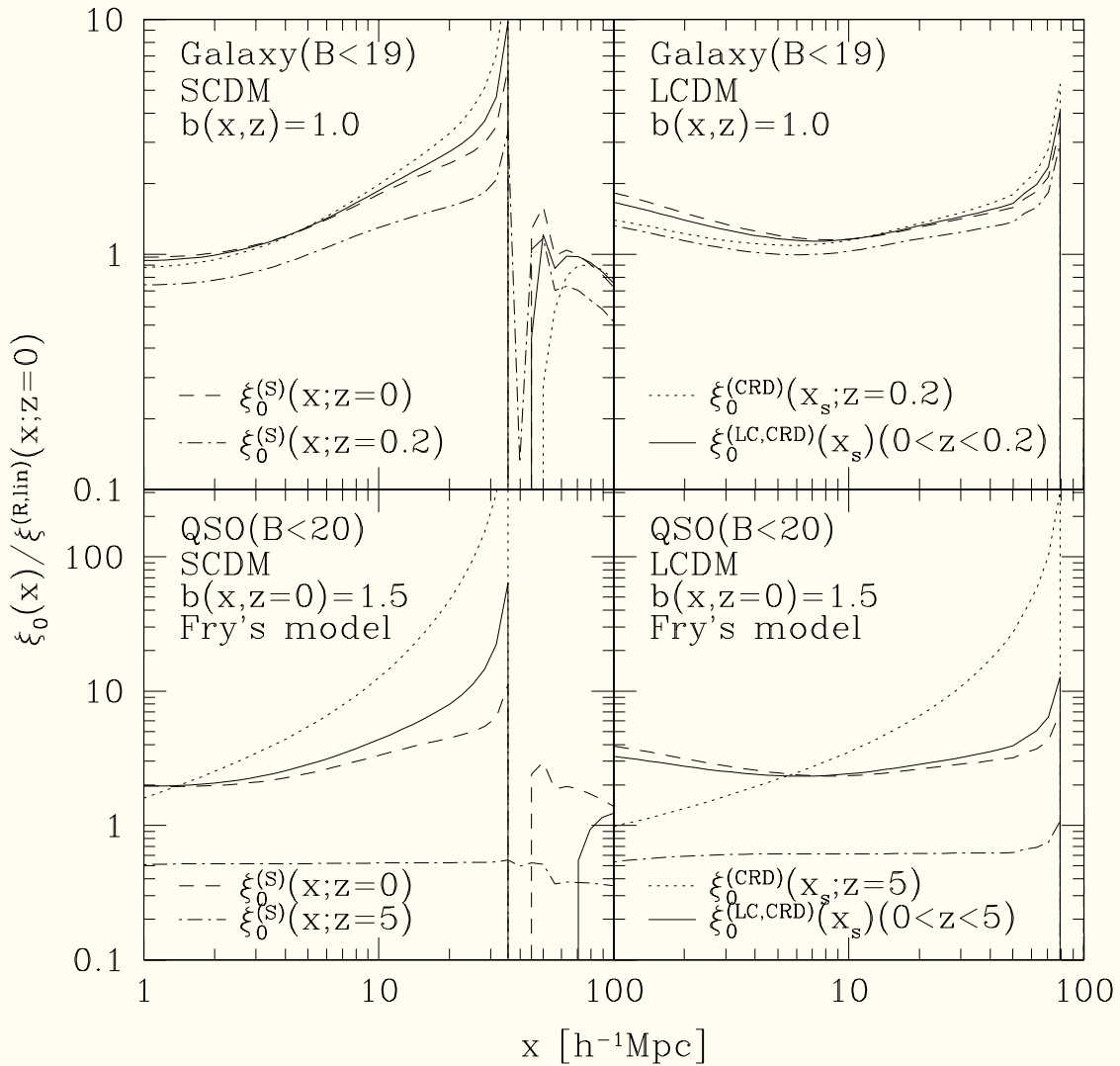


Figure 8: (YS, Magira & Yamamoto 1999)

## 4 Summary and conclusions

In the statistical analysis of clustering at high redshifts, the cosmological redshift-space distortion and light-cone effects play an important role both as signals and as noises.

We have constructed a theoretical model to describe those physical effects for the two-point spatial clustering statistics (the gravitational lensing effect is important for the angular statistics, but not for the spatial one). except for the bias model, which is unlikely to be described by physics.

### **correction for the systematic effects**

feasible for a given cosmological model (fluctuation spectrum and amplitude, cosmological parameters, bias model, etc.)

### **cosmological parameters**

can be constrained from the SDSS and 2dF QSO samples provided that the bias is linear.

### **scale-dependence and nonlinearity of bias**

with  $\Omega_0$  and  $\lambda_0$  accurately determined from MAP and PLANCK, the cosmological redshift-space distortion and light-cone effects are essential to model the bias from observations.