

Stable clustering solutions and the nonlinear stochastic biasing models for dark matter halos

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1 Stable clustering solution

1.1 Review of the conventional stable solution

★ Asymptotic behavior of two-point correlation functions at nonlinear regimes in the Einstein–de Sitter model
(Peebles 1976; Davis & Peebles 1977; Peebles 1980)

★ **similarity solutions in the Einstein–de Sitter model**

(1) one-particle distribution function

$$f(\mathbf{x}, \mathbf{p}, t) = t^{-3\beta} \hat{f}(\mathbf{x}/t^{\beta-1/3}, \mathbf{p}/t^\beta)$$

(2) two-point correlation function

$$\xi(x, t) = \hat{\xi}(x/t^{\beta-1/3})$$

(3) $P_{\text{initial}}(k) \propto k^n$ + linear theory ($\xi \propto x^{-(n+3)} t^{4/3}$)

$$\alpha \equiv \beta - \frac{1}{3} = \frac{4}{3(n+3)}$$

★ **stable clustering ansatz**

(1) particle pair conservation

$$\frac{\partial}{\partial t} \left\{ \bar{n} a^3 \int_0^x 4\pi x^2 dx [1 + \xi(x, t)] \right\} + 4\pi a^2 x^2 \bar{n} [1 + \xi(x, t)] \langle v_{21}(x, t) \rangle = 0$$

$$\rightarrow \frac{\langle v_{21}(x, a) \rangle}{\dot{a}x} = -\frac{a}{3[1 + \xi(x, a)]} \frac{\partial \bar{\xi}(x, a)}{\partial a}$$

with $\bar{\xi}(x, t) \equiv \frac{3}{x^3} \int_0^x x'^2 \xi(x', t) dx'$

(2) stability condition (ansatz)

In ($\xi \gg 1$), the average proper separation of pairs remains constant:

$$\langle \dot{r}_{21} \rangle = 0 = \langle v_{21}(x, t) \rangle + \dot{a}x$$

(3) stable clustering solution

$$\frac{\partial}{\partial t}(1 + \xi) = \frac{\dot{a}}{ax^2} \frac{\partial}{\partial x}[x^3(1 + \xi)].$$

Thus for $\xi \gg 1$, $\xi = a^3 g(ax)$, and this is consistent with the similarity solution only if

$$\xi(x, t) \propto x^{-\frac{3(n+3)}{n+5}} t^{\frac{4}{n+5}} \propto x^{-\frac{3(n+3)}{n+5}} a^{\frac{6}{n+5}}$$

This solution has been widely applied in modeling the nonlinear gravitational clustering (Hamilton et al. 1991; Peacock & Dodds 1994; Jain, Mo & White 1995) and in understanding the pair-wise velocity dispersions and thus the redshift-space distortion (e.g., YS & Jing 1997; Mo, Jing & Börner 1998; Caldwell et al. 1999).

\downarrow However

the validity of the solution in non - Einstein – de Sitter model has not been examined carefully yet.

1.2 Indications from previous N-body results

Scale-dependence of scaled two-point correlation functions

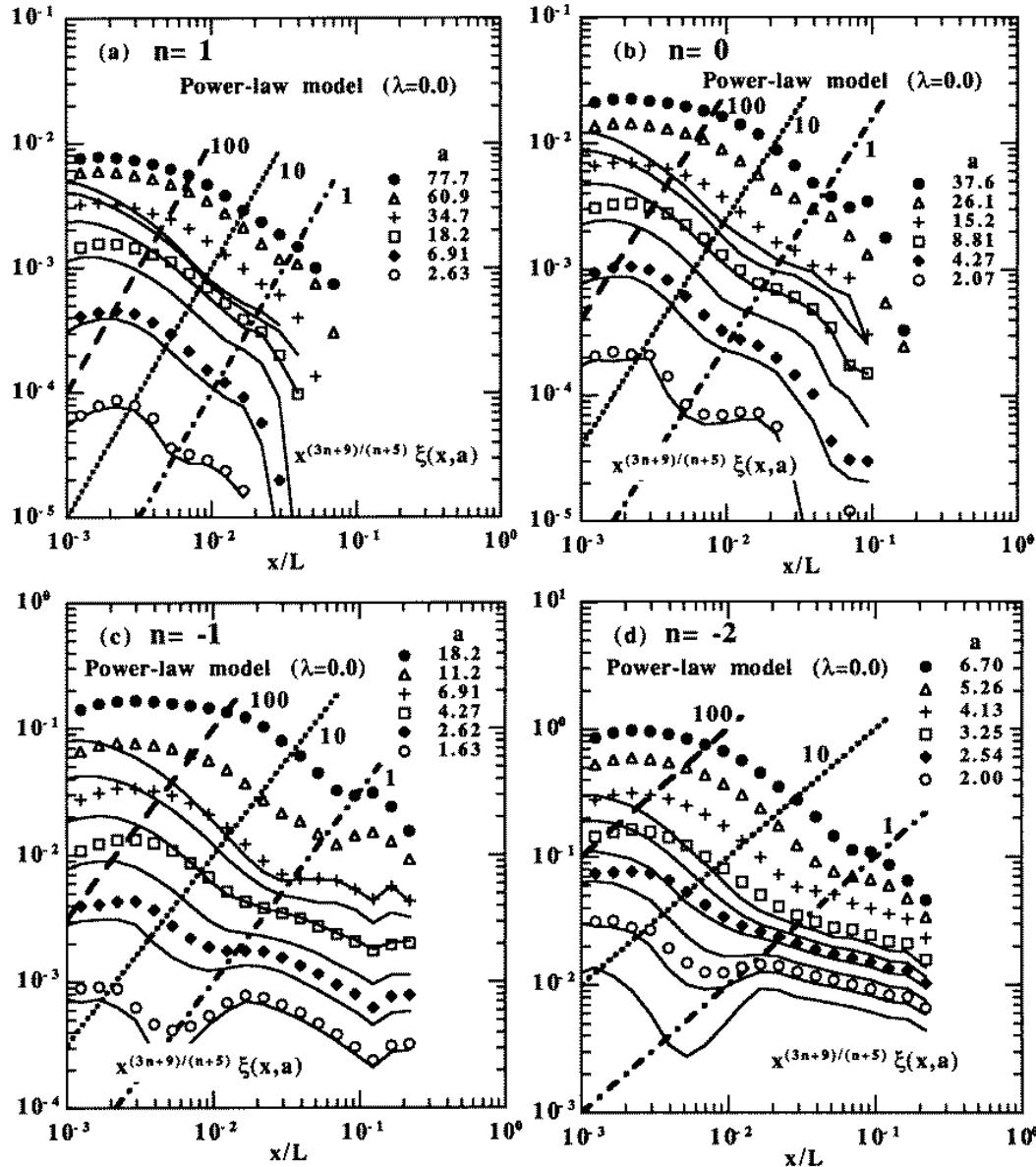


Figure 1: Cosmological N-body simulations (Sugino et al. 1991) for scale-free models ($N = 64^3$): $P_{\text{initial}}(k) \propto k^n$ in $\lambda_0 = 0$ ($\Omega_0 = 1$: symbols and $\Omega_0 = 0.2$: solid lines) universes. (YS 1993)

Evolution of scaled two-point correlation functions

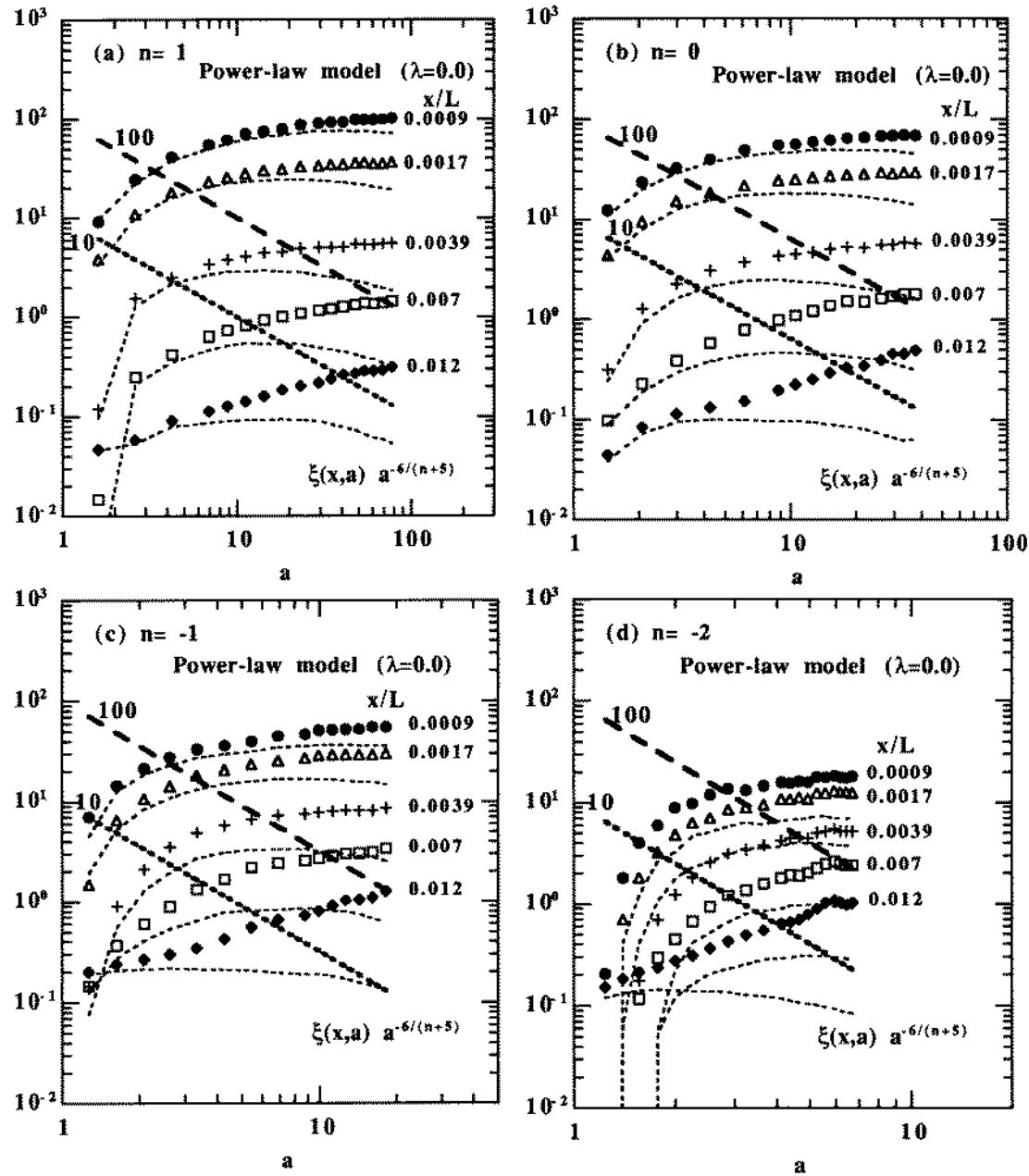


Figure 2: Cosmological N-body simulations (Sugino et al. 1991) for scale-free models ($N = 64^3$): $P_{\text{initial}}(k) \propto k^n$ in $\lambda_0 = 0$ ($\Omega_0 = 1$: symbols and $\Omega_0 = 0.2$: dotted lines) universes. (YS 1993)

1.3 Quasi-stable clustering solutions

* Find the similarity solution in the universe where the scale factor $a(t) \propto t^p$, and the linear growth rate $D(t) \propto t^q$.

(1) similarity

$$\xi(x, t) = \hat{\xi}(x/t^\alpha)$$

(2) + $P_{\text{initial}}(k) \propto k^n$ + linear theory ($\xi \propto x^{-(n+3)}t^{2q}$)

$$\alpha = \frac{2q}{n+3}$$

(3) + stability condition (ansatz)

For $\xi \gg 1$, we obtain

$$\xi(x, t) \propto x^{-\frac{3(n+3)}{n+3+2q/p}} a^{\frac{6q/p}{n+3+2q/p}}$$

* Quasi-stable clustering solutions (?)

If $a(t)$ and $D(t)$ change the effective slope p and q sufficiently slowly, we expect that

$$\xi(x, t) \propto x^{-\frac{3(n+3)}{n+3+2f}} a^{\frac{6f}{n+3+2f}}$$

in the strongly nonlinear regime, where

$$f \equiv \frac{d \ln D / d \ln t}{d \ln a / d \ln t} = \frac{d \ln D}{d \ln a} \sim \Omega(z)^{0.6} + \frac{\lambda(z)}{70} \left[1 + \frac{\Omega(z)}{2} \right]$$

(YS, Taruya & Sugino 2000, in preparation)

1.4 Comparison with the latest N-body results

Two-point correlation functions at $z = 0$

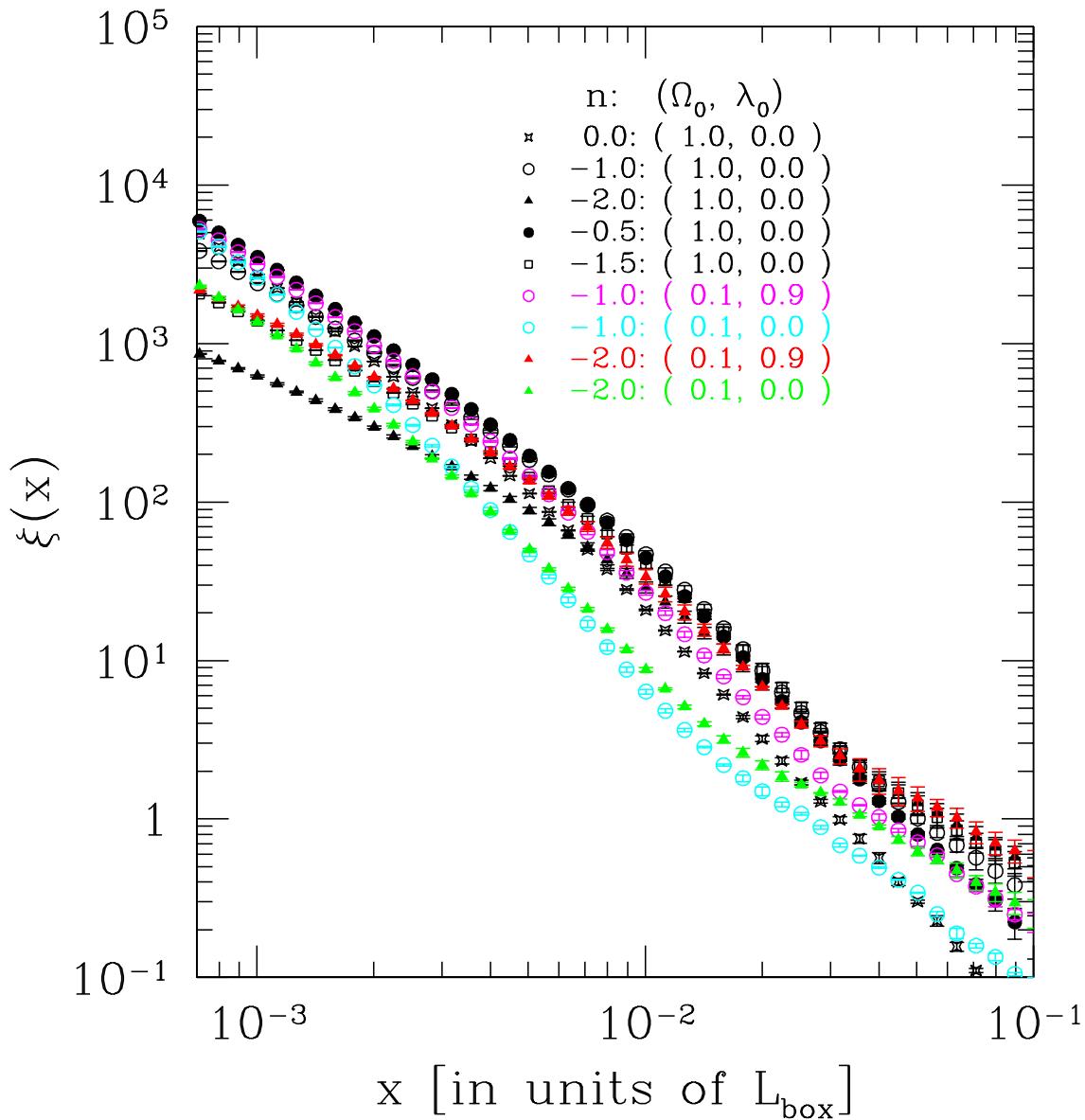


Figure 3: Cosmological N-body simulations (Jing 2000) for scale-free models: $P_{\text{initial}}(k) \propto k^n$ in $\Omega_0 = 1, \lambda_0 = 0$ (black: $N = 256^3$) and $\Omega_0 = 0.1$ (color: $N = 200^3$) models. (YS, Taruya & Sugiyohara 2000)

Conventional scaling: stable solutions at $z = 0$

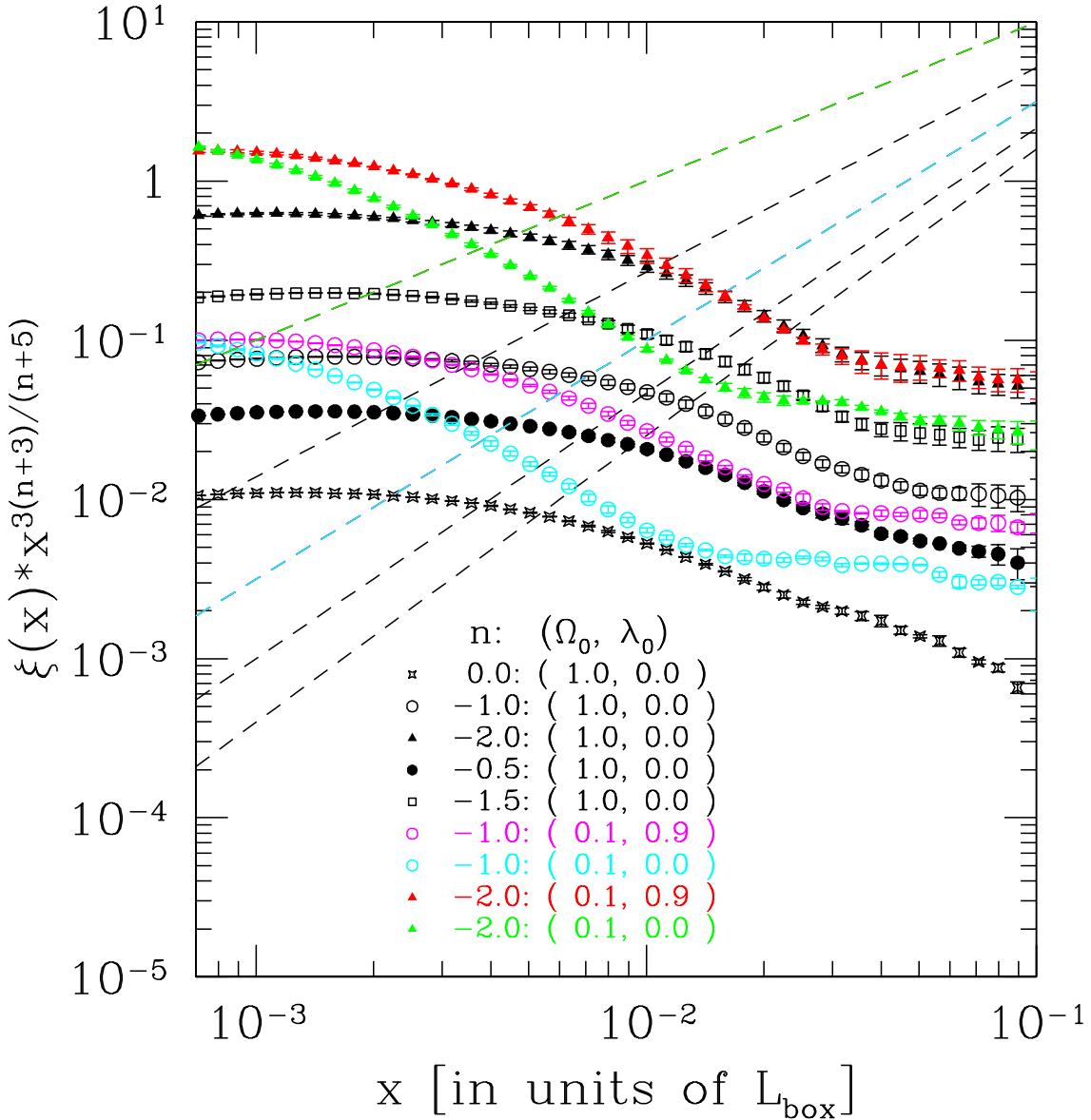


Figure 4: Cosmological N-body simulations (Jing 2000) for scale-free models: $P_{\text{initial}}(k) \propto k^n$ in $\Omega_0 = 1, \lambda_0 = 0$ (black: $N = 256^3$) and $\Omega_0 = 0.1$ (color: $N = 200^3$) models. Dashed lines corresponds to contours of $\xi(x) = 100$ for each model. (YS, Taruya & Sugino 2000)

New scaling: quasi-stable solutions at $z = 0$

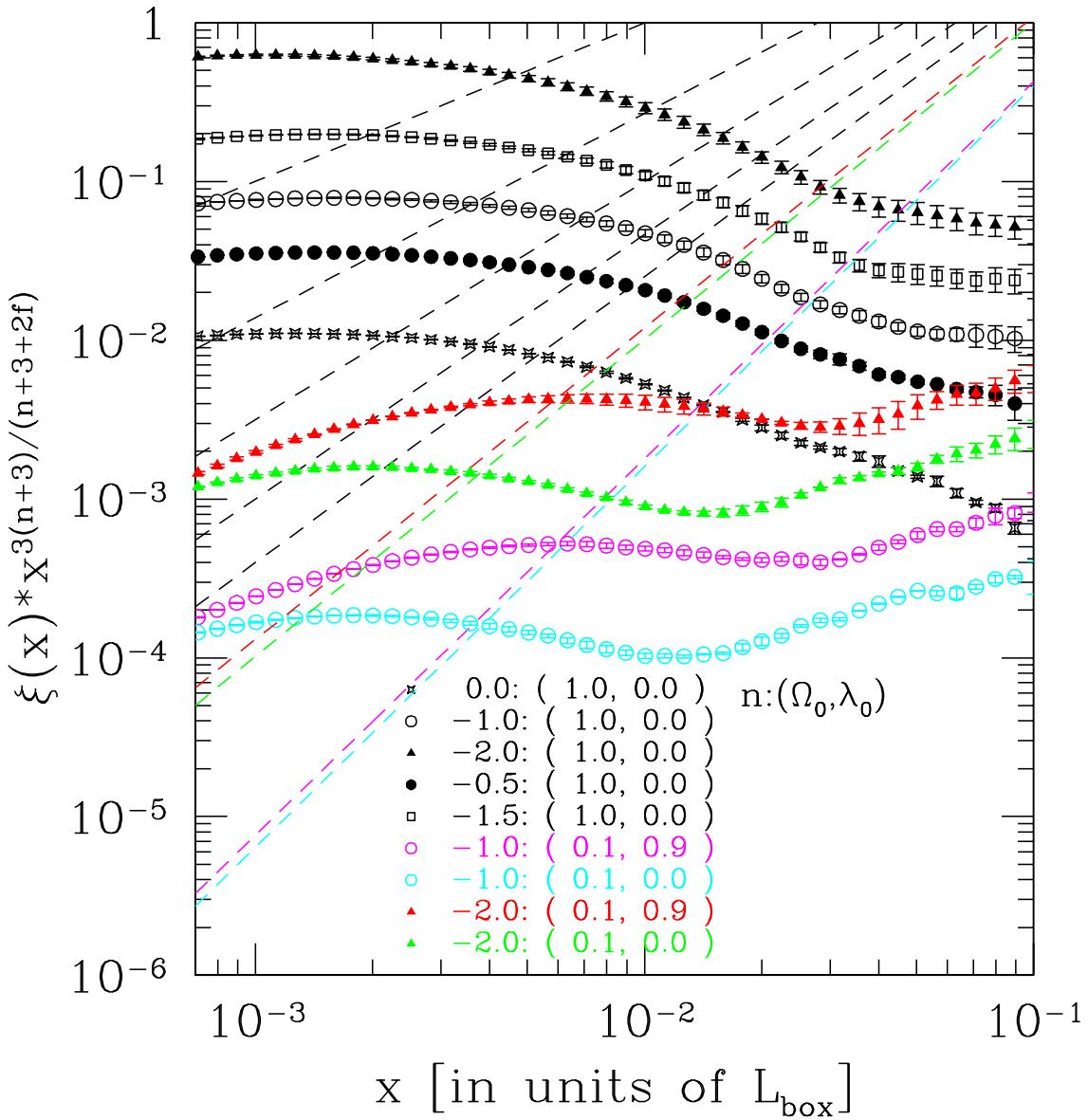


Figure 5: Cosmological N-body simulations (Jing 2000) for scale-free models: $P_{\text{initial}}(k) \propto k^n$ in $\Omega_0 = 1, \lambda_0 = 0$ (black: $N = 256^3$) and $\Omega_0 = 0.1$ (color: $N = 200^3$) models. Dashed lines corresponds to contours of $\xi(x) = 100$ for each model. (YS, Taruya & Sugino 2000)

1.5 Validity of the assumptions

Quasi-stable solutions are in good agreement with simulations in $\Omega_0 = 0.1$ & $\lambda_0 = 0$, but not in $\Omega_0 = 0.1$ & $\lambda_0 = 0.9$ models.

→ Why ?

Effective power-law indices: $a \propto t^p$, $D \propto t^q$

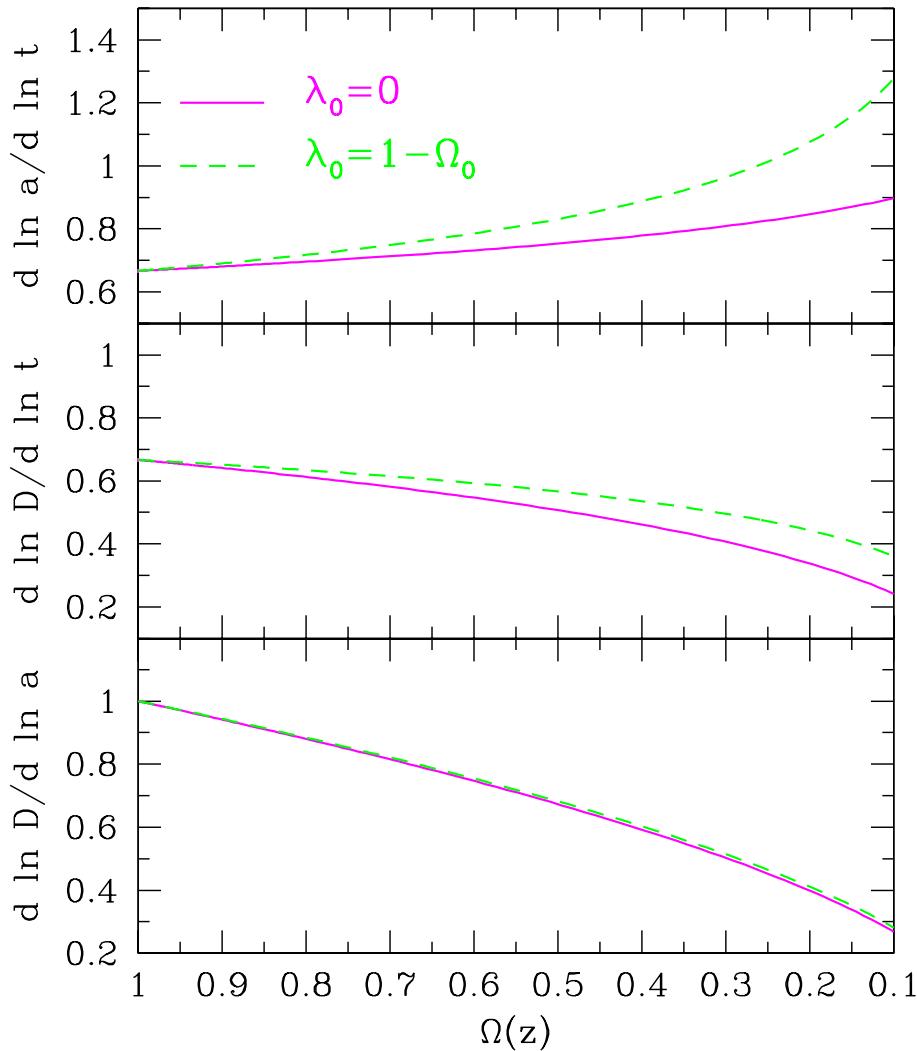


Figure 6: Evolution of the effective power-law indices for the scale factor and the linear growth rate.
(YS, Taruya & Sugino 2000)

Effective Power-law indices change more rapidly in spatially-flat universes.

Logarithmic derivatives of $p(t)$, $q(t)$ and $f(t)$

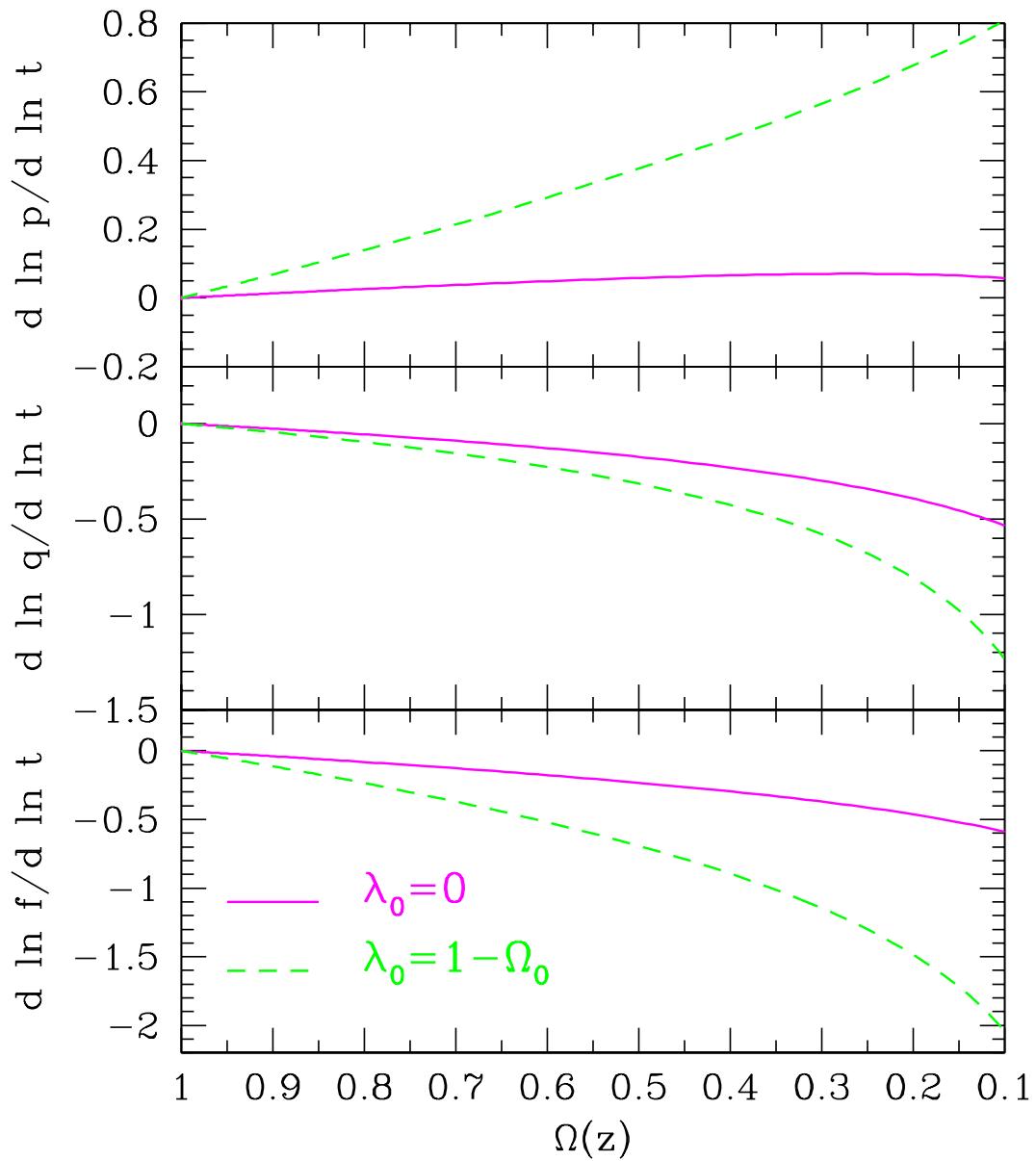


Figure 7: Logarithmic derivatives of the effective power-law indices for the scale factor and the linear growth rate.
(YS, Taruya & Sugino 2000)

2 Nonlinear stochastic biasing

Cosmological information from **galaxy/QSO** redshift surveys → **needs to understand biasing** i.e., **luminous** galaxy distribution \neq **dark** mass distribution

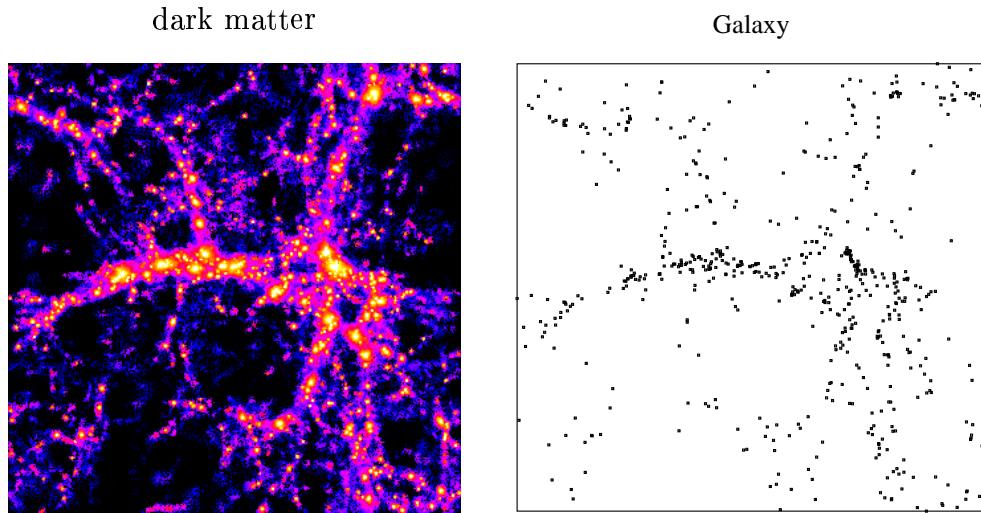


Figure 8: SPH simulation. (Yoshikawa, Taruya, Jing & YS 2000)

★ **general nonlocal stochastic nonlinear bias:**

$$b_{\text{obj}}(\mathbf{x}, z|R) \equiv \frac{\delta_{\text{obj}}(\mathbf{x}, z|R)}{\delta_{\text{mass}}(\mathbf{x}, z|R)} = \frac{\mathcal{F}[\mathbf{x}, z, R, \delta_{\text{mass}}(\mathbf{x}, z|R), \vec{\mathcal{A}}(\mathbf{x}', z'|R'), \dots]}{\delta_{\text{mass}}(\mathbf{x}, z|R)}$$

– **local stochastic nonlinear bias:**

$$b_{\text{obj}}(\mathbf{x}, z|R) = B_l[\mathbf{x}, z, R, \delta_{\text{mass}}(\mathbf{x}, z|R), \vec{\mathcal{A}}(\mathbf{x}, z|R), \dots]$$

– **local deterministic nonlinear bias:**

$$b_{\text{obj}}(\mathbf{x}, z|R) = B_{\text{ld}}[z, R, \delta_{\text{mass}}(\mathbf{x}, z|R)]$$

– **local deterministic linear bias:**

$$b_{\text{obj}}(\mathbf{x}, z|R) = B_{\text{ldl}}(z, R)$$

3 An analytic model of stochastic biasing

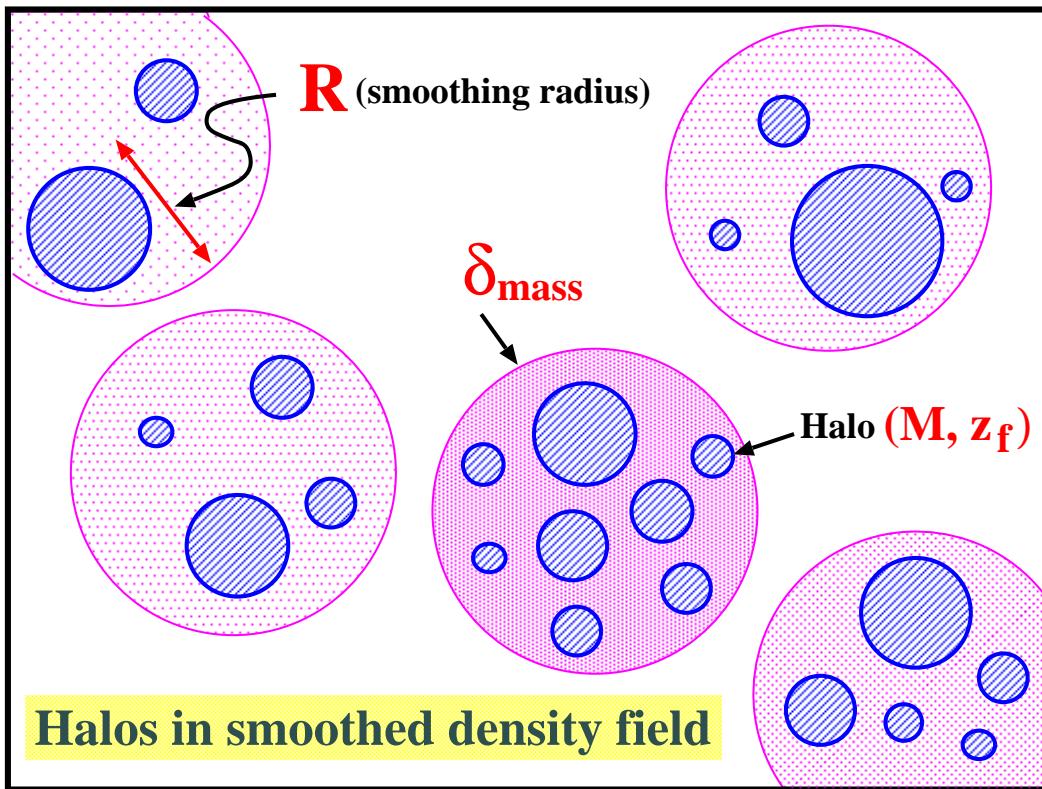


Figure 9: A schematic picture of *halo* biasing (Taruya & YS 2000)

Halo number density fluctuation δ_{halo}

Analytic model (Mo & White 1996) :
extended PS theory + spherical collapse model

$$\delta_{\text{halo}} = \Delta_h(R, z | \delta_{\text{mass}}, M, z_f)$$

Probability distribution of M

Mass function (Press & Schechter 1974): $n(M, z; \delta_c)$

Probability distribution of z_f

Distribution function (Lacey & Cole 1996): $\frac{\partial p}{\partial z_f}(z_f | M, z)$

3.1 Probability distribution functions

Fluctuation of halo number density

$$\delta_{\text{halo}} = (1 + \delta_{\text{mass}}) \frac{n(M_1, \delta_c | M_0, \delta_0; z)}{n(M_1, z; \delta_c)} - 1$$

with $\delta_c = 1.69 D(z)/D(z_f)$.

$n(M_1, z; \delta_1) dM_1 \dots$ Original PS mass function

$n(M_1, \delta_1 | M_0, \delta_0; z) dM_1 \dots$ Conditional mass function

Conditional probability

$$\begin{aligned} P(\delta_{\text{halo}} | \delta_{\text{mass}}) d\delta_{\text{halo}} &= \mathcal{N}^{-1} \int \int_{\mathcal{C}(M, z_f)} dM dz_f \\ &\times \frac{\partial p}{\partial z_f}(z_f | M, z) n(M, z; \delta_{c,0}) \end{aligned}$$

Integral constraint

$$\mathcal{C}(M, z_f) = \{ (M, z_f) \mid \delta_{\text{halo}} \leq \delta_{\text{halo}}(R, z | \delta_{\text{mass}}, M, z_f) \leq \delta_{\text{halo}} + d\delta_{\text{halo}}, M_{\min} \leq M \leq M_{\max}, z \leq z_f \leq \infty \}$$



Joint probability of halos and mass:

$$P(\delta_{\text{halo}}, \delta_{\text{mass}}) = P(\delta_{\text{halo}} | \delta_{\text{mass}}) P(\delta_{\text{mass}})$$



we assume “log-normal” distribution here

3.2 Formation epoch z_f distribution function

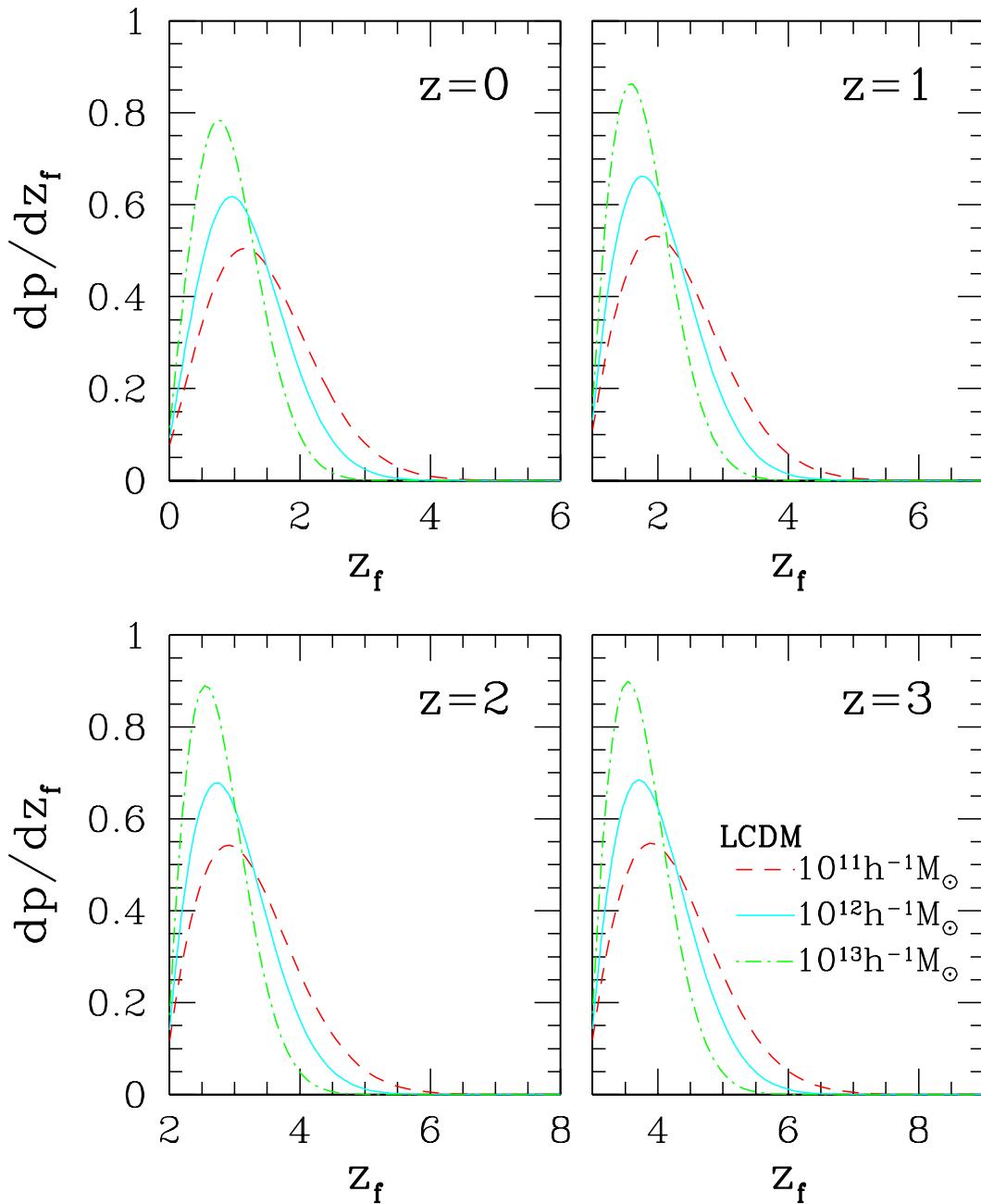


Figure 10: Formation epoch distribution function $\frac{\partial p}{\partial z_f}(z_f | M, z)$ on the basis of the extended PS theory by Lacey & Cole (1993) and Kitayama & YS (1996). (Taruya & YS 2000)

3.3 Evolution of joint probability distribution

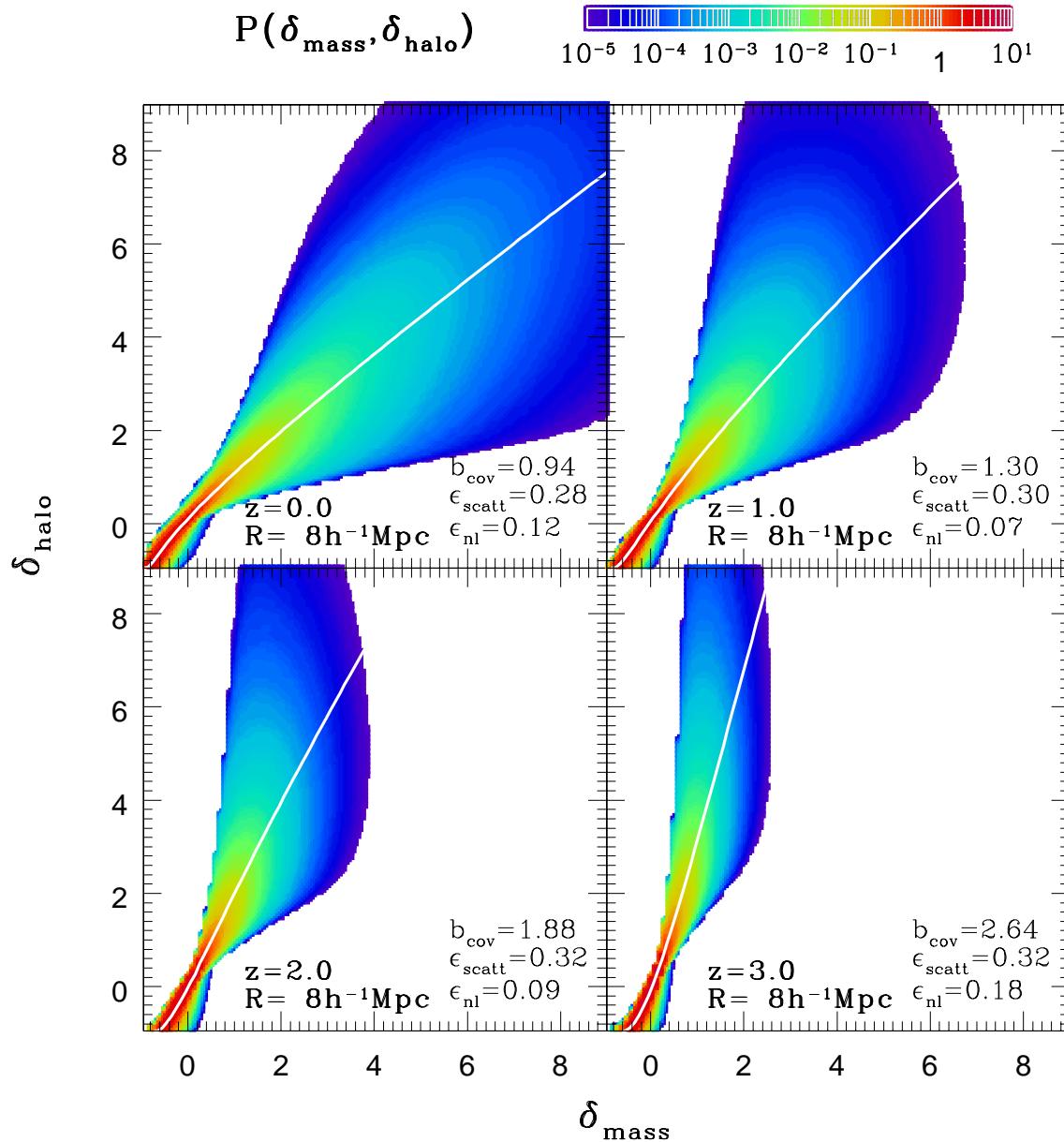


Figure 11: Color contour plot of joint probability distribution functions $P(\delta_{\text{mass}}, \delta_{\text{halo}})$ smoothed over $R = 8h^{-1}\text{Mpc}$ at $z = 0, 1, 2$ and 3 . (Taruya & YS 2000)

3.4 Origin of stochasticity in halo biasing

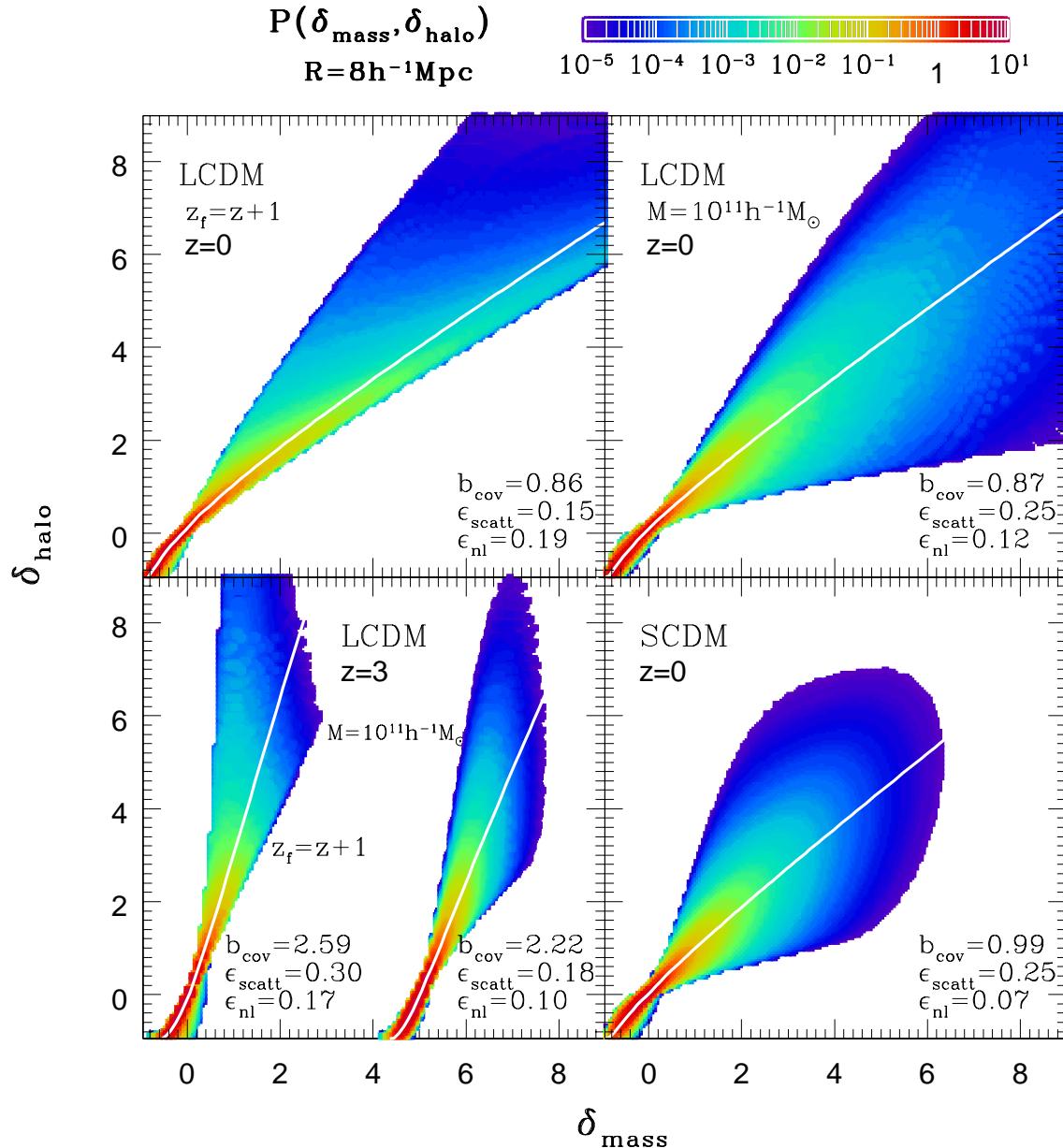


Figure 12: Color contour plot of joint probability distribution functions $P(\delta_{\text{mass}}, \delta_{\text{halo}})$ smoothed over $R = 8h^{-1}\text{Mpc}$. (Taruya & YS 2000)

3.5 Second-order moments of biasing parameters

conditional mean of δ_{halo}

$$\bar{\delta}_{\text{halo}}(\delta_{\text{mass}}) \equiv \int d\delta_{\text{halo}} P(\delta_{\text{halo}}|\delta_{\text{mass}}) \delta_{\text{halo}}$$

degree of nonlinearity

$$\epsilon_{\text{nl}}^2 = \frac{\langle \delta_{\text{mass}}^2 \rangle \langle \bar{\delta}_{\text{gal}}^2 \rangle}{\langle \delta_{\text{mass}} \bar{\delta}_{\text{gal}} \rangle} - 1$$

degree of stochasticity

$$\epsilon_{\text{scatt}}^2 = \frac{\langle \delta_{\text{mass}}^2 \rangle \langle (\delta_{\text{gal}} - \bar{\delta}_{\text{gal}})^2 \rangle}{\langle \delta_{\text{mass}} \bar{\delta}_{\text{gal}} \rangle}$$

linear regression

$$b_{\text{cov}} = \frac{\langle \delta_{\text{mass}} \bar{\delta}_{\text{gal}} \rangle}{\langle \delta_{\text{mass}}^2 \rangle}$$

variance

$$b_{\text{var}}^2 \equiv \frac{\langle \delta_{\text{halo}}^2 \rangle}{\langle \delta_{\text{mass}}^2 \rangle} = b_{\text{cov}} (1 + \epsilon_{\text{scatt}}^2 + \epsilon_{\text{nl}}^2)^{1/2}$$

cross-correlation

$$r_{\text{corr}} \equiv \frac{\langle \delta_{\text{halo}} \delta_{\text{mass}} \rangle}{\sqrt{\langle \delta_{\text{halo}}^2 \rangle \langle \delta_{\text{mass}}^2 \rangle}} = \frac{1}{\sqrt{1 + \epsilon_{\text{scatt}}^2 + \epsilon_{\text{nl}}^2}}$$

3.6 Scale-dependence of biasing parameters

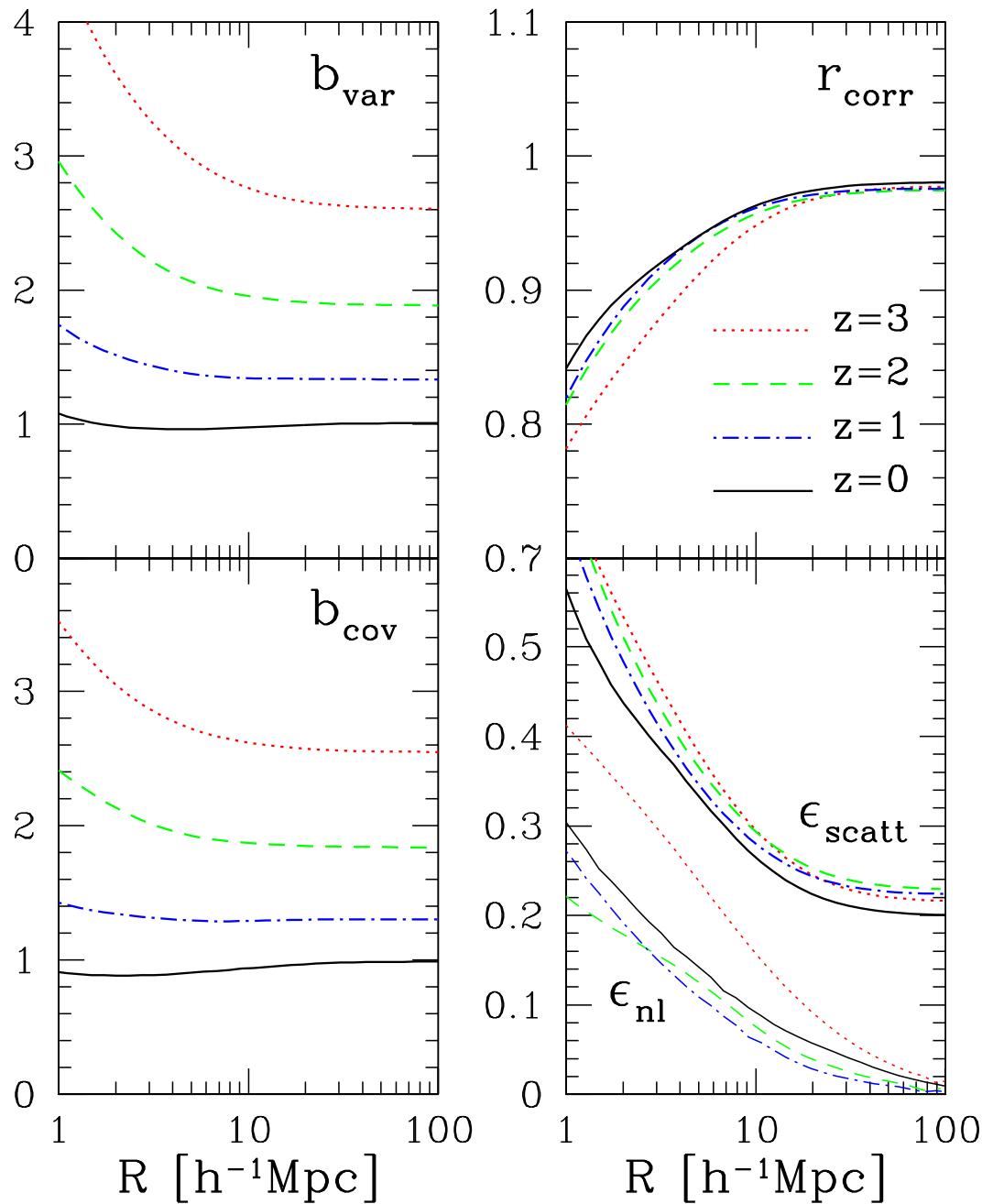


Figure 13: Scale-dependence of the halo biasing parameters.
(Taruya & YS 2000)

3.7 Evolution of biasing parameters

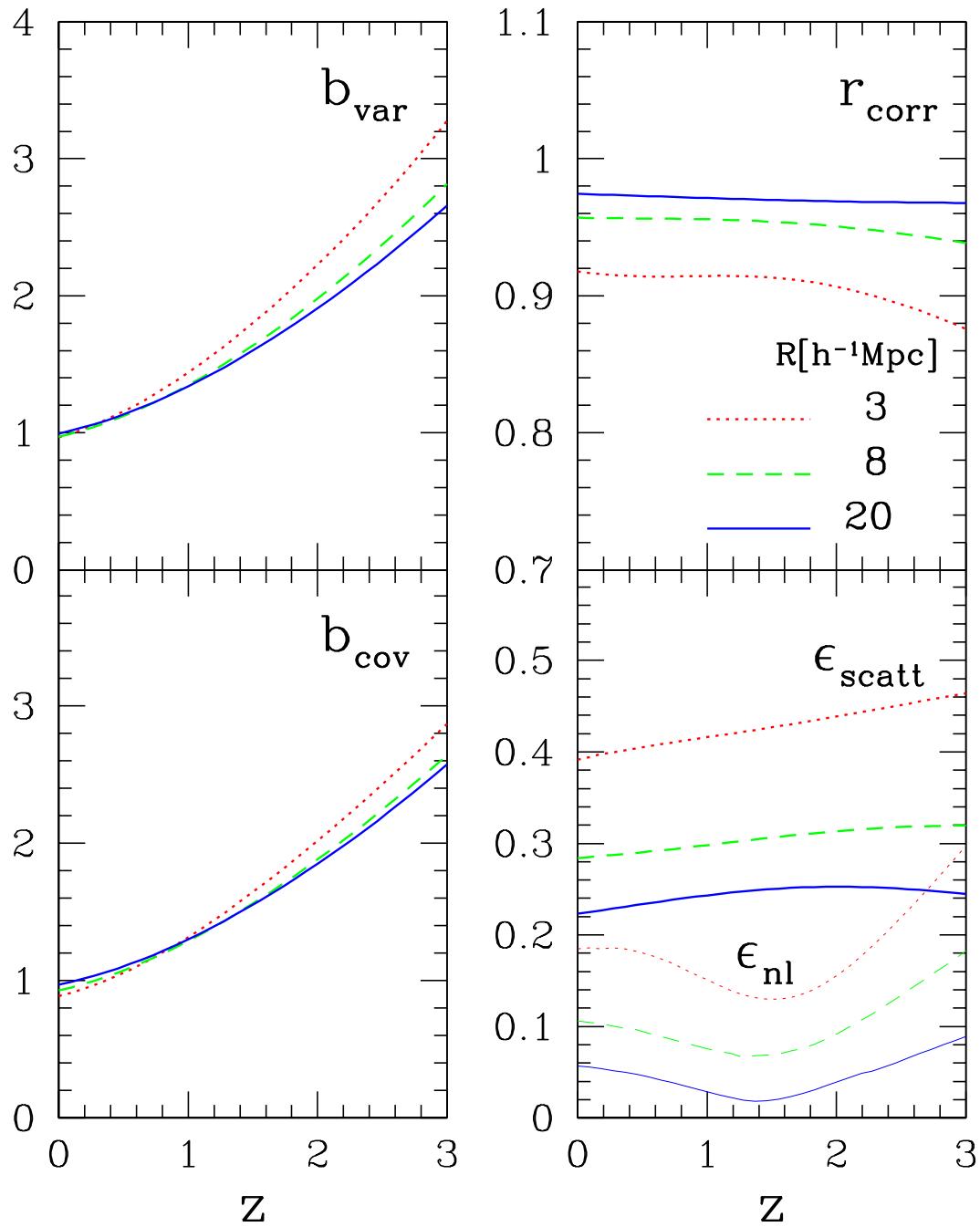


Figure 14: Evolution of the halo biasing parameters.
(Taruya & YS 2000)

4 Summary and conclusions

(1) We generalized the stable clustering solution in the Einstein – de Sitter universe and found **quasi-stable clustering solutions**:

$$\xi(x, t) \propto x^{-\frac{3(n+3)}{n+3+2f}} a^{\frac{6f}{n+3+2f}},$$

where

$$f \equiv \frac{d \ln D / d \ln t}{d \ln a / d \ln t} = \frac{d \ln D}{d \ln a} \sim \Omega(z)^{0.6} + \frac{\lambda(z)}{70} \left[1 + \frac{\Omega(z)}{2} \right]$$

(2) We proposed **a first physical model of non-linear stochastic biasing for halos** resulting from formation epoch “ z_f ” and mass “ M ” of dark halos, and analytically derived the joint probability $P(\delta_{\text{mass}}, \delta_{\text{halo}})$.

★ While a possible volume exclusion effect of halos and non-gravitational processes, which are not taken into account in our model, are essential in the small-scale clustering < a few Mpc, our current predictions turn out to describe the galaxy biasing in numerical simulations quite well.

(Kravtsov & Klypin 1999; Somerville et al. 2000)

→ the next talk by Yoshikawa (吉川耕司)

- ★ application to the genus statistics of clusters
(Hikage, Taruya & YS, in preparation.)
- ★ generalization to the two-point statistics of halos
(Kayo, Taruya & YS, in preparation.)