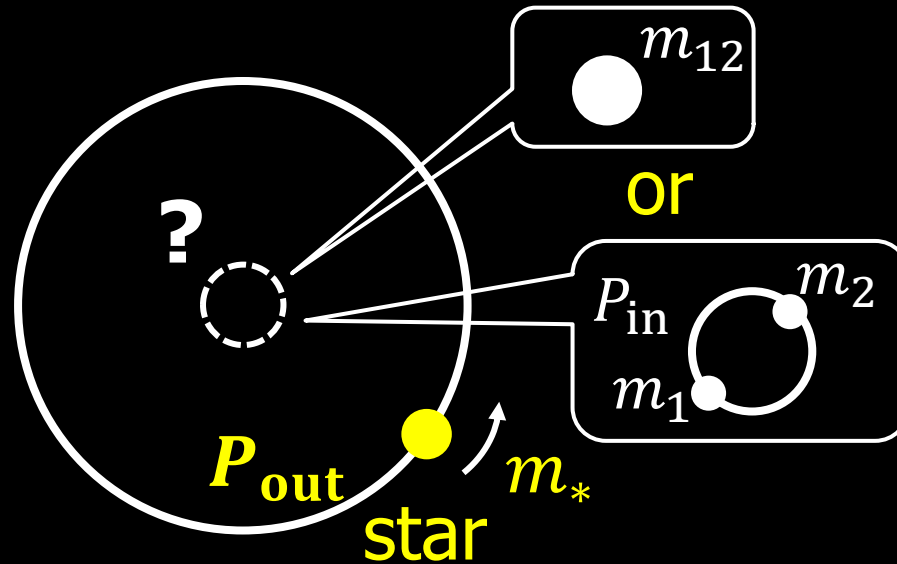


# Radial velocity modulation of a tertiary star orbiting an inner binary black hole



Yasushi Suto (須藤靖)

Department of Physics & Research Center for the Early Universe, The University of Tokyo

14:20 March 15, 2023 @Academia Sinica Institute of Astronomy and Astrophysics

# My past talks at Taiwan

- Talk at the 8th Taipei Astrophysics Workshop AMiBA2001: high-z clusters, missing baryons and CMB polarization@National Taiwan University and Grand Formosa (June11-15, 2001) "Clustering of Dark Matter Halos on the Light-cone"
- Matter and Energy in Clusters of Galaxies@National Central University, Chung-Li, Taiwan (April 23-27, 2002) "Density profiles and clustering of dark halos and clusters of galaxies"
- DENET-Taiwan HSC collaboration meeting@ASIAA, Taipei (March 7-8, 2011) "DENET and Sumire collaboration"
- Seminar@ASIAA (February 2, 2012) "Cosmological implications of inhomogeneities in intra-cluster gas"
- Colloquium@ASIAA (February 3, 2012) "Colors of a second Earth"

# Today's talk

1. Three-body dynamics: old and new problem
2. Hierarchical three-body systems
3. Secular-perturbation theory in the three-body problem and von Zeipel-Lidov-Kozai effect
4. Search for star-blackhole binaries in optical surveys: a star-blackhole binary or a star-binary blackhole triple?
5. Dynamical signature of triple systems with an inner binary black-hole

# **1 Three-body dynamics: old and new problem**

# Celestial mechanics and the three-body problem = “superstring theory” before the 20<sup>th</sup> century

- Great mathematicians had seriously worked on the topic
  - Joseph-Louis Lagrange (1736-1813)
  - Pierre-Simon Laplace (1749-1827)
  - Johann Carl Friedrich Gauß (1777-1855)
  - Carl Gustav Jacob Jacobi (1804-1851)
  - William Rowan Hamilton (1805-1865)
  - Jules-Henri Poincaré (1854-1912)
- If quantum theory and relativity had not been discovered, three-body problem could have been the frontier in mathematical physics at the current epoch that the best scientists would choose to work on
  - what can “we” do then? ⇒ amazing three-body systems + accurate numerical simulations that those great people never expected in their epochs

# Ubiquity of triple systems in astronomy

## ■ Stellar systems

- more than 70% of OBA-type stars and 50% of FGK-type stars belong to binary/multiple systems (e.g., Alpha Centauri)

## ■ (Exo)Planetary systems

- planets around binary stars, multi-planets, satellites,,,

## ■ Compact objects

- Possible pathway towards binary BHs detected by GW
- Binaries (stars, BHs) around a supermassive BH in galaxies
- Triples of compact objects, e.g., pulsar-WD binary + tertiary WD (Ransom et al. 2014)



# Diversities triggered by triple dynamics

*Alpha Centauri was a triple system, two suns tightly orbiting one another, and a third, more remote, circling them both.*

*What would it be like to live on a world with three suns in the sky?*

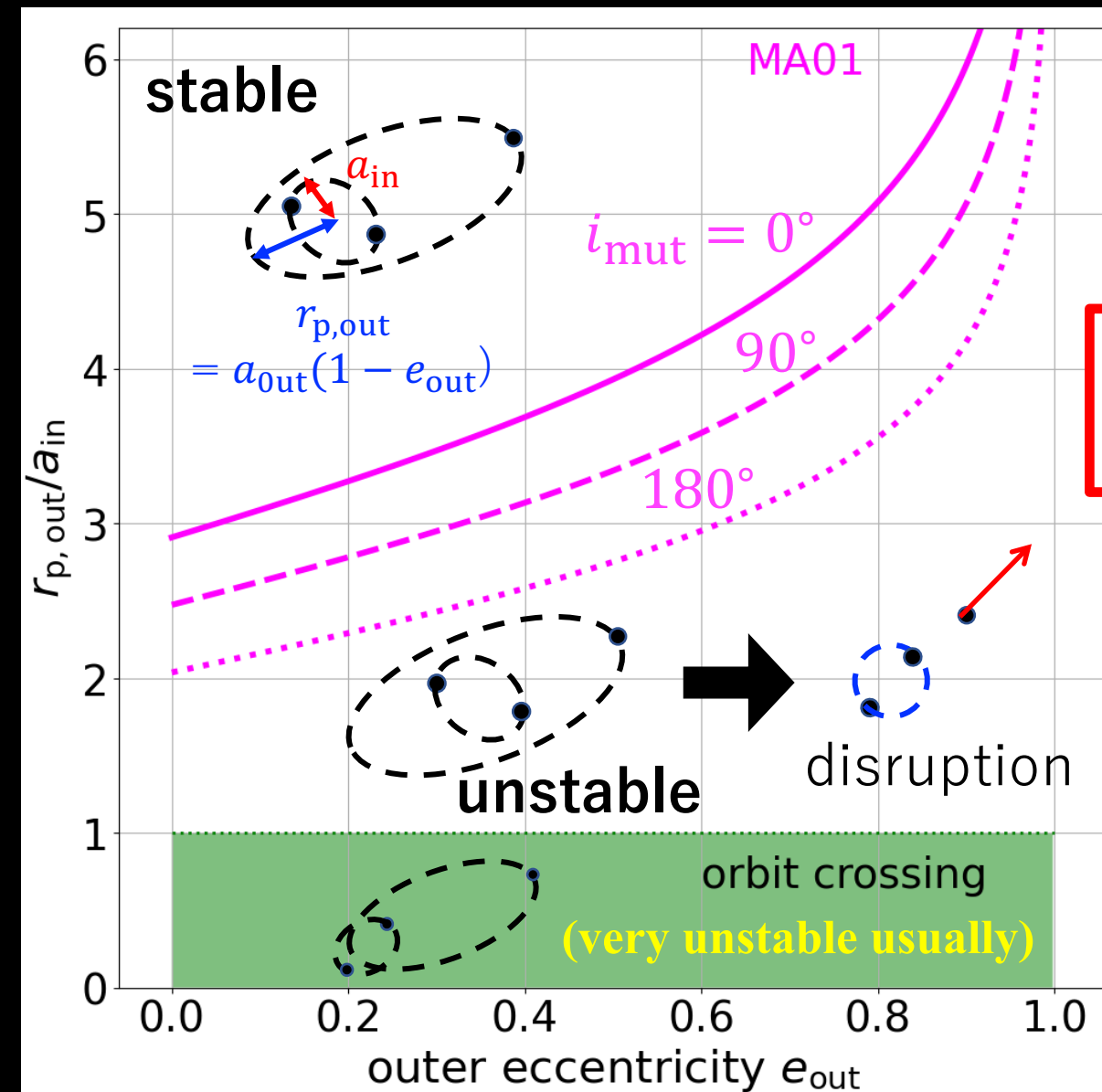
*— Carl Sagan "Contact"*



## **2 Hierarchical three-body systems**



# Triples are unstable in general $\Leftrightarrow$ diversity



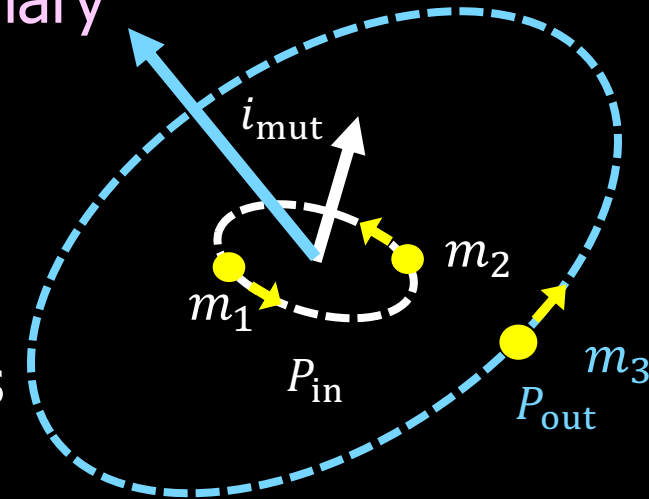
- Stability criterion (Mardling & Aarseth 2001)

$$\left(\frac{r_{p,out}}{a_{in}}\right)_{MA} \equiv 2.8 \left(1 - 0.3 \frac{i_{mut}}{\pi}\right) \left[\left(1 + \frac{m_3}{m_{12}}\right) \frac{(1 + e_{out})}{\sqrt{1 - e_{out}}}\right]^{2/5}$$

- Well-known and widely used, but its implication is often misinterpreted...
- Lyapunov (chaoticity of local trajectory) vs. Lagrange (escape of a body from the system) stability
- Hayashi, Trani & YS; ApJ 939(2022)81  
ApJ 943(2023)58

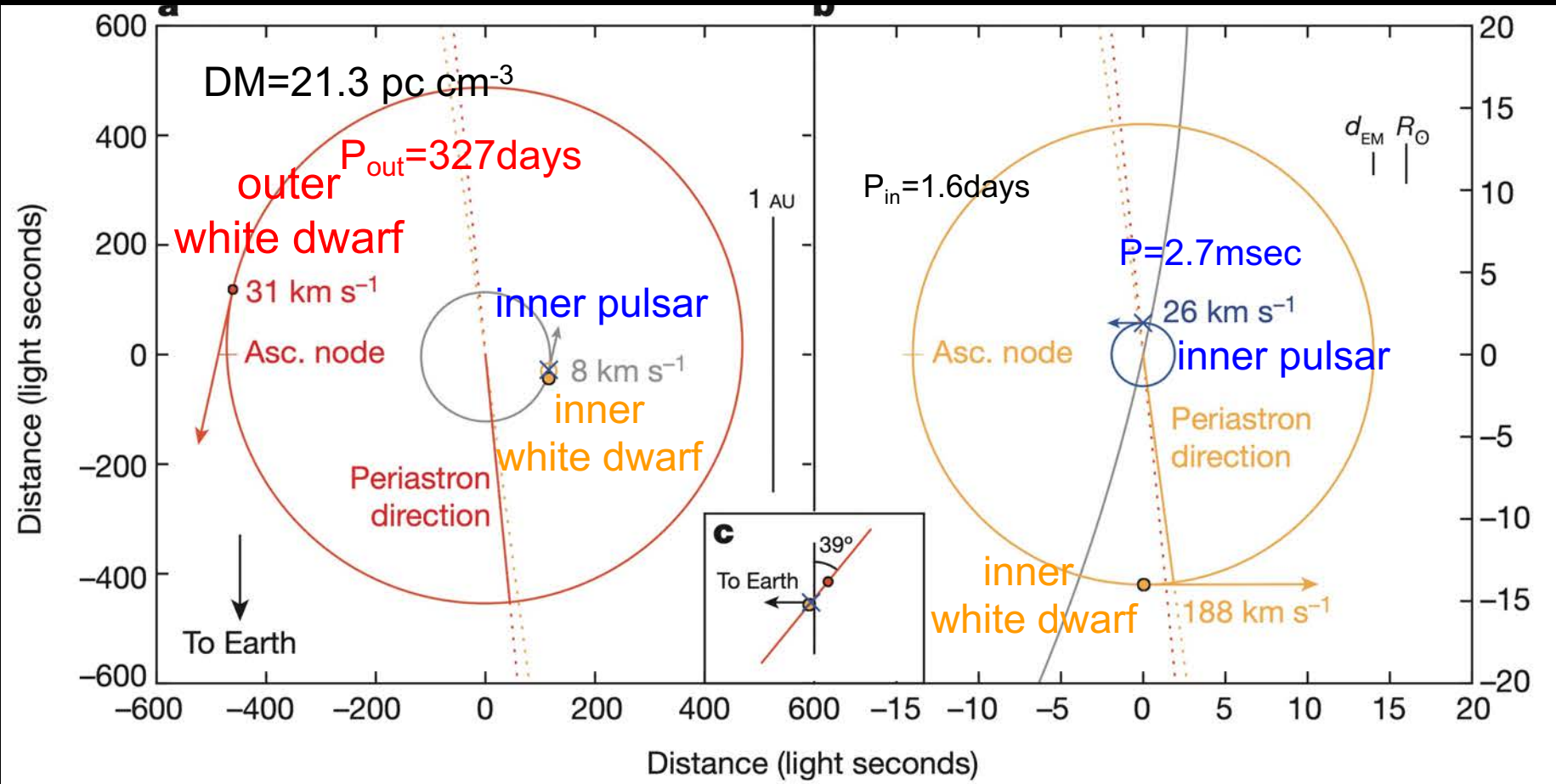
# Hierarchical three-body systems

- Gravitational three-body systems are unstable in general
  - stable three-body systems are mostly hierarchical: tight binary + distant tertiary orbiting the center-of-mass of the inner binary
  - observed three-body systems are likely to be hierarchical
- Stable systems are inevitably associated with (undemocratic) hierarchies
  - quite universal in biological, astronomical and social systems
    - quarks and leptons – atoms – molecules – DNAs – cells – organs – animals – villages – cities – nations – planets – stars – star clusters – galaxies – galaxy clusters – universe(s) – multiverse(s)
  - non-intuitive (counter-intuitive) dynamical behavior of hierarchical triples triggers unexpectedly broad diversities in astronomical phenomena (e.g., ZKL effect)



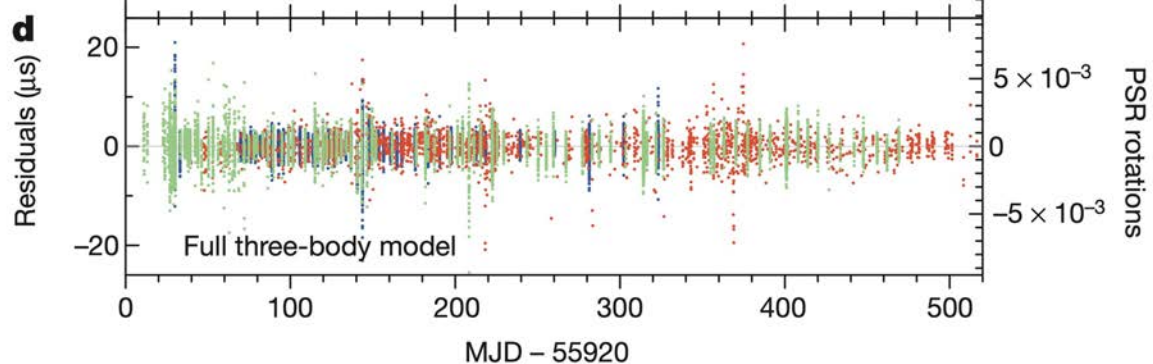
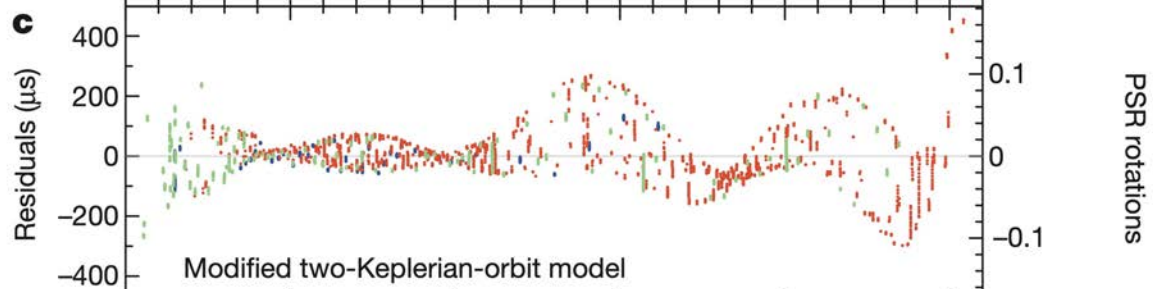
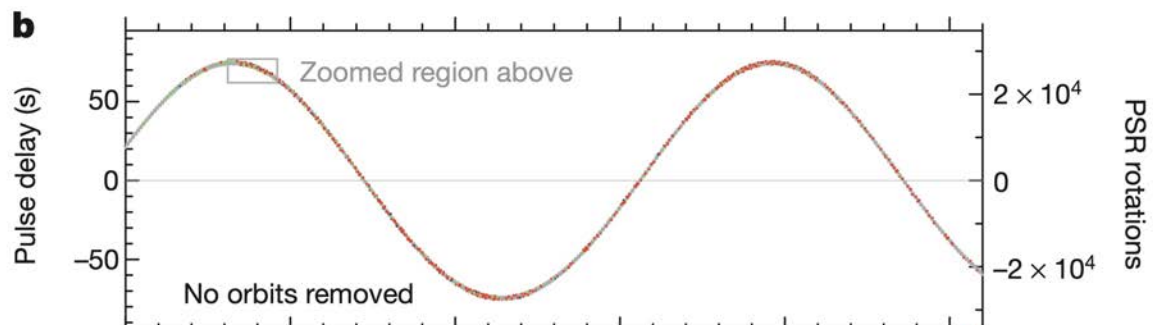
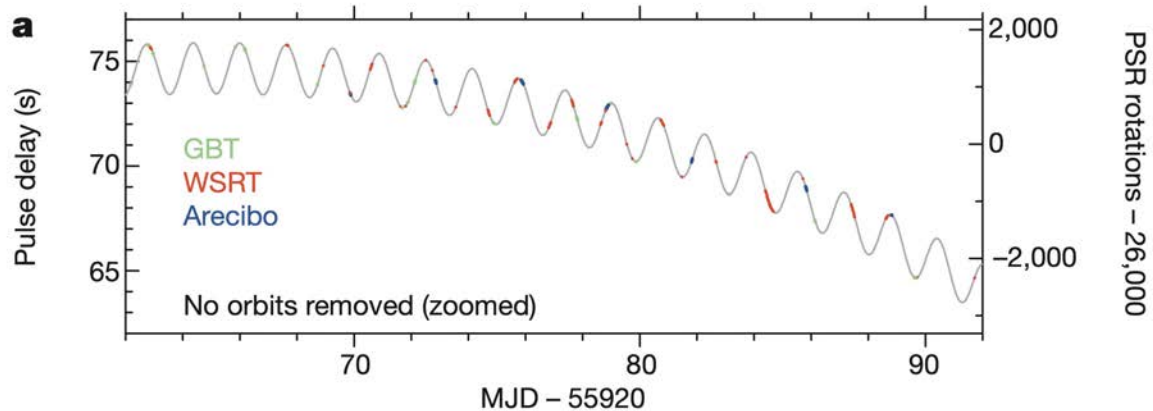
# A millisecond pulsar in a stellar triple system

S. M. Ransom<sup>1</sup>, I. H. Stairs<sup>2</sup>, A. M. Archibald<sup>3,4</sup>, J. W. T. Hessels<sup>3,5</sup>, D. L. Kaplan<sup>6,7</sup>, M. H. van Kerkwijk<sup>8</sup>, J. Boyles<sup>9,10</sup>, A. T. Deller<sup>3</sup>, S. Chatterjee<sup>11</sup>, A. Schechtman-Rook<sup>7</sup>, A. Berndsen<sup>2</sup>, R. S. Lynch<sup>4</sup>, D. R. Lorimer<sup>9</sup>, C. Karako-Argaman<sup>4</sup>, V. M. Kaspi<sup>4</sup>, V. I. Kondratiev<sup>3,12</sup>, M. A. McLaughlin<sup>9</sup>, J. van Leeuwen<sup>3,5</sup>, R. Rosen<sup>1,9</sup>, M. S. E. Roberts<sup>13,14</sup> & K. Stovall<sup>15,16</sup>



Ransom et al.  
Nature  
505 (2014)  
520

# Ransom et al. Nature 505(2014)520 NS-WD binary + WD



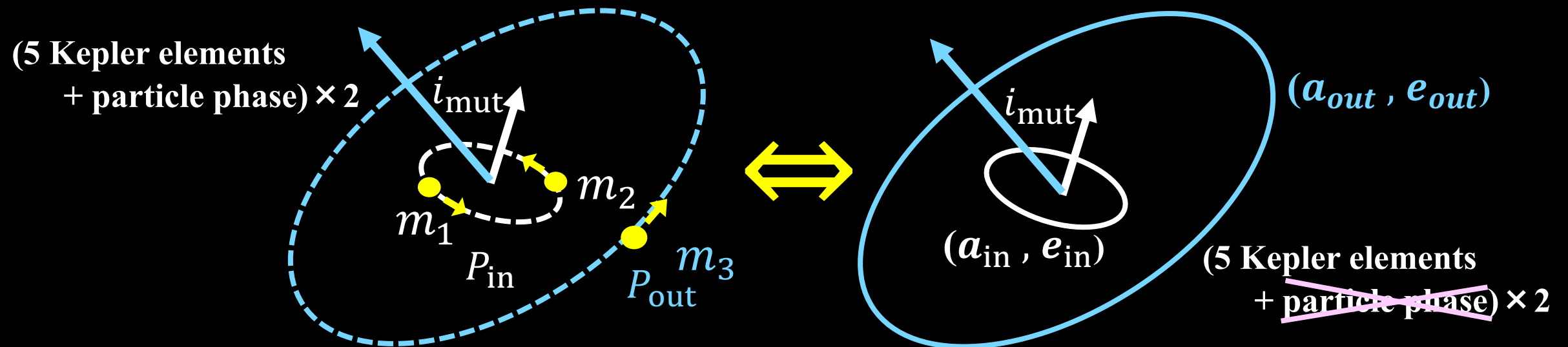
## PSR J0337+1715 parameters

inner orbital period (pulsar+WD)	1.629401788(5) day
outer orbital period (WD)	327.257541(7) day
pulsar spin period	2.73258863244(9) msec
mutual orbital inclination	0.0120(17) deg.
	<b>highly circular &amp; coplanar !</b>
Pulsar mass	1.4378(13) $M_{\odot}$
Inner WD mass	0.19751(15) $M_{\odot}$
Outer WD mass	0.4101(3) $M_{\odot}$

# **3 Secular-perturbation theory in the three-body problem and von Zeipel-Lidov-Kozai effect**

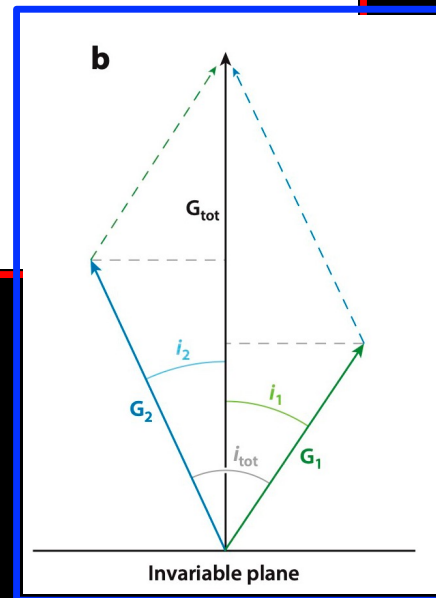
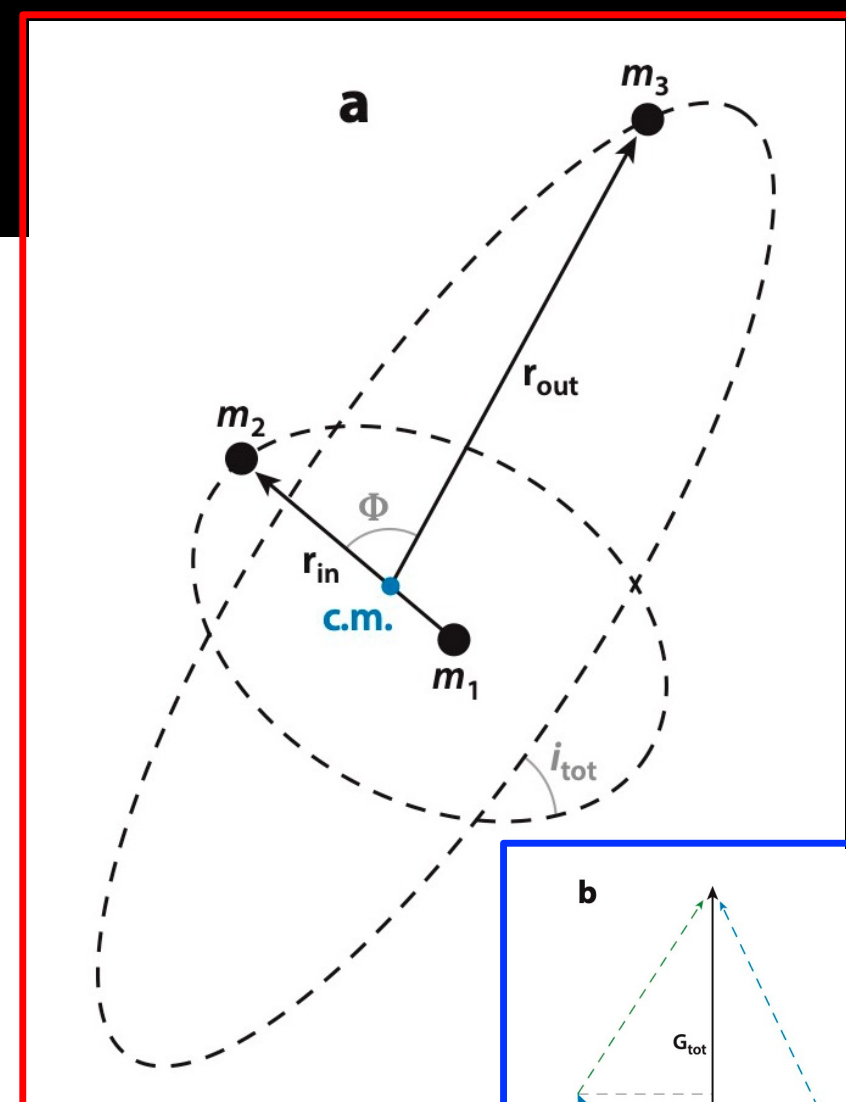
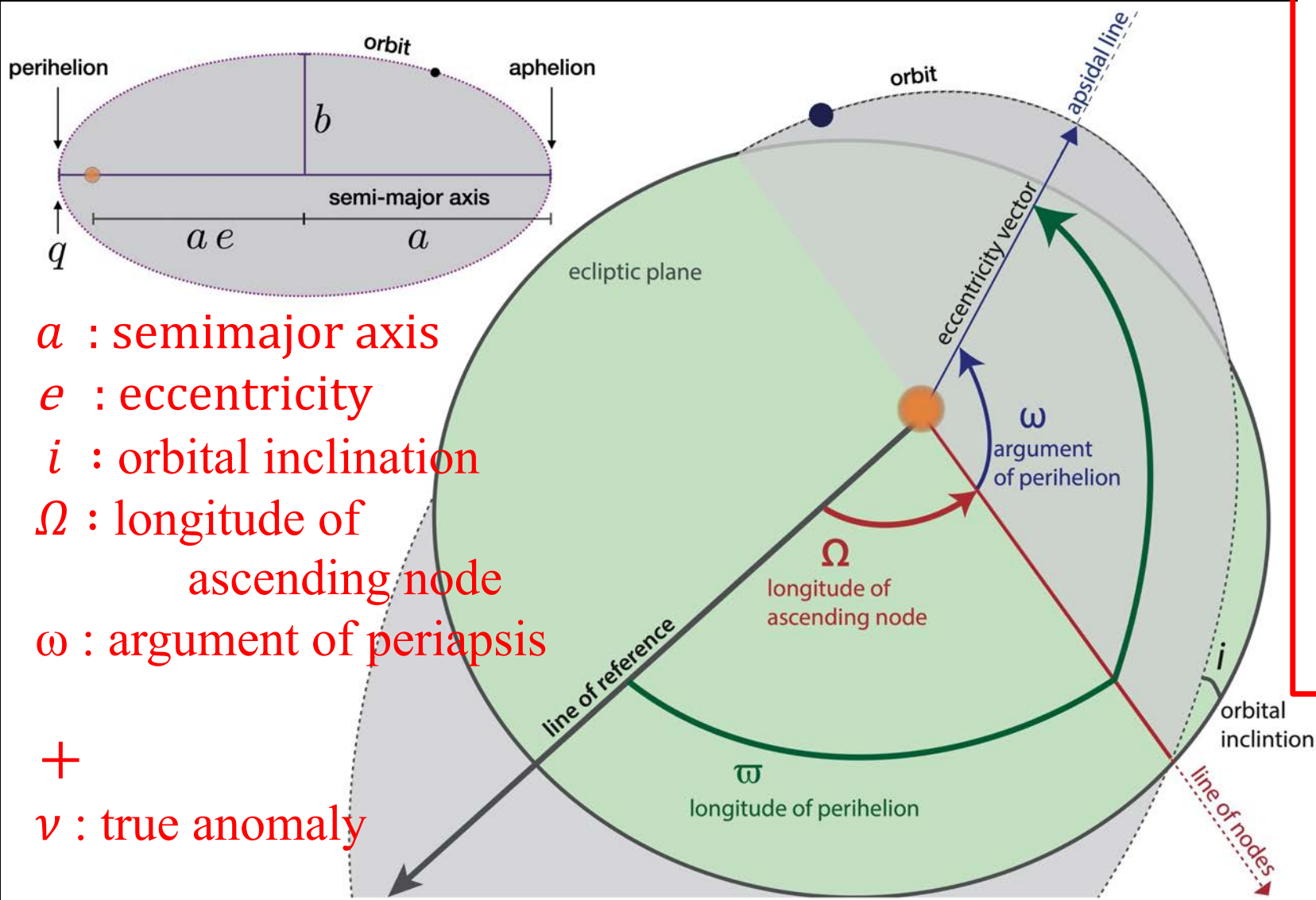
# Secular approximation to triple dynamics

- Very different timescales involved:  $P_{in} \ll P_{out}$ 
  - time-consuming for accurate numerical integration
- **perturbative expansion in terms of  $a_{in}/a_{out} (\ll 1)$** 
  - long-time numerical integration by approximating the particle-particle interaction with the ring-ring interaction over appropriate time-averaging of particles on their orbits





# Kepler orbital elements





# Legendre expansion of Hamiltonian

Kepler motion for inner orbit

interaction between inner and outer orbits

$$\mathcal{H} = \frac{k^2 m_1 m_2}{2a_1} + \frac{k^2 m_3 (m_1 + m_2)}{2a_2} + \frac{k^2}{a_2} \sum_{n=2}^{\infty} \left(\frac{a_1}{a_2}\right)^n M_n \left(\frac{\mathbf{r}_{\text{in}}}{a_1}\right)^n \left(\frac{a_2}{\mathbf{r}_{\text{out}}}\right)^{n+1} P_n(\cos \Phi)$$

Kepler motion for outer orbit

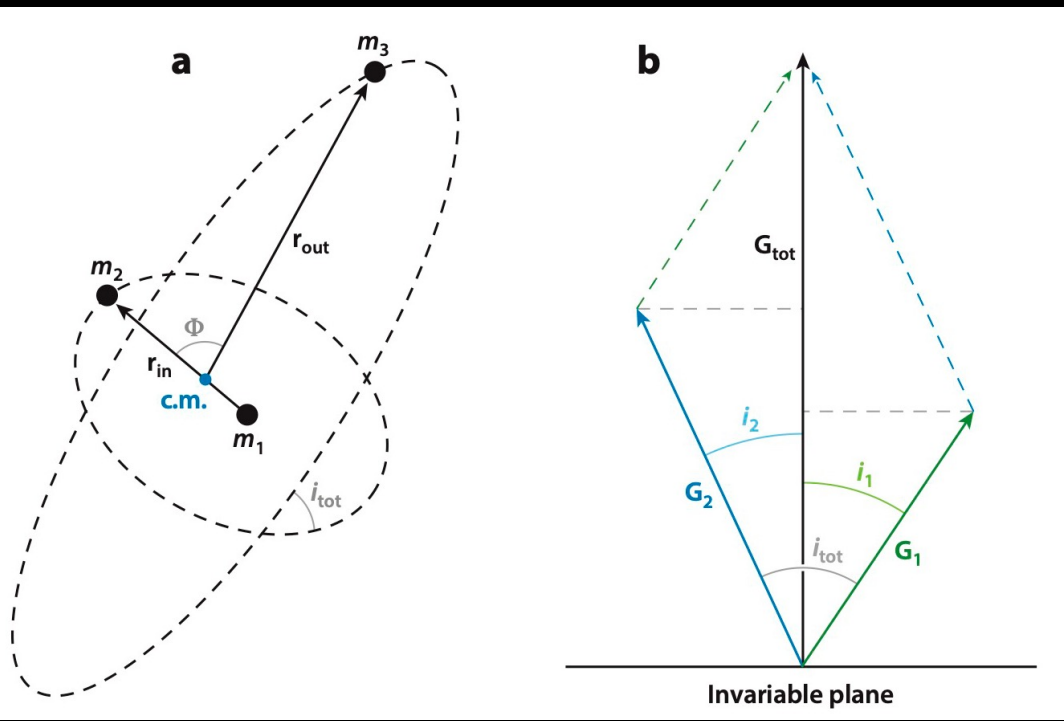
coupling constant

$$M_n = m_1 m_2 m_3 \frac{m_1^{n-1} - (-m_2)^{n-1}}{(m_1 + m_2)^n}$$

$$\mathcal{H} = \mathcal{H}_{\text{Kep,in}} + \mathcal{H}_{\text{Kep,out}} + \mathcal{H}_{\text{int}}$$

$$\mathcal{H}_{\text{int}} = \mathcal{H}_{\text{quad}} + \mathcal{H}_{\text{oct}} + \dots$$

⇒ approximation by double-averaging of the Hamiltonian over the inner and outer orbits



# double-averaged quadrupole and octupole Hamiltonians

$$\langle \mathcal{H}_{\text{quad}} \rangle = \frac{\mu_{12} \Phi_0}{16} [(2 + 3e_1^2)(3 \cos^2 i_{\text{tot}} - 1) + 15e_1^2 \sin^2 i_{\text{tot}} \cos 2\omega_1]$$

$$\langle \mathcal{H}_{\text{oct}} \rangle = -\frac{15}{64} \mu_{12} \Phi_0 e_1 \epsilon_{\text{oct}} [A \cos \phi + 10 \cos i_{\text{tot}} \sin^2 i_{\text{oct}} (1 - e_1^2) \sin \omega_1 \sin \omega_2]$$

$$\epsilon_{\text{oct}} = \frac{m_1 - m_2}{m_1 + m_2} \frac{a_1}{a_2} \frac{e_2}{1 - e_2^2}$$

$$\mu_{12} \Phi_0 = \frac{m_1 m_2}{m_1 + m_2} \frac{G m_3 a_1^2}{a_2^3 (1 - e_2^2)^{3/2}}$$

- octupole term vanishes if  $m_1 = m_2$  (equal-mass inner binary) or  $e_2 = 0$  (circular outer orbit)
- both  $a_{\text{in}}$  and  $a_{\text{out}}$  are conserved in the secular approximation  $\Rightarrow$  no energy exchange between inner and outer orbits (angular momentum exchange alone)

$$A = 4 + 3e_1^2 - \frac{5}{2} \sin^2 i_{\text{tot}} (2 + 5e_1^2 - 7e_1^2 \cos 2\omega_1)$$

$$\cos \phi = -\cos \omega_1 \cos \omega_2 - \cos i_{\text{tot}} \sin \omega_1 \sin \omega_2$$

# von Zeipel-Lidov-Kozai effect

- Takashi Ito and Katsuhito Ohtsuka (2019) "The Lidov–Kozai Oscillation and Hugo von Zeipel" *Monogr. Environ. Earth Planets*, 7, 1–113
- von Zeipel, H. (1910) "Sur l'application des séries de M. Lindstedt à l'étude du mouvement des comètes périodiques", *Astronomische Nachrichten*, 183, 345–418
- M.L. Lidov (1961) "Evolution of the orbits of artificial satellites of planets as affected by gravitational perturbation from external bodies" *Artificial Earth Satellite*, 8, 5–45
- Kozai, Yoshihide (1962) "Secular perturbations of asteroids with high inclination and eccentricity" *The Astronomical Journal*, 67, 591–598
- Naoz, S. (2016) "The eccentric Kozai–Lidov effect and its applications", *Annual Reviews of Astronomy and Astrophysics*, 54, 441–489

# standard and eccentric ZKL effects

## ■ standard ZKL

- test particle limit ( $m_2=0$ ) and circular outer orbit ( $e_{\text{out}}=0$ )
- angular momentum of the inner orbit along the total (outer) orbital axis is conserved

$$j_{1,z} = \sqrt{1 - e_1^2} \cos i_1 = \sqrt{1 - e_{1,\text{init}}^2} \cos i_{1,\text{init}}$$

- inner eccentricity and inclination periodically change with

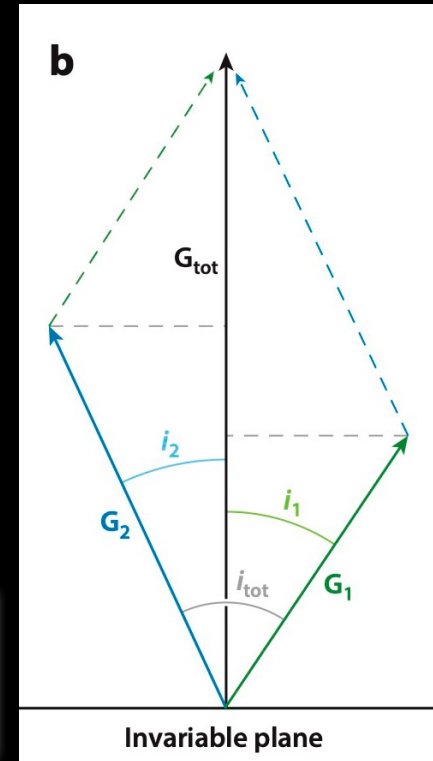
$$e_{1,\text{max}} = \sqrt{1 - \frac{5}{3} \cos^2 i_{1,\text{init}}}$$

if

$$39.2^\circ < i_{1,\text{init}} < 140.8^\circ$$

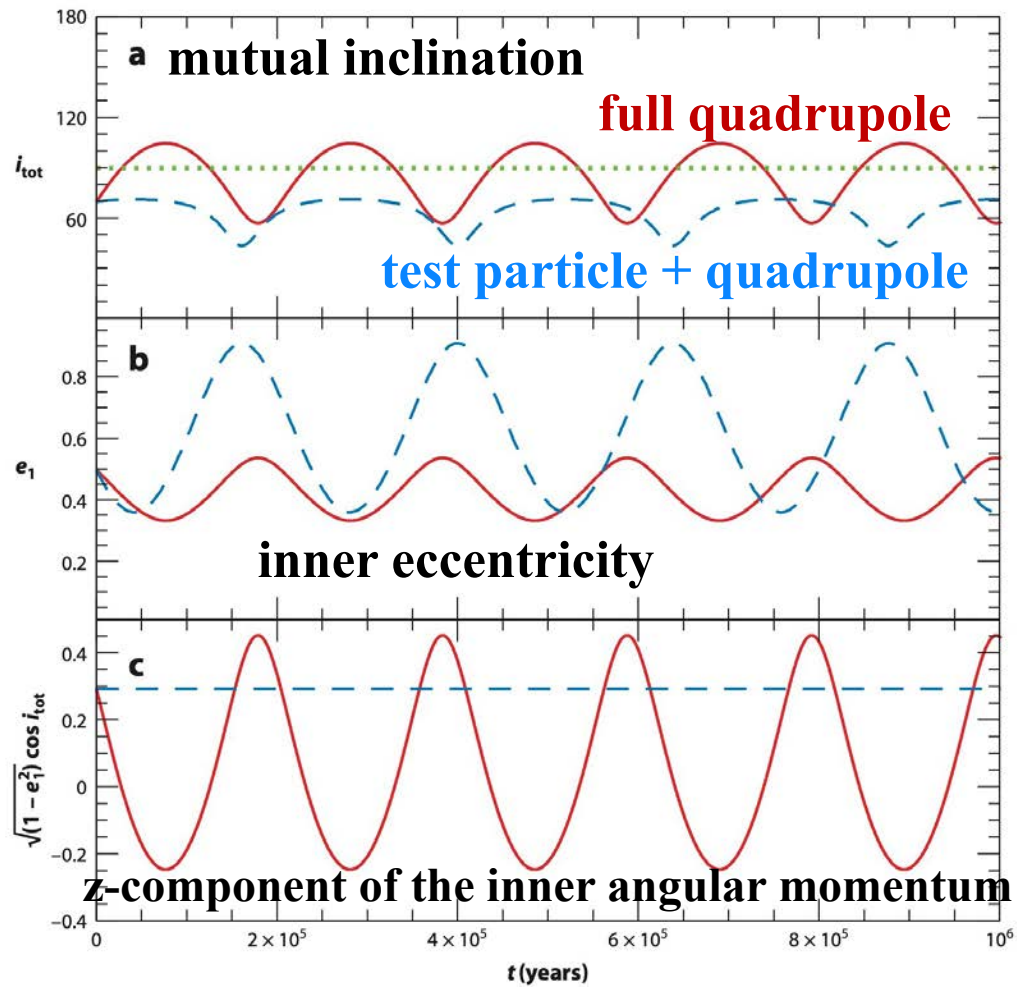
## ■ eccentric ZKL

- $e_{\text{out}} \sim 1 \Rightarrow$  more drastic effect due to the octupole term



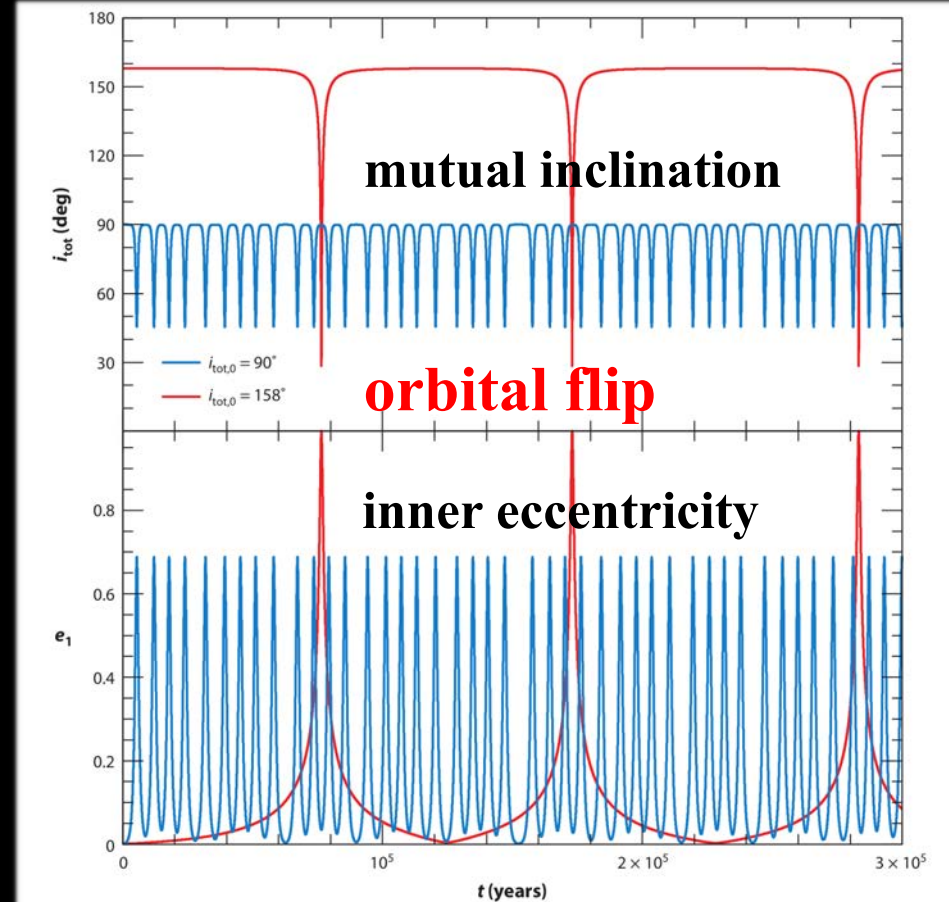
# examples of ZKL effects in secular approximation

Naoz, S: ARAA 54(2016)441



**Figure 3**

Comparison between the test particle quadrupole (TPQ) formalism (*dashed blue lines*) and the full quadrupole calculation (*solid red lines*). The system has an inner binary with  $m_1 = 1.4 M_\odot$  and  $m_2 = 0.3 M_\odot$ , and the outer body has mass  $m_3 = 0.01 M_\odot$ . The orbit separations are  $a_1 = 5$  AU and  $a_2 = 50$  AU. The system was set initially with  $e_1 = 0.5$  and  $e_2 = 0$ ,  $\omega_1 = 120^\circ$  and  $\omega_2 = 0$ , and relative inclination  $i_{\text{tot}} = 70^\circ$ . The panels



**Figure 4**

Small-mass outer perturber that induces large eccentricity excitation away from the nominal range of the Kozai angles of  $39.2^\circ$ – $140.77^\circ$ . We consider  $m_1 = 1 M_\odot$ ,  $m_2 = 0.5 M_\odot$ ,  $m_3 = 0.05 M_\odot$ ,  $a_1 = 0.5$  AU, and  $a_2 = 5$  AU. Both outer and inner eccentricities are set initially to zero, and also set initially are  $\omega_1 = 90^\circ$  and  $\omega_2 = 0^\circ$ . We show two examples: The first shows the eccentricity excitations for as expected initial mutual inclination of  $i_{\text{tot}} = 90^\circ$ , where in this case  $i_1 = 25.01^\circ$  and  $i_2 = 64.99^\circ$ . This produces eccentricity excitation with  $e_{1,\text{max}} = 0.689$ . We also consider an example for which the mutual inclination is set initially to be  $i_{\text{tot}} = 158^\circ$ . In this case  $i_1 = 17.12^\circ$  and  $i_2 = 140.88^\circ$ . The latter parameters are adapted from Martin & Triaid (2015b), which leads to maximum inner eccentricity of  $e_{1,\text{max}} = 0.99$ . Note that in both examples  $i_2$  is close to the nominal Kozai angles range.



# effects of octupole and general relativity

Naoz, S: ARAA 54(2016)441

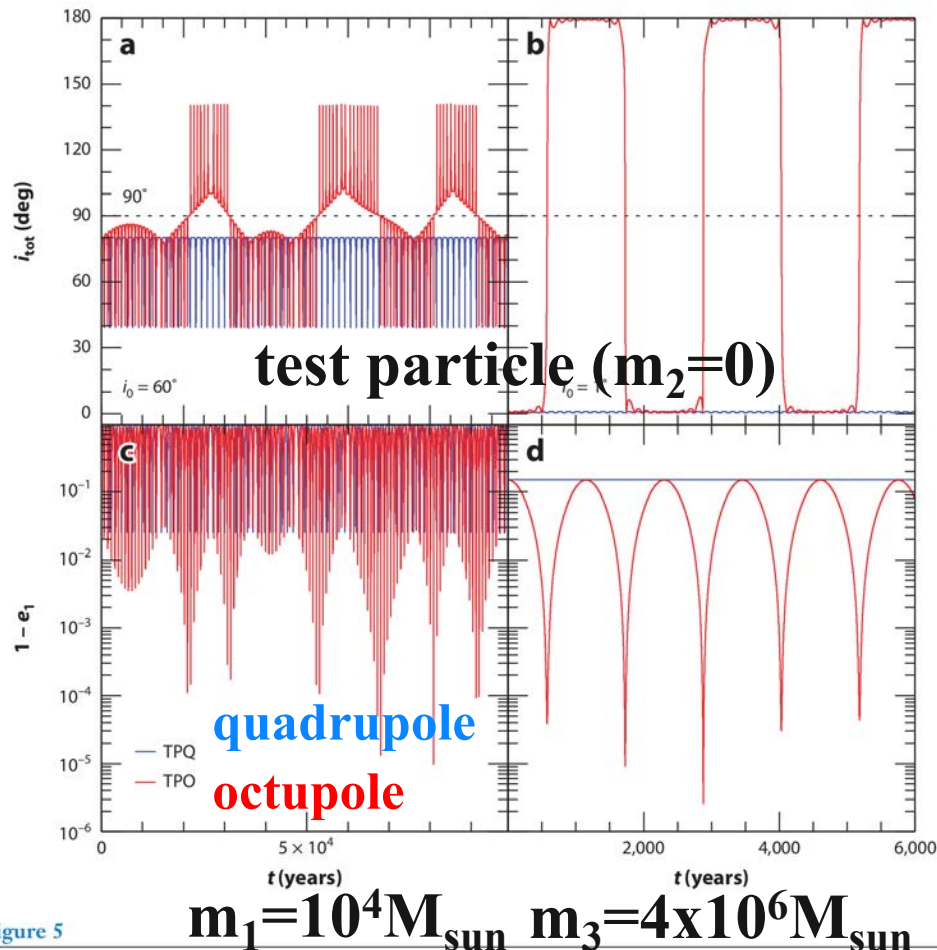


Figure 5

Time evolution example of the test particle octupole (TPO) approximation (red lines) and the test particle quadrupole (TPQ) approximation (blue lines). Panels *a* and *c* show high-inclination ( $i > 39.2^\circ$ ) flip, whereas panels *b* and *d* show low-inclination ( $i < 39.2^\circ$ ) flip (high inclination  $i > 39.2^\circ$ , low inclination  $i < 39.2^\circ$ ). In this example, we consider the time evolution of a test particle at 135 AU around a  $10^4 M_\odot$  intermediate black hole located 0.03 pc from the massive black hole in the center of our galaxy ( $4 \times 10^6 M_\odot$ ). In panels *a* and *c* the system initially is set with  $e_1 = 0.01$ ,  $e_2 = 0.7$ ,  $i = 60^\circ$ ,  $\Omega_1 = 60^\circ$ , and  $\omega_1 = 0^\circ$ . In panels *b* and *d* the system is initially set with  $e_1 = 0.85$ ,  $e_2 = 0.85$ ,  $i = 1^\circ$ ,  $\Omega_1 = 180^\circ$ , and  $\omega_1 = 0^\circ$ . In panels *a* and *b*, we show the inclination, and in panels *c* and *d* the inner orbit eccentricity as  $1 - e_1$ .

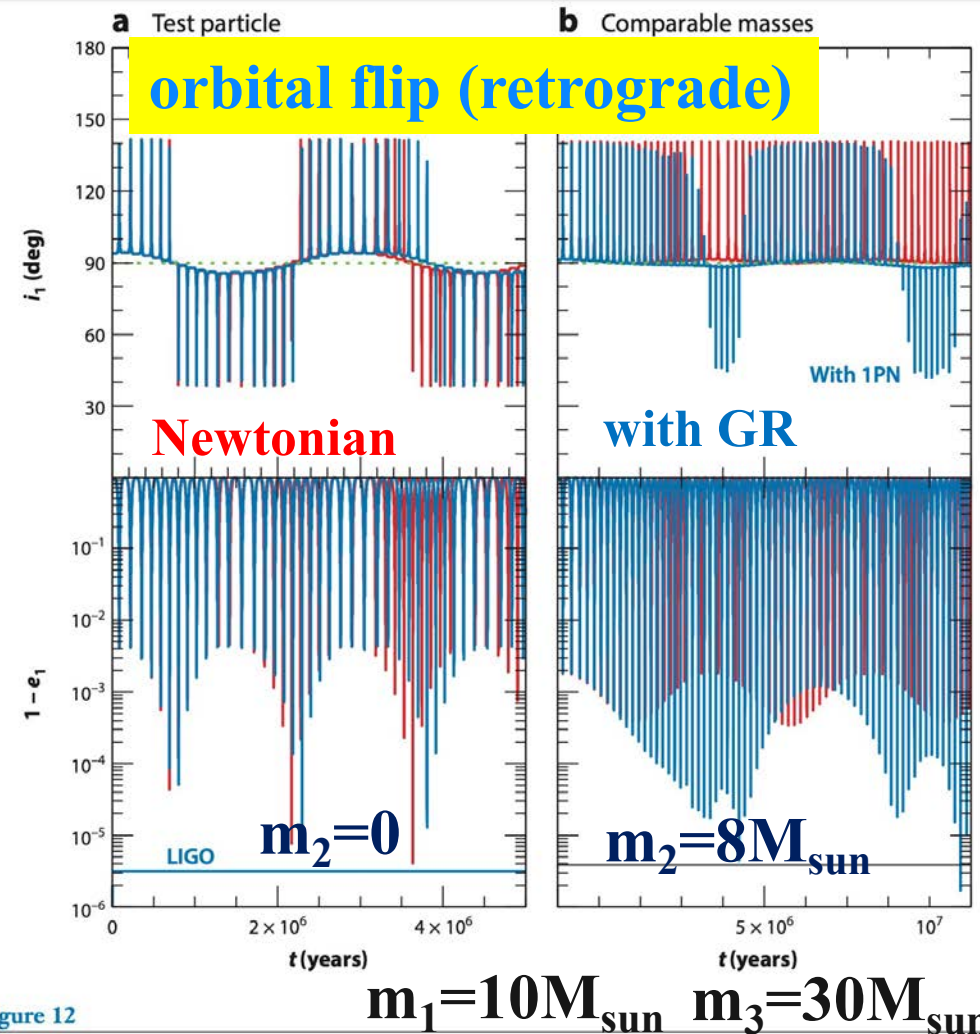


Figure 12

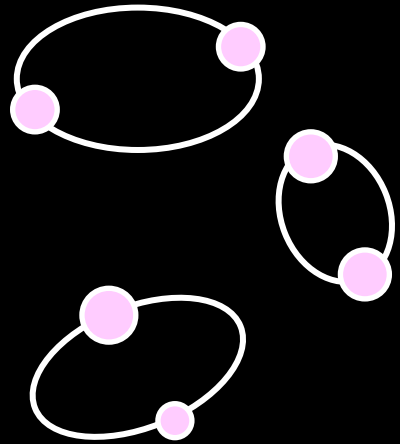
Comparison between the test particle approximation and a comparable-mass system in the presence of general relativity. The systems in the right and left panels have the same parameters and initial conditions apart from  $m_2$ , which is set to zero in panel *a* and  $m_2 = 8 M_\odot$  in panel *b*. The other parameters are:  $m_1 = 10 M_\odot$ ,  $m_3 = 30 M_\odot$ ,  $a_1 = 10$  AU,  $a_2 = 502$  AU,  $e_1 = 0.001$ ,  $e_2 = 0.7$ ,  $\omega_1 = \omega_2 = 240^\circ$ , and  $i_{\text{tot}} = 94^\circ$ . Red lines correspond to pure Newtonian evolution, and blue lines include general relativity (GR).

**4 Search for star-blackhole binaries in  
optical surveys:  
a star-blackhole binary  
or  
a star-binary blackhole triple?**

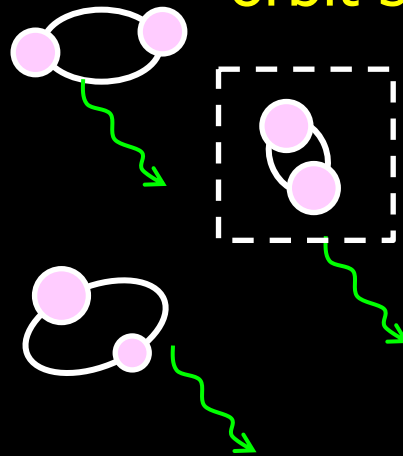


# Generic picture of binary BH evolution

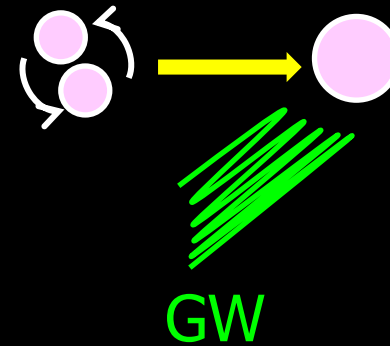
binary black holes form in wide orbits



orbit shrinking



merger

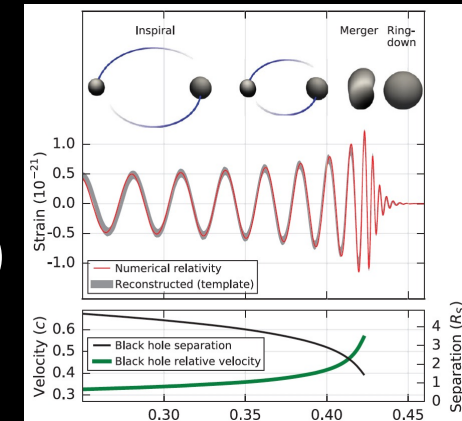


weak GW (low-frequency)

BBHs would spend longer time in wide orbits before merging

Abundant longer orbital-period BBHs may remain undetected (e.g.  $\sim 10$  day orbital period  $\sim 10^{-6}$  Hz).

Detection strategy complementary to GW ?

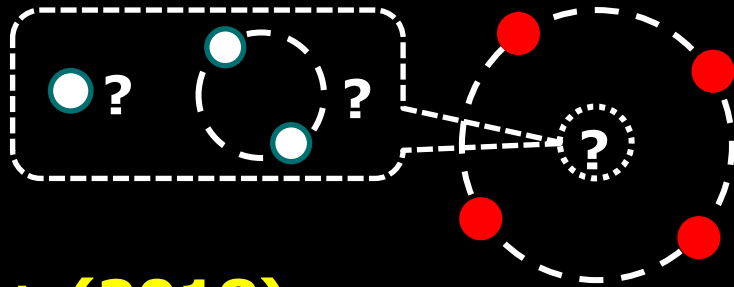


LIGO/Virgo

# Proposals to search for star-BH binaries

## Gaia mission (2013-)

Astrometry of stars in Galaxy  
~  $10^9$  stars eventually  
RV with 200-350m/s precision  
for brightest stars (Katz 2018)



## Yamaguchi+ (2018)

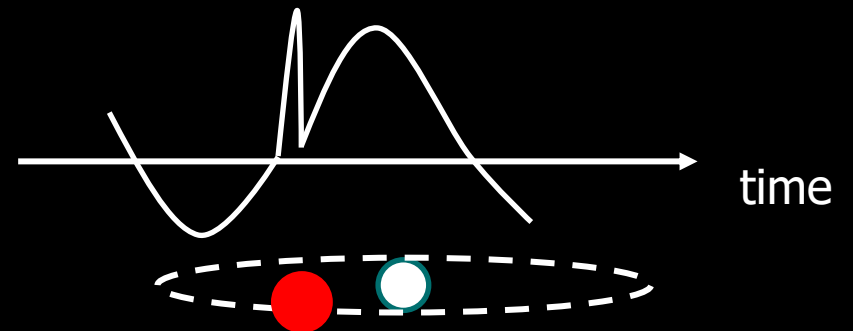
5-year mission may detect  
200-1000 star-BH binaries

## TESS mission (2018-)

photometry of nearby stars (~ 12mag)  
transit planets

## Masuda & Hotokezaka (2019)

Light curve modulation  
(relativistic effects, tidal deformation)  
⇒ (10 – 100) star-BH binaries may be  
identified

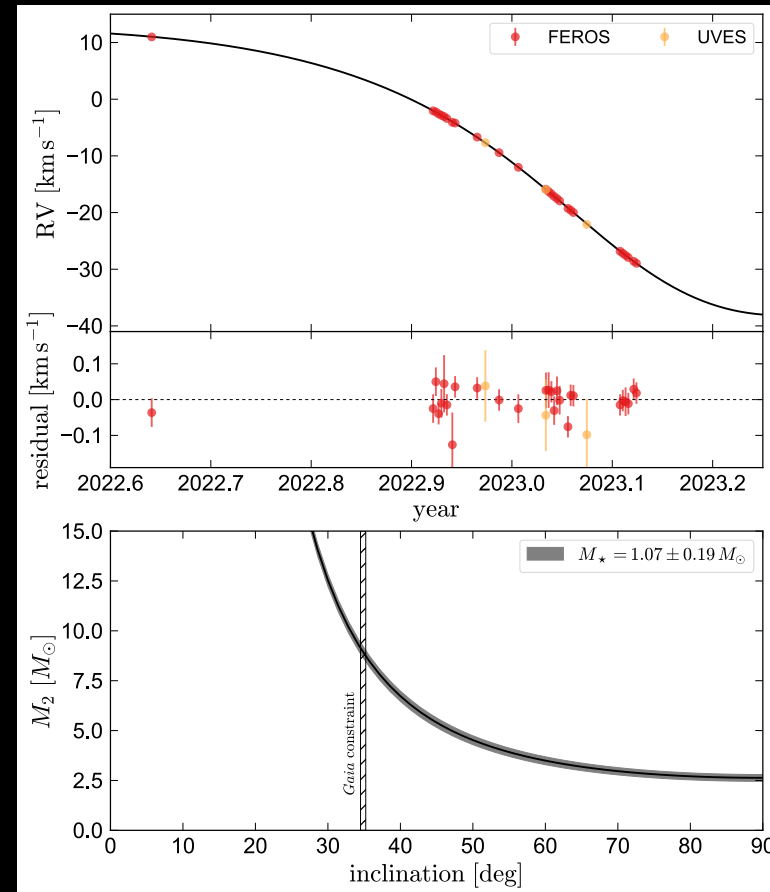
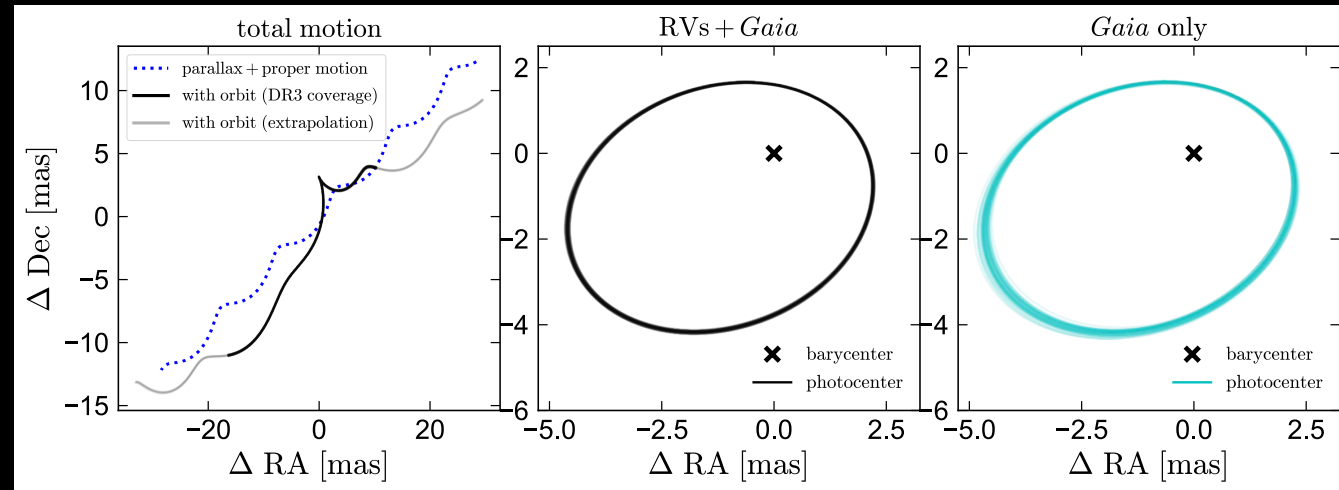
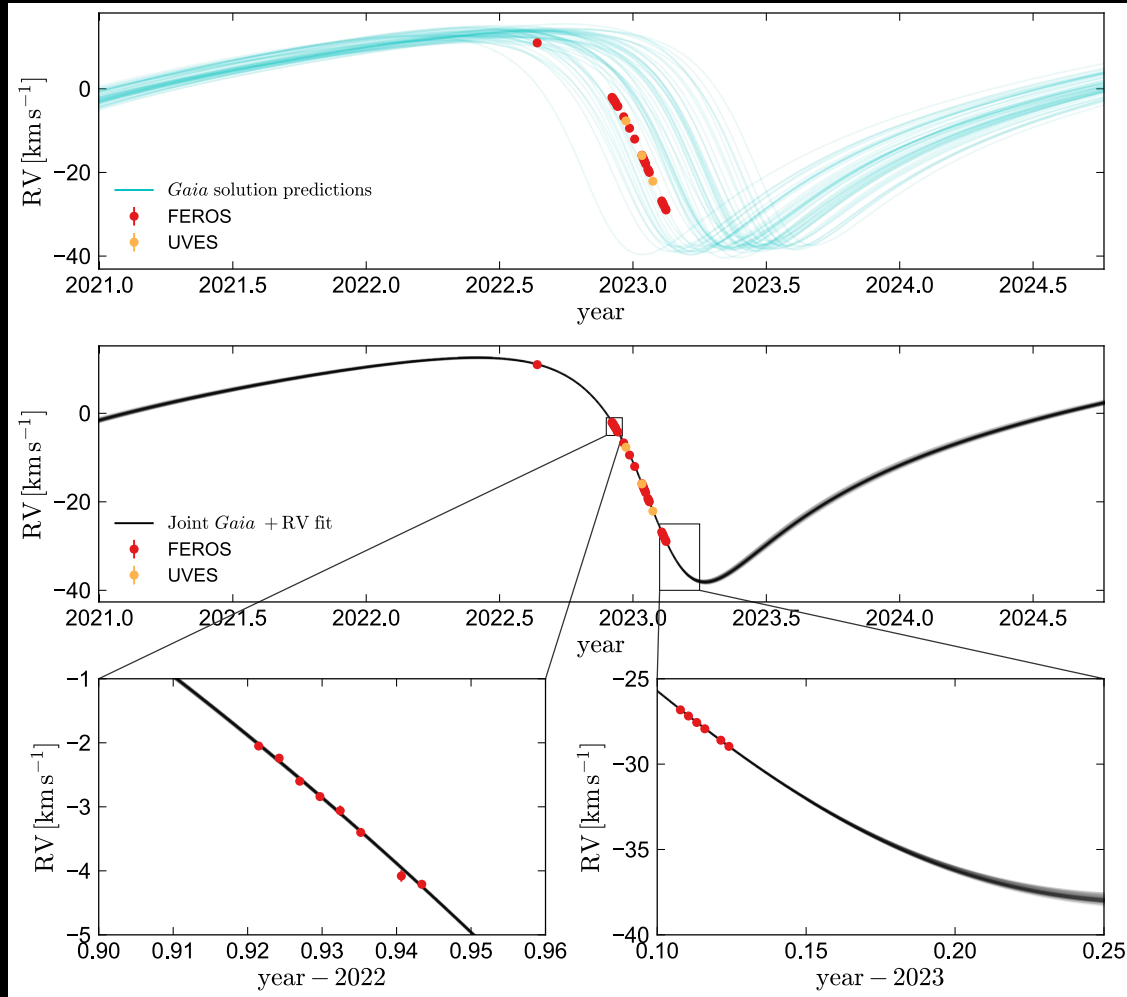


**Some of them may be indeed a star-binary BH triple!**  
**Can precise radial velocity follow-up unveil the inner BBH?**

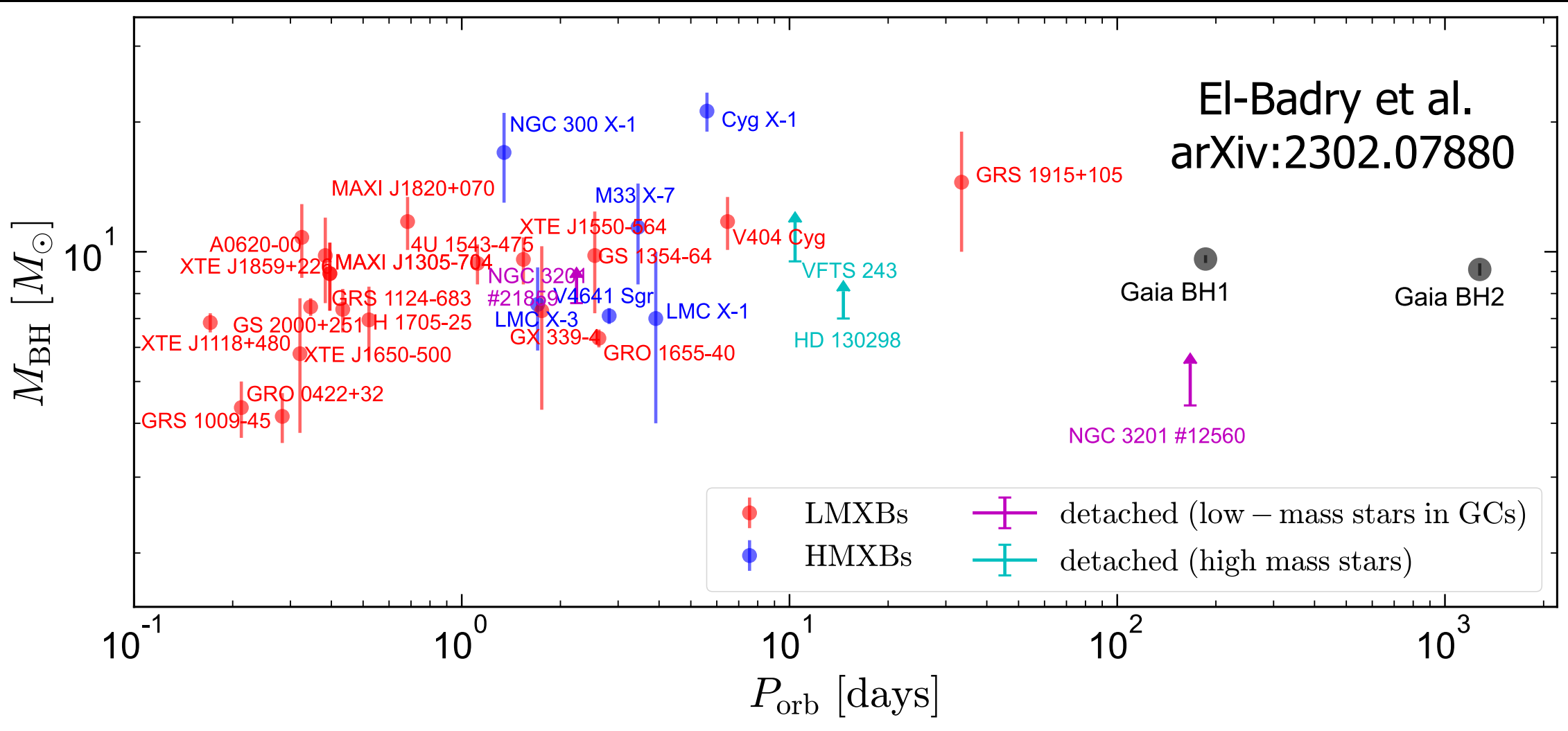


# Gaia BH-2

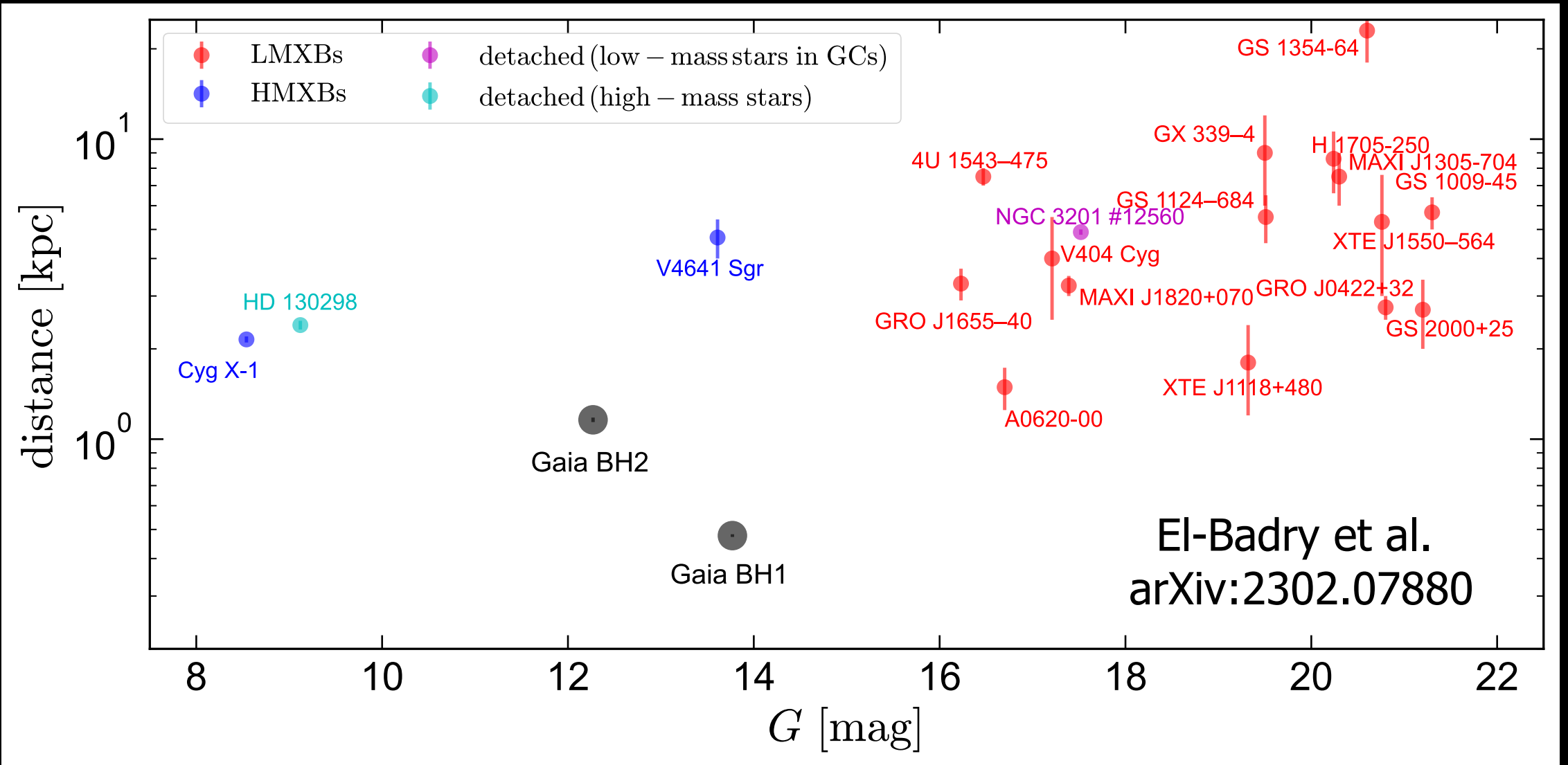
$1 M_{\odot}$  red giant +  $9 M_{\odot}$  BH  
( $P_{\text{orb}}=1277$ days) at  $d=1.16$ kpc  
eccentricity  $\sim 0.52$



El-Badry et al.  
arXiv:2302.07880



**Comparison of Gaia BH1 and BH2 to other known BHs. Red and blue symbols correspond to accreting BHs with low- and high-mass companions. Magenta symbols show detached binaries in the globular cluster NGC 3201, and cyan points show detached binaries in which the luminous star is a high-mass ( $>20 M_{\odot}$ ) star.**

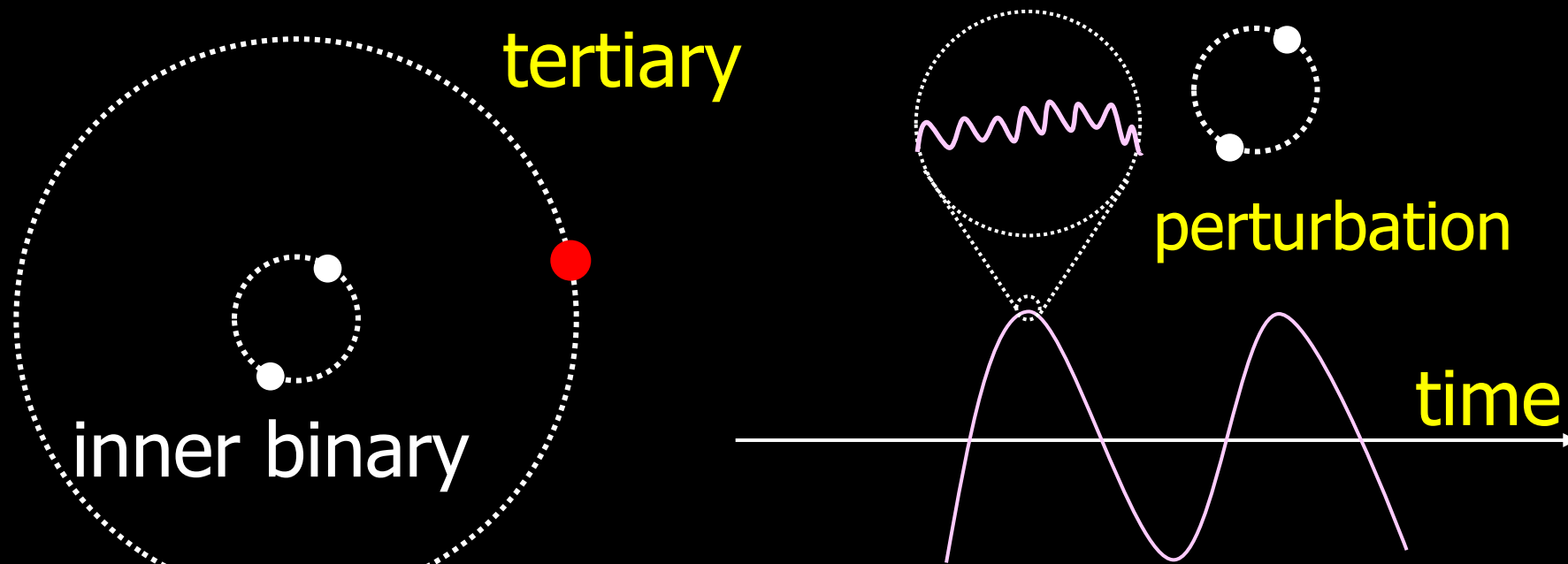


**Comparison of Gaia BH1 and BH2 (black points) to known Galactic BHs in the plane of distance and quiescent optical magnitude**

# **5 Dynamical signature of triple systems with an inner binary black-hole**



# Radial velocity modulation of a tertiary star due to an inner binary



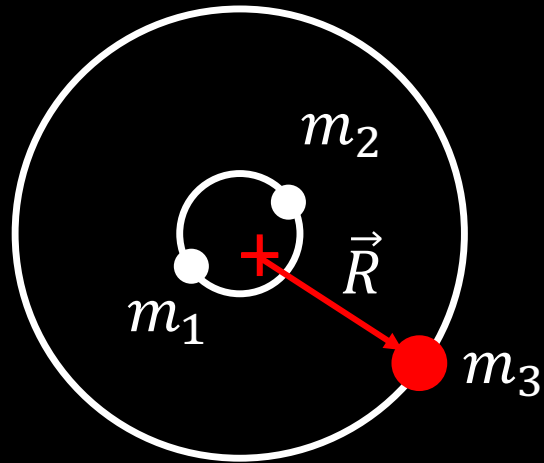
Toshinori Hayashi  
(林 利憲)

## Kepler motion of the tertiary

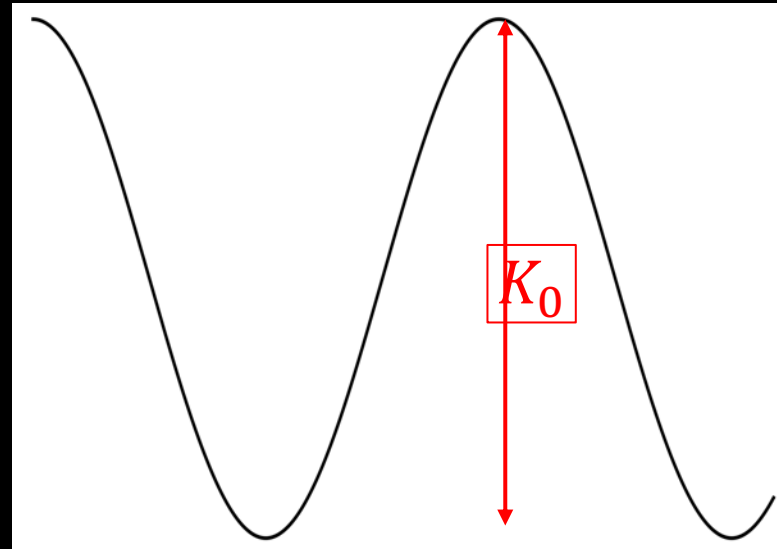
Hayashi, Wang + YS: ApJ 890(2020)112

Hayashi + YS: ApJ 897(2020)29

# RV modulations for coplanar triples



RV =



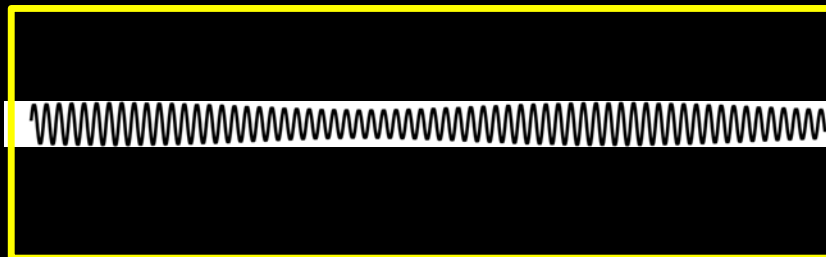
period:  $P_{out}$



period:  $P_{out}/2$

first order in  $e_{out}$

$$\left( \sqrt{\frac{m_1}{m_2}} + \sqrt{\frac{m_2}{m_1}} \right)^{-2} \left( \frac{a_{in}}{a_{out}} \right)^{3.5} K_0$$



period:  $P_{in}/2$   
inner binary

# Approximate expressions for RV of the tertiary star

$$V_{\text{RV}}(t) = V_{\text{Kep}}^{(0)}(t) + \delta V_{\text{Kep}}(t) + V_{\text{bin}}(t)$$

Morais & Correia (2008)

Hayashi & YS (2020)

$$\nu_{-3} \equiv 2\nu_{\text{in}} - 3\nu_{\text{out}},$$

$$\nu_{-1} \equiv 2\nu_{\text{in}} - \nu_{\text{out}}.$$

## (i) Unperturbed Kepler motion

$$V_{\text{Kep}}^{(0)}(t) = K_0 \sin I_{\text{out}} \cos[\nu_{\text{out}} t + f_{\text{out},0} + \omega_{\text{out}}]$$

$$K_0 \equiv \frac{m_1 + m_2}{m_1 + m_2 + m_*} a_{\text{out}} \nu_{\text{out}},$$

## (ii) Perturbation to the Kepler motion

$$\delta V_{\text{Kep}}(t) = K_1 \sin I_{\text{out}} \cos[\nu_{\text{out}} t + f_{\text{out},0} + \omega_{\text{out}}]$$

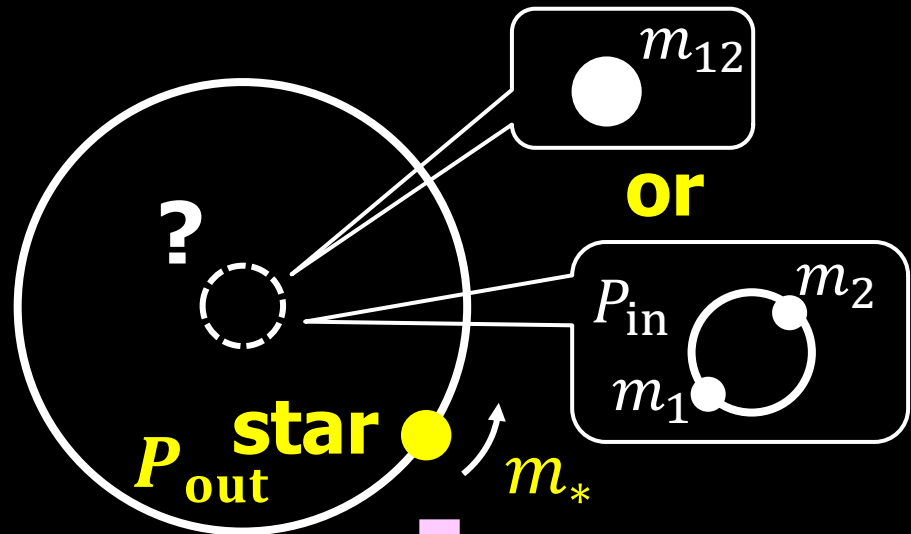
$$K_1 \equiv \frac{3}{4} K_0 \left( \frac{a_{\text{in}}}{a_{\text{out}}} \right)^2 \frac{m_1 m_2}{(m_1 + m_2)^2}.$$

## (iii) Modulation by the inner binary

$$\begin{aligned} V_{\text{bin}}(t) = & -\frac{15}{16} K_{\text{bin}} \sin I_{\text{out}} \cos[(2\nu_{\text{in}} - 3\nu_{\text{out}})t \\ & + 2(f_{\text{in},0} + \omega_{\text{in}}) - 3(f_{\text{out},0} + \omega_{\text{out}})] \\ & + \frac{3}{16} K_{\text{bin}} \sin I_{\text{out}} \cos[(2\nu_{\text{in}} - \nu_{\text{out}})t \\ & + 2(f_{\text{in},0} + \omega_{\text{in}}) - (f_{\text{out},0} + \omega_{\text{out}})], \end{aligned}$$

$$K_{\text{bin}} \equiv \frac{m_1 m_2}{(m_1 + m_2)^2} \sqrt{\frac{m_1 + m_2 + m_*}{m_1 + m_2}} \left( \frac{a_{\text{in}}}{a_{\text{out}}} \right)^{7/2} K_0,$$

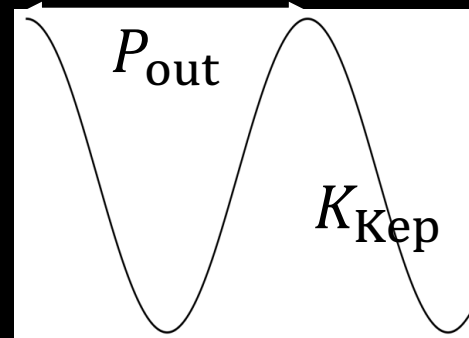
# RV modulations for non-coplanar triples



**high-precision RV follow-up**

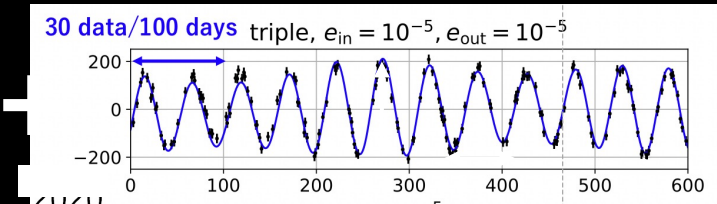
Keplerian motion RV  
+ RV variations by inner binary

## (i) Coplanar triple



Kepler motion + Short-term RV variations  
(inner-binary perturbation)

$$\text{Amp} \sim K_{Kep} \left( \frac{P_{in}}{P_{out}} \right)^{\frac{7}{3}}$$

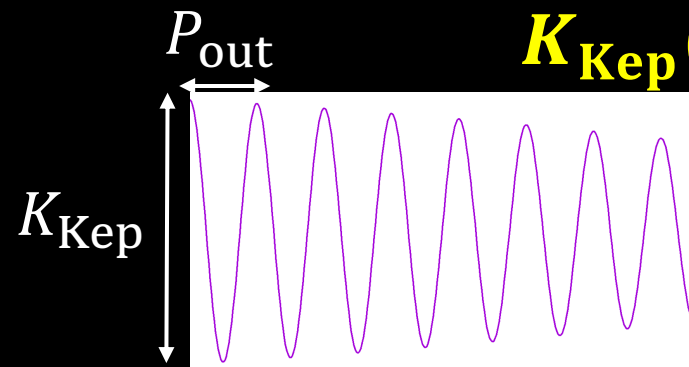


period  $\sim P_{in}/2$

## (ii) Non-coplanar triple

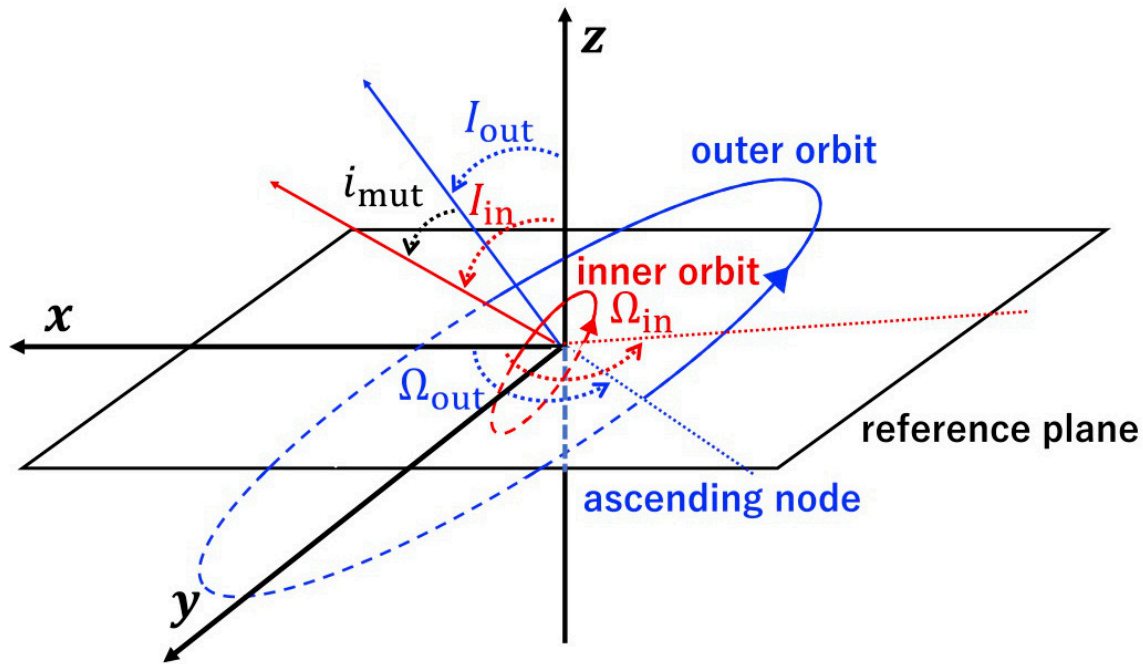
Inclination  $I_{out}(t)$  modulated in the Kozai-Lidov timescale

$$K_{Kep}(t) = K_0 \sin I_{out}(t)$$



Amplitude of Kepler RV  
varies with the timescale

# Parameters for simulated triple systems



direct N-body (N=3) simulation  
without secular approximation  
**Hayashi & YS 2020, ApJ, 897, 29**

Model	$I_{out}$ (deg)	$I_{in}$ (deg)	$i_{mut}$ (deg)	$m_1$ ( $M_{\odot}$ )	$m_2$ ( $M_{\odot}$ )	$e_{in}$
P1010	90	90	0	10	10	$10^{-5}$
PE1010	90	90	0	10	10	0.2
R1010	90	270	180	10	10	$10^{-5}$
O1010	0	90	90	10	10	$10^{-5}$
I1010	0	45	45	10	10	$10^{-5}$
P0218	90	90	0	18	2	$10^{-5}$
PE0218	90	90	0	18	2	0.2
R0218	90	270	180	18	2	$10^{-5}$
O0218	0	90	90	18	2	$10^{-5}$
I0218	0	45	45	18	2	$10^{-5}$

**Note.** P, PE, R, O, and I indicate prograde, prograde eccentric, retrograde, orthogonal, and inclined orbits.

$P_{out} = 78.9$  days

$P_{in} = 10$  days

equal-mass binary  $10M_{\odot} + 10M_{\odot}$

unequal-mass binary  $2M_{\odot} + 18M_{\odot}$



# Coplanar circular triples

Prograde equal-mass

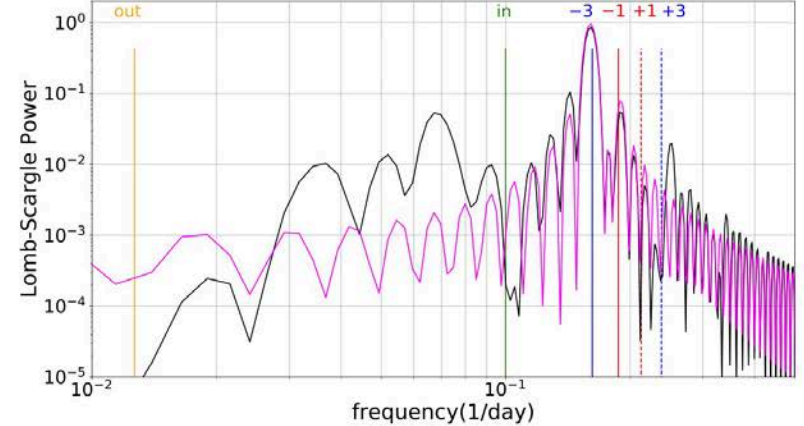
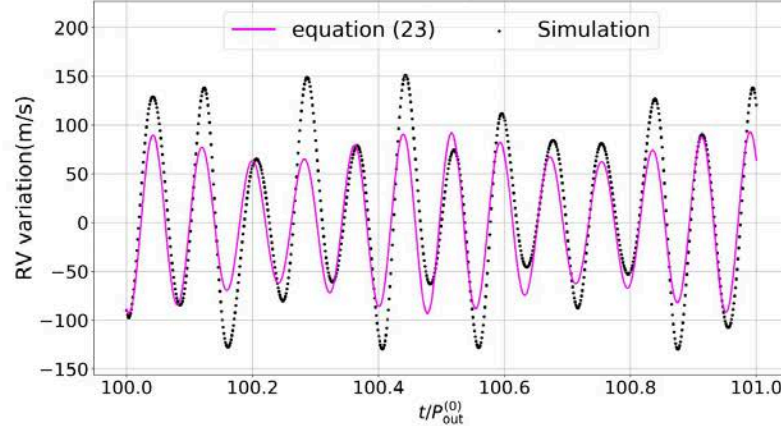
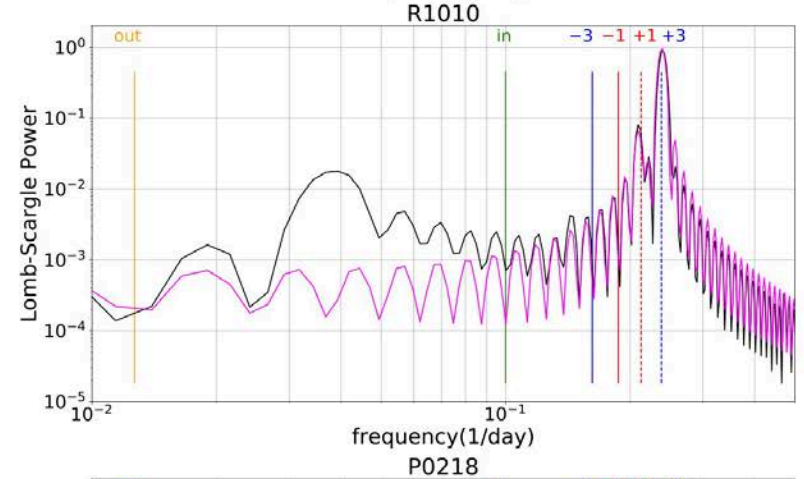
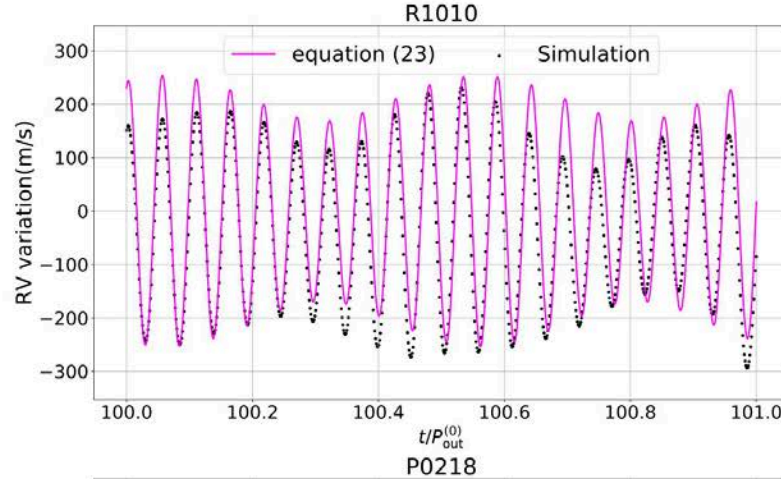
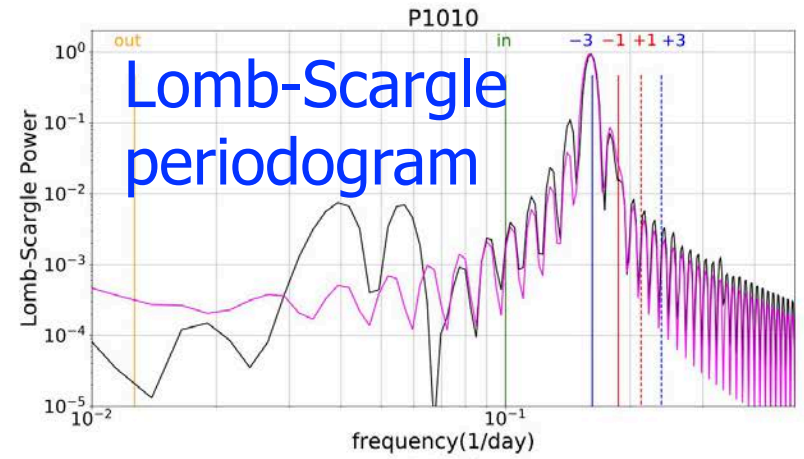
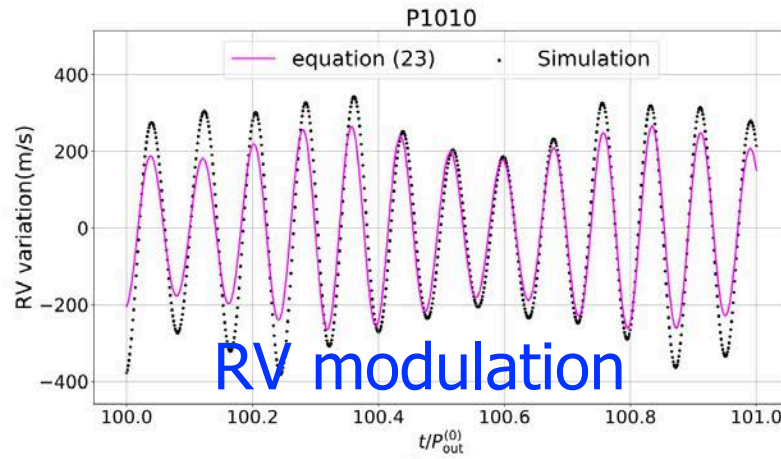
Simulation against Perturbative model (Morais & Correia 2008, 2012)

Retrograde equal-mass

$$\nu_{-3} \equiv 2\nu_{\text{in}} - 3\nu_{\text{out}},$$

$$\nu_{-1} \equiv 2\nu_{\text{in}} - \nu_{\text{out}}.$$

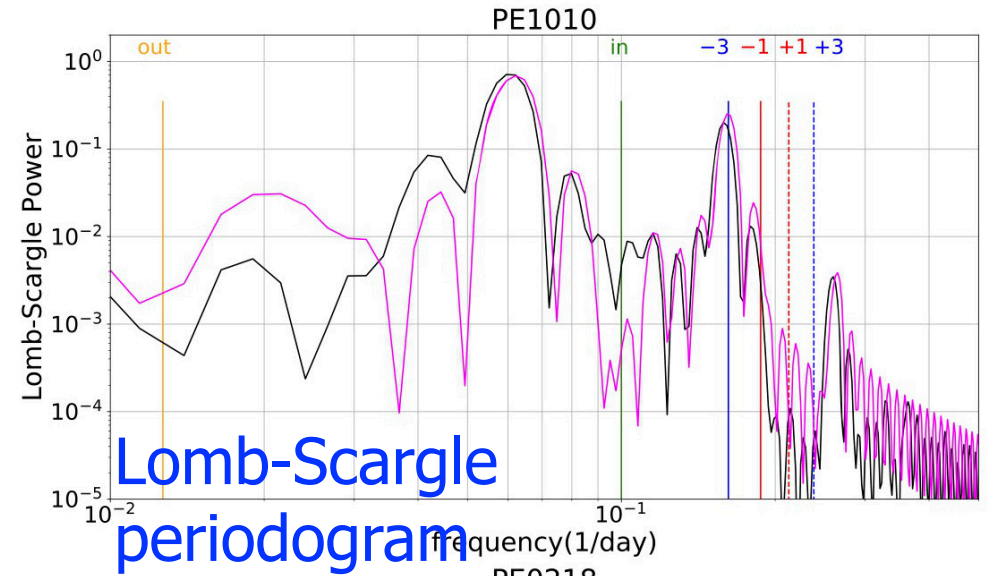
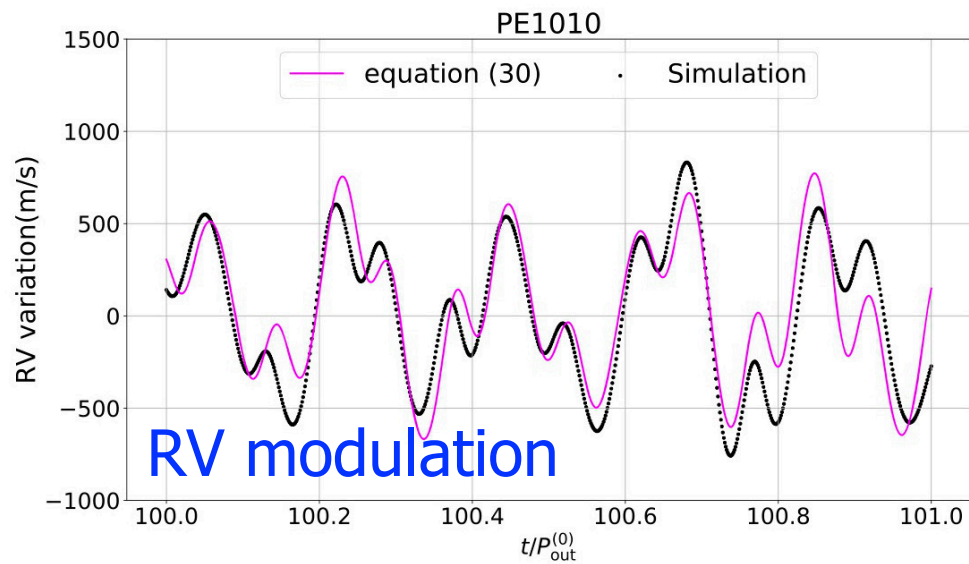
Prograde unequal-mass



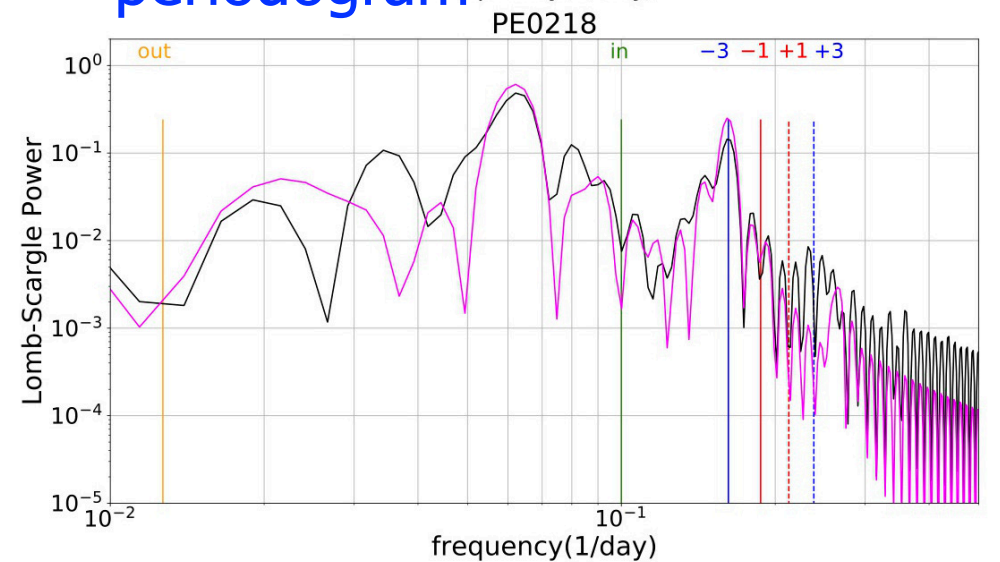
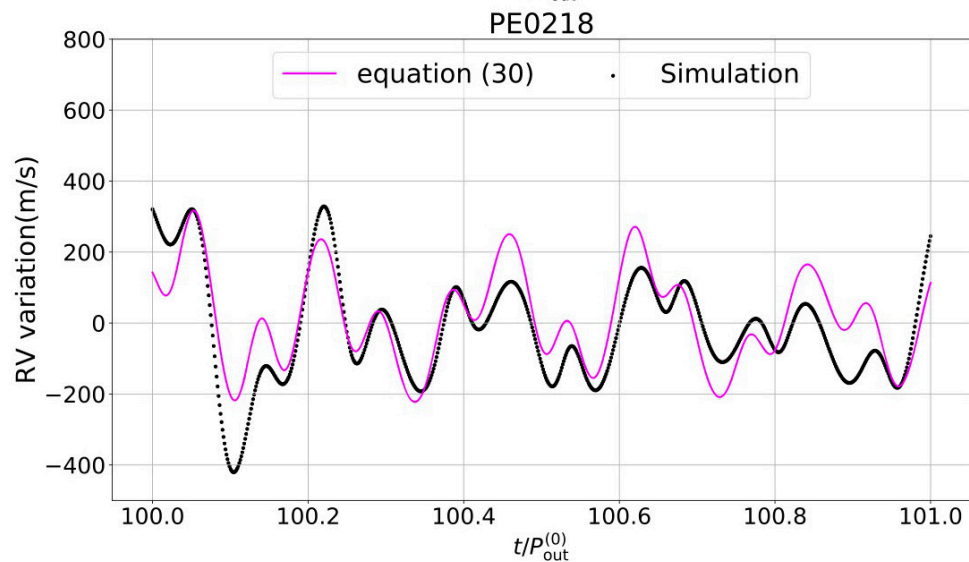
# Coplanar eccentric triples

Simulation against Perturbative model (Morais & Correia 2008, 2012)

Prograde  
equal-mass



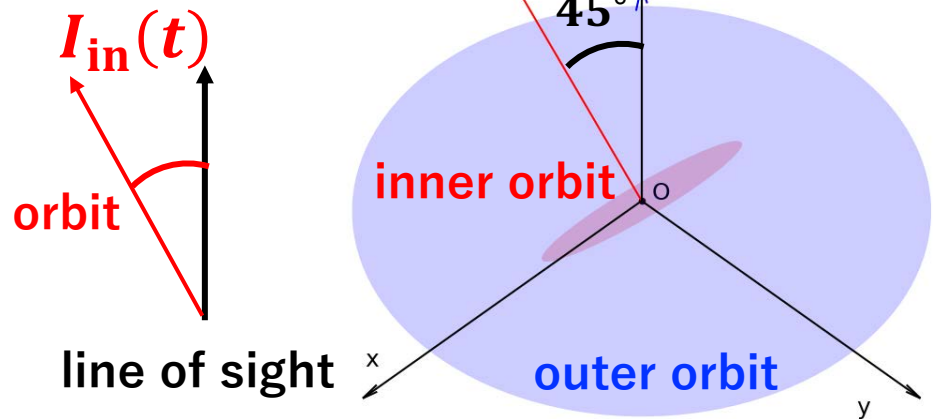
Prograde  
unequal-mass





# Evolution of inclination for non-coplanar triples

$i_{\text{mut}} = 45^\circ$   $t = 0P_{\text{out}}^{(0)}$



$P_{\text{out}} = 78.9$  days

$P_{\text{in}} = 10$  days

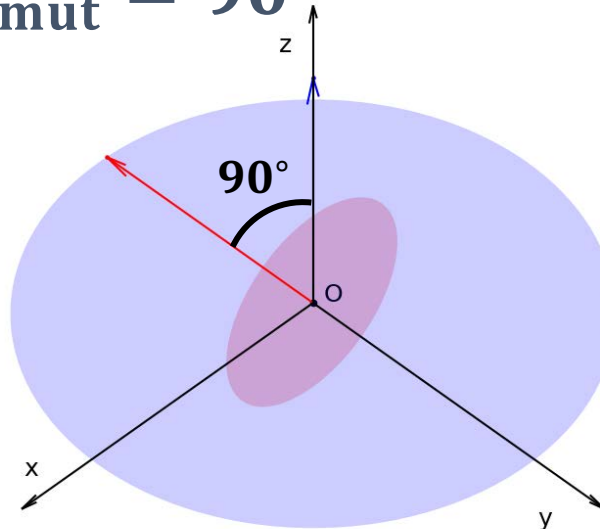
$m_1 = m_2 = 10M_\odot$

$m_* = 3M_\odot$

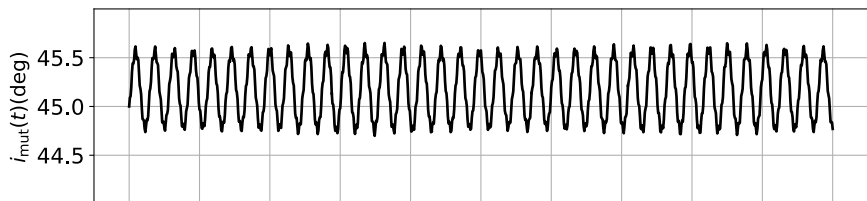
$e_{\text{out}} = 0.03$

$e_{\text{in}} = 10^{-5}$

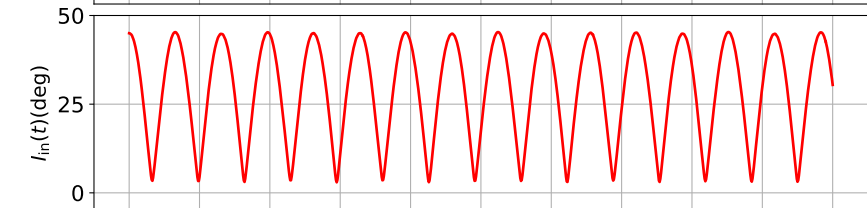
$i_{\text{mut}} = 90^\circ$   $t = 0P_{\text{out}}^{(0)}$



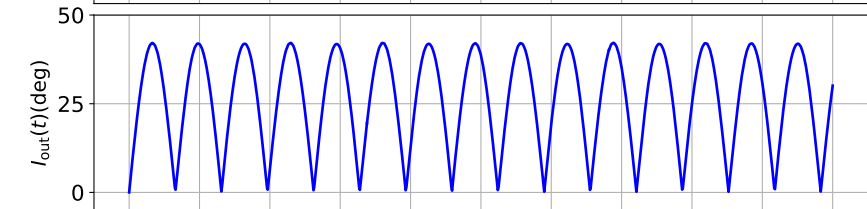
$i_{\text{mut}}(t)$



$I_{\text{in}}(t)$

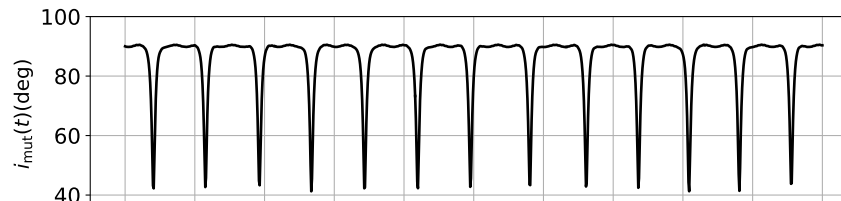


$I_{\text{out}}(t)$

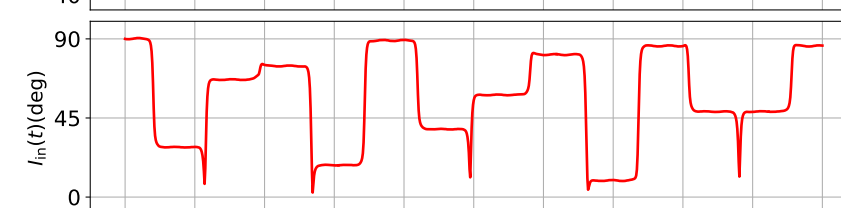


$t/P_{\text{out}}^{(0)}$

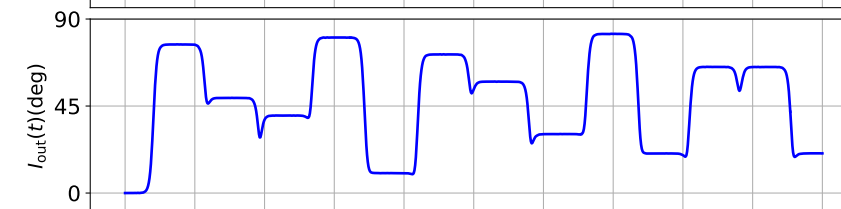
$i_{\text{mut}}(t)$



$I_{\text{in}}(t)$



$I_{\text{out}}(t)$



$t/P_{\text{out}}^{(0)}$

# Evolution of inclination for non-coplanar triples

$t = 0P_{\text{out}}^{(0)}$

$t = 0P_{\text{out}}^{(0)}$

$i_{\text{mut}} = 45^\circ$

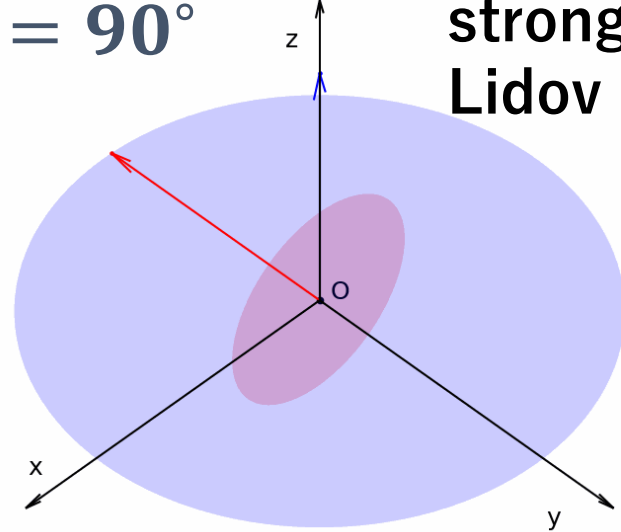
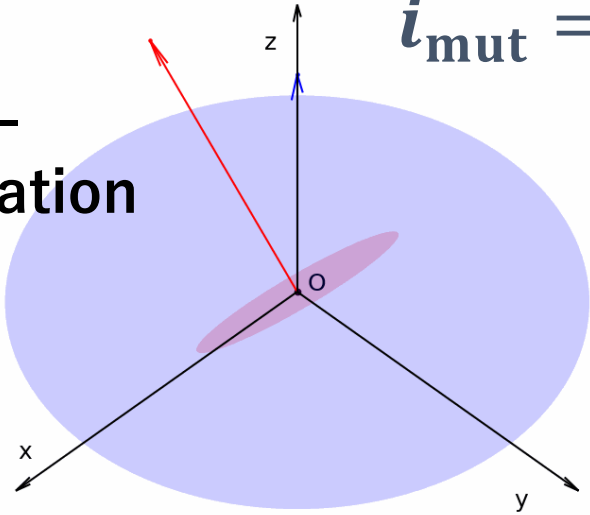
$i_{\text{mut}} = 90^\circ$

strong Kozai-Lidov oscillation

weak Kozai-Lidov oscillation

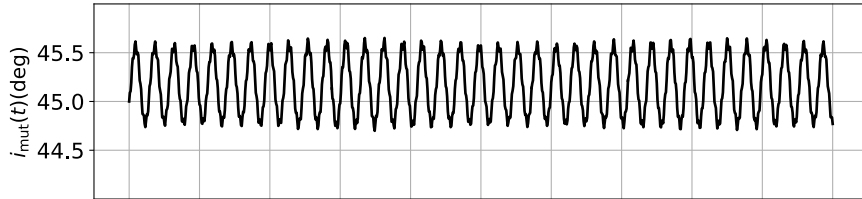
⇒ small-amplitude regular precession

⇒ large-amplitude sporadic precession

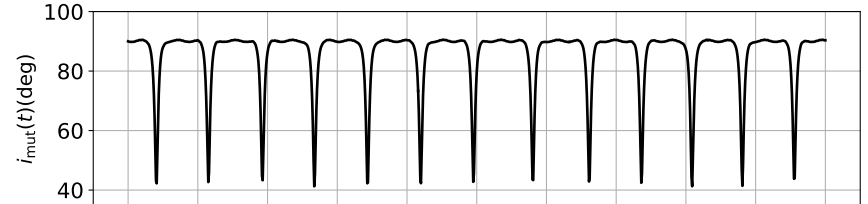


$$K_{\text{Kep}} = K_0 \sin I_{\text{out}}(t)$$

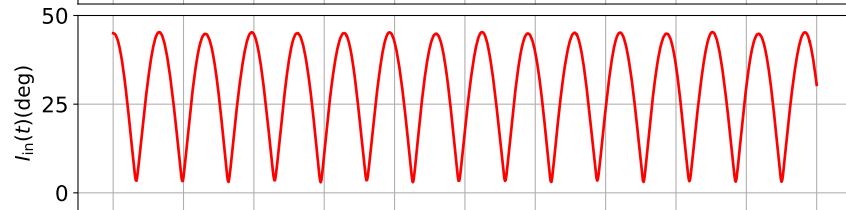
$i_{\text{mut}}(t)$



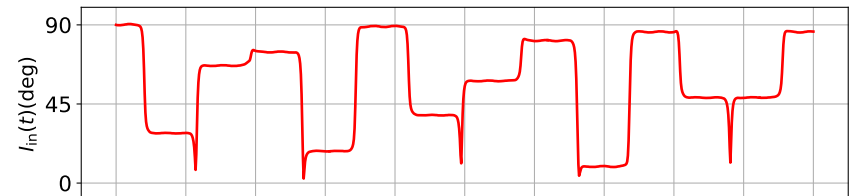
$i_{\text{mut}}(t)$



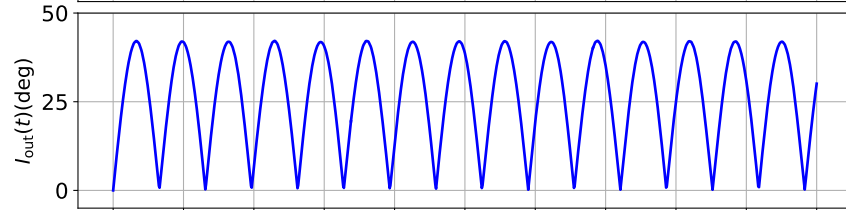
$I_{\text{in}}(t)$



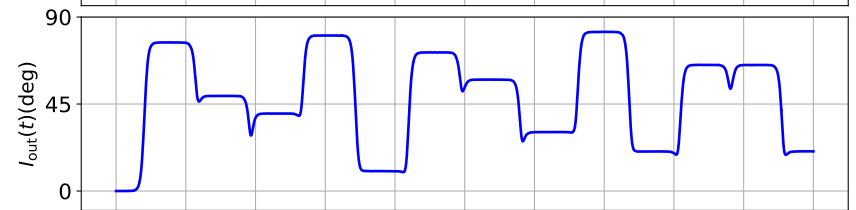
$I_{\text{in}}(t)$



$I_{\text{out}}(t)$



$I_{\text{out}}(t)$



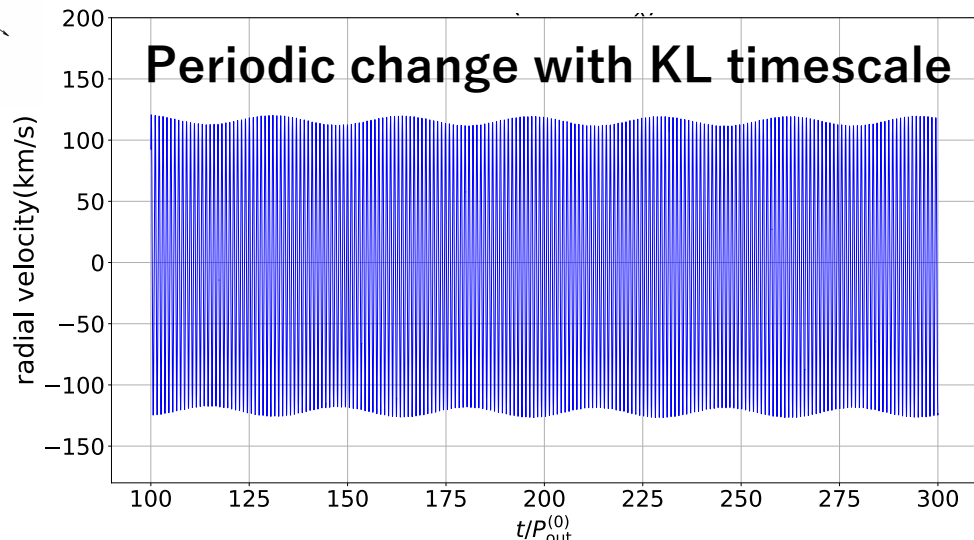
$t/P_{\text{out}}^{(0)}$

$t/P_{\text{out}}^{(0)}$

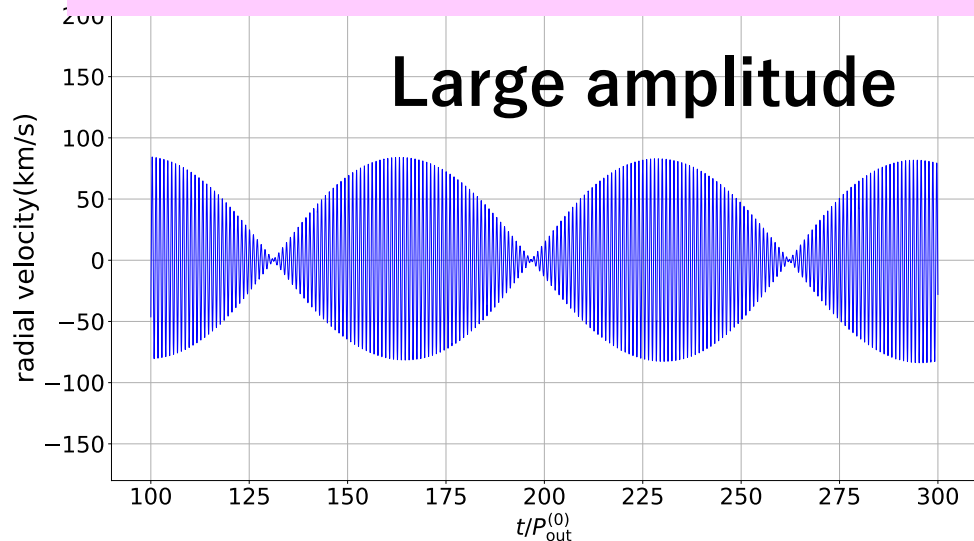
# Evolution of radial velocity for non-coplanar triples

$i_{\text{mut}} = 45^\circ$        $K_{\text{Kep}} = K_0 \sin I_{\text{out}}(t)$

**x-direction (near edge-on) total RV**

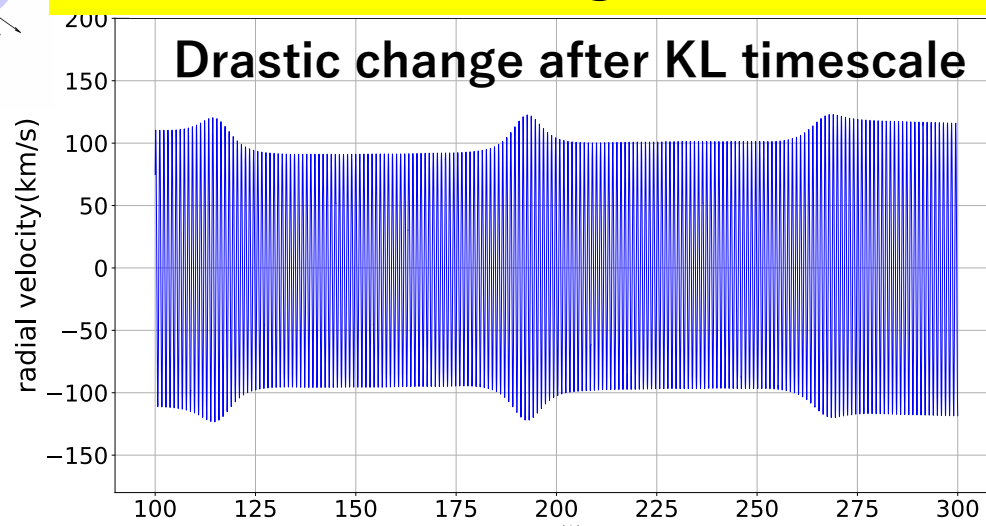


**z-direction (near face-on) total RV**

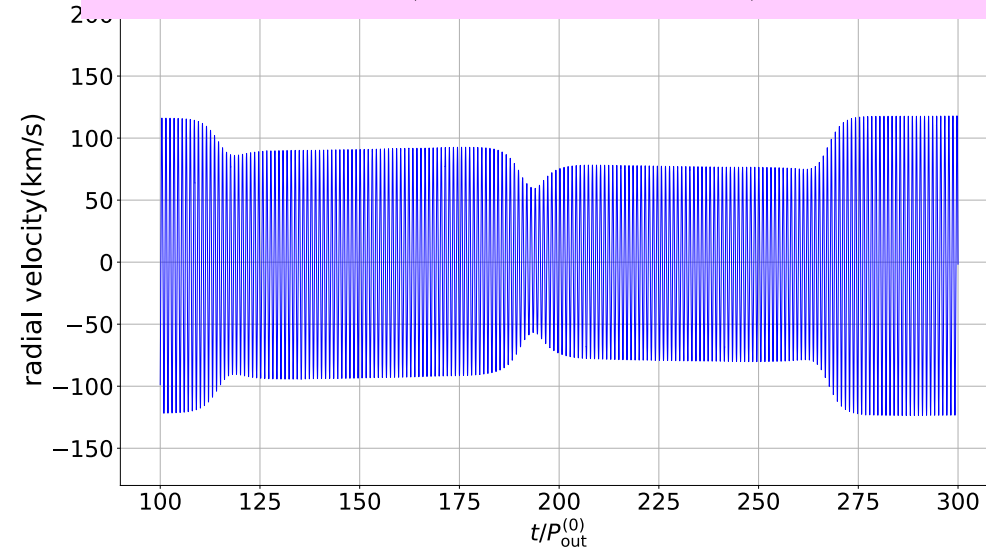


$i_{\text{mut}} = 90^\circ$

**x-direction (near edge-on) total RV**



**z-direction (near face-on) total RV**



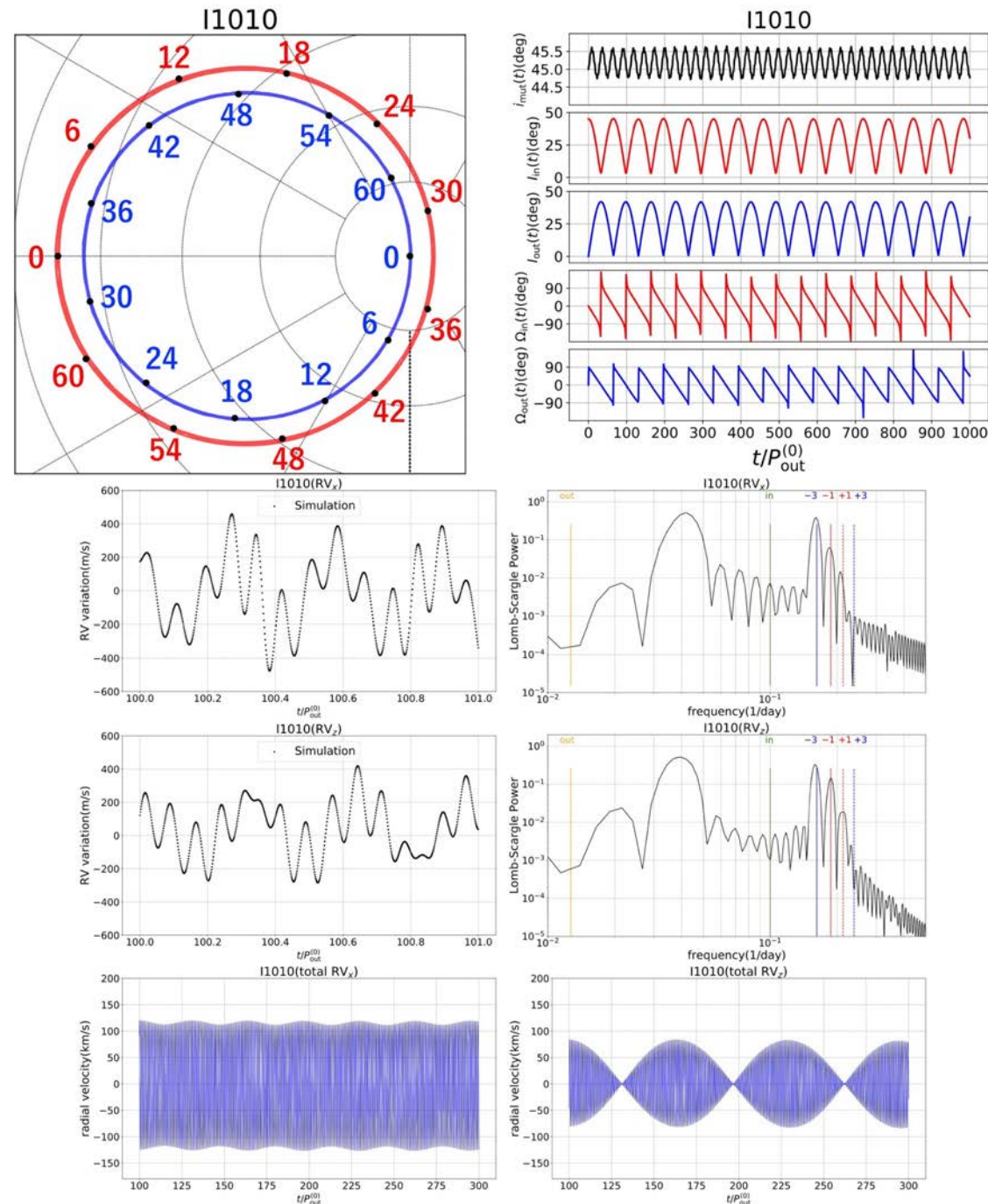
# Inclined equal-mass binary

## Precession timescale

$$\frac{P_\Omega}{P_{\text{out}}} \approx \frac{80.7}{\cos i_{\text{mut}}} \left( \frac{m_1 + m_2 + m_*}{23 M_\odot} \right) \left( \frac{m_*}{3 M_\odot} \right)^{-1} \times \left( \frac{P_{\text{out}}}{78.9 \text{ days}} \right) \left( \frac{P_{\text{in}}}{10.0 \text{ days}} \right)^{-1}$$

## Kozai-Lidov timescale

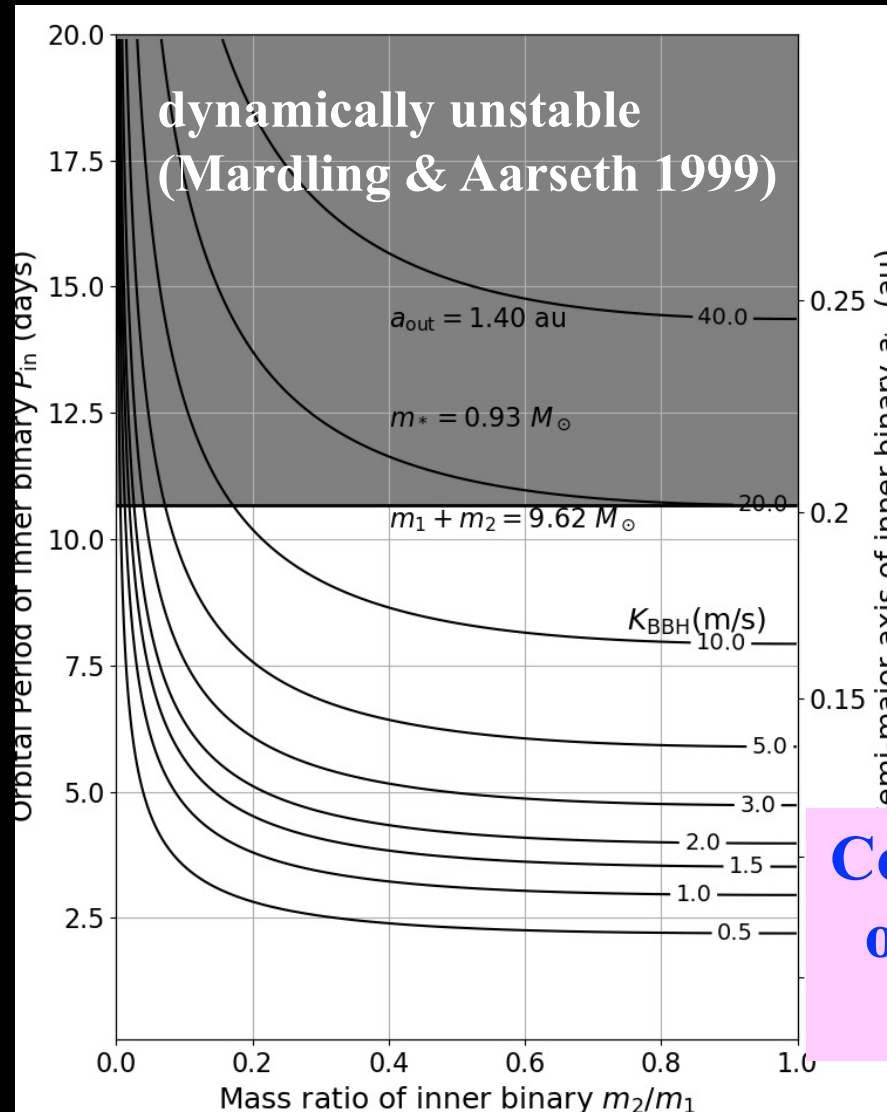
$$\frac{T_{\text{KL}}}{P_{\text{out}}} = \frac{m_1}{m_*} \left( \frac{P_{\text{out}}}{P_{\text{in}}} \right) (1 - e_{\text{out}}^2)^{3/2} \approx 26 \left( \frac{m_1}{10 M_\odot} \right) \left( \frac{m_*}{3 M_\odot} \right)^{-1} \times \left( \frac{P_{\text{out}}}{78.9 \text{ days}} \right) \left( \frac{P_{\text{in}}}{10 \text{ days}} \right)^{-1}$$



# Constraints on the binarity of Gaia BH1 and BH2 from short-term RD modulations

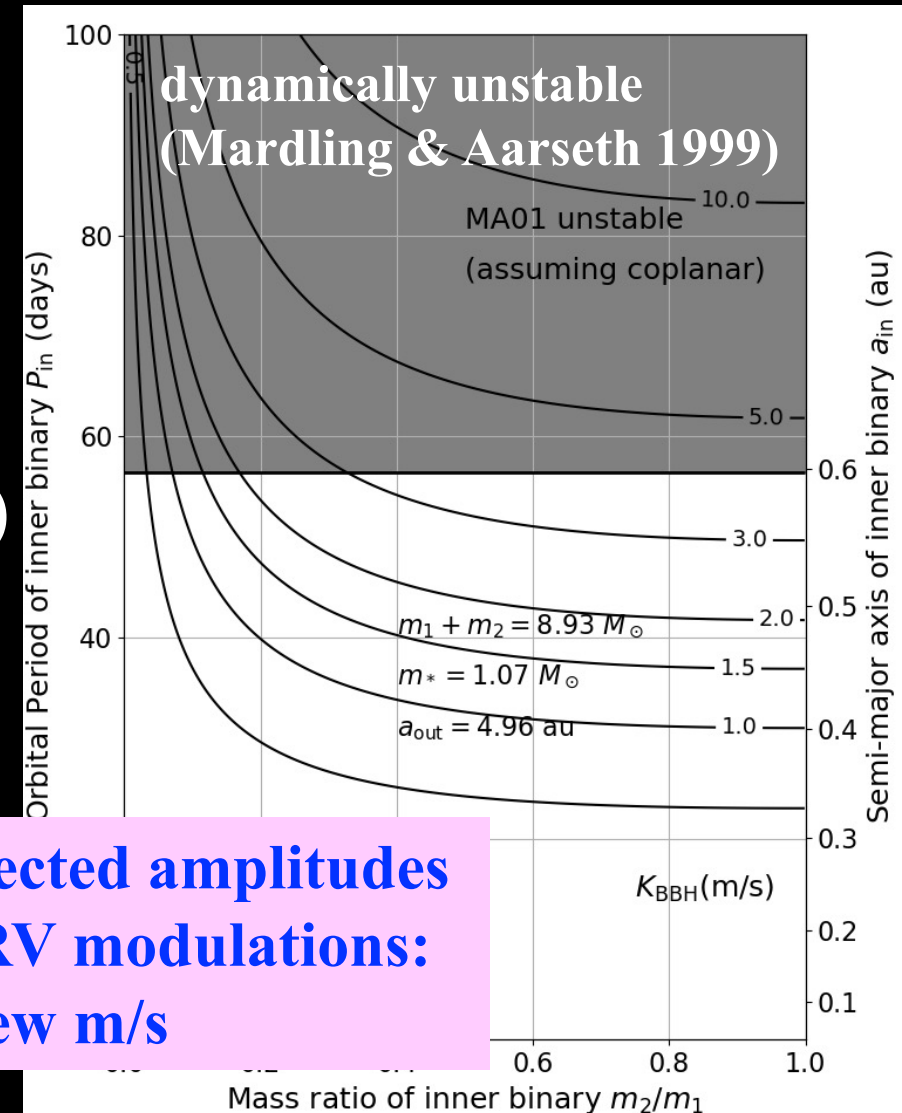
**Gaia BH1**

**$P_{in}$ (days)**



**Gaia BH2**

**$P_{in}$ (days)**



**Contours of expected amplitudes of short-term RV modulations:  
~ a few m/s**



# Conclusions: signature of inner binary black holes in triple systems

- Radial velocity (RD) monitoring of future star-black hole binary candidates may reveal inner binary black holes (instead of single black holes) in those systems
  - short-term RD variations [Hayashi, Wang + YS: ApJ 890\(2020\)112](#)
    - periodic modulations of  $O(1)$  percent of the Kepler orbital velocity amplitude with a half inner orbital period
  - long-term RD variations in inclined triples [Hayashi + YS: ApJ 897\(2020\)29](#)
    - the semi-amplitude of the Kepler orbital velocity modulated periodically by the precession (and the ZKL oscillation) of the inner and outer orbits over  $(10-100)(P_{\text{out}}/P_{\text{in}})P_{\text{out}}$