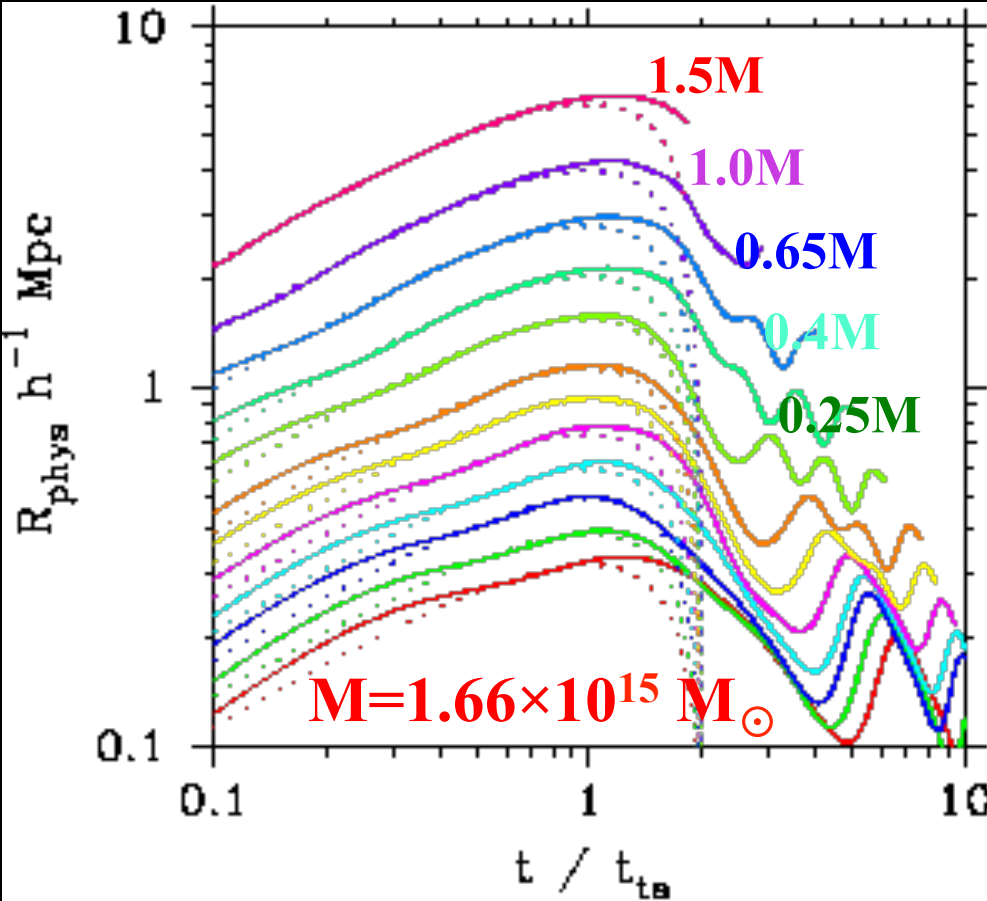


# Beyond the spherical dust collapse model



Evolution of radii of different mass shells in a simulated halo

**Yasushi Suto**  
*Department of Physics  
and  
RESCEU (Research Center  
for the early Universe)  
The University of Tokyo*

Kyoto workshop

“Vlasov-Poisson: towards numerical methods without particles”  
June 2, 2015@Yukawa Institute, Kyoto University

# Collaborators

- This talk is based on my collaboration with
  - **Daichi Suto** (Univ. of Tokyo)
  - Ken Osato (Univ. of Tokyo),
  - Tetsu Kitayama (Toho Univ.)
  - Shin Sasaki (Tokyo Metropolitan Univ.)
- Still one-going and preliminary work !

# Spherical dust collapse (SDC) model

- **The** most basic model of structure formation
- Everybody knows that it is just a simple approximation, but still widely used even in precision cosmology:
  - e.g., Dark matter halo abundance vs. cluster mass and temperature functions to determine cosmological parameters
- **Attempts for improvement**
  - Non-sphericity (e.g., Jing & Suto 2002)
  - inhomogeneities (e.g., Kawahara et al. 2007)
  - shell-crossing and velocity dispersions (this talk)

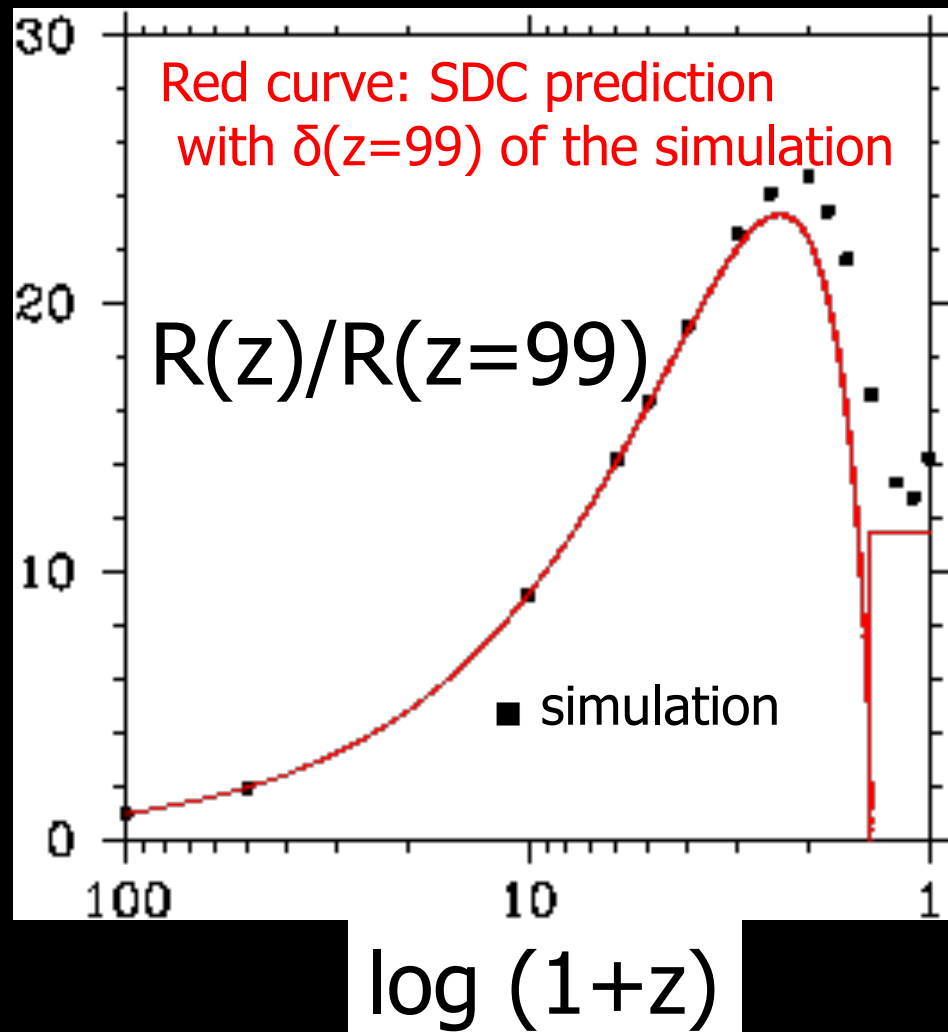
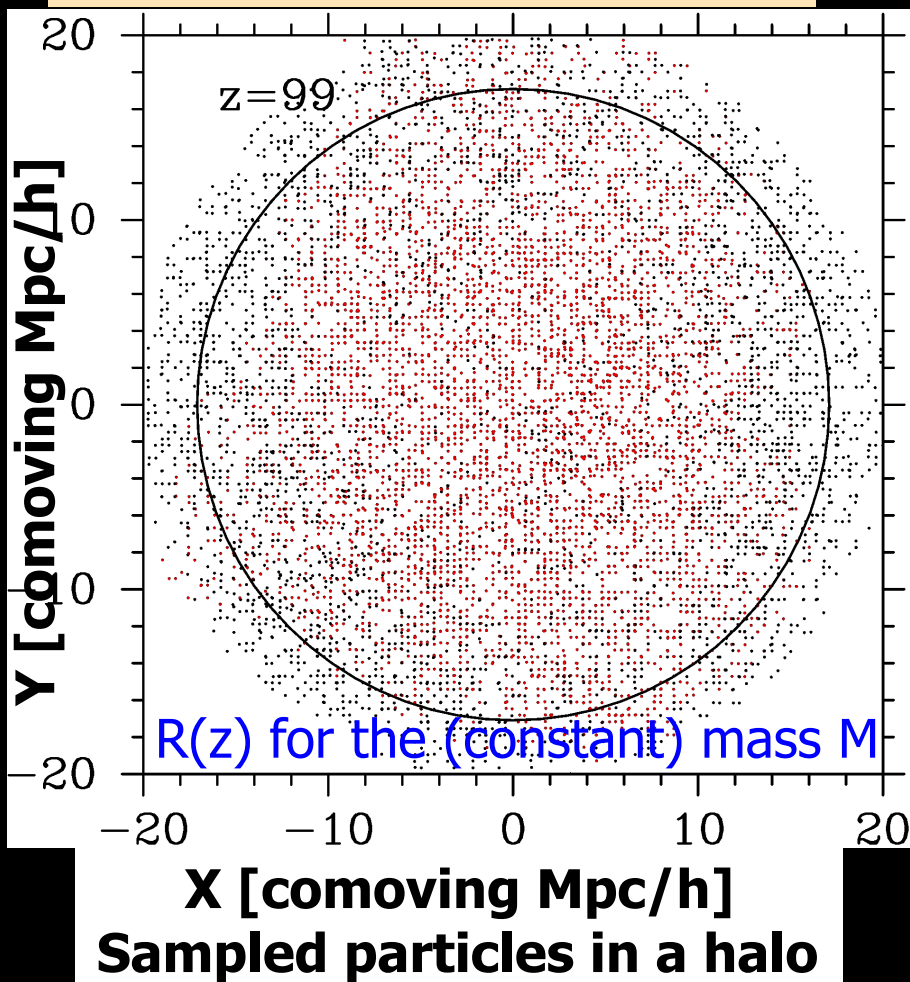
# Comparison of the SDC model predictions against N-body results

- **Dark matter only simulations with GADGET-2**
  - $\Lambda$ CDM with WMAP9 cosmological parameters
  - $N=1024^3$  in  $L=360 \text{ Mpc } h^{-1}$
  - $m=3.4 \times 10^9 M_{\odot}$
- **FOF halos identified at  $z=0$** 
  - compute **the spherical mass  $M$**  and radius  $R$  of spherical overdensity of  $\Delta=\rho/\rho_m=355.4$
  - Identifies the center-of-mass of the  $z=0$  FOF halo particles at  $z$ , and compute the radius  **$R(z)$**  enclosing the mass  $M$  at  $0 < z < z_{\text{initial}} = 99$

# The most massive halo with $M = 1.66 \times 10^{15} M_{\odot}$

Red: FOF particles at  $z=0$

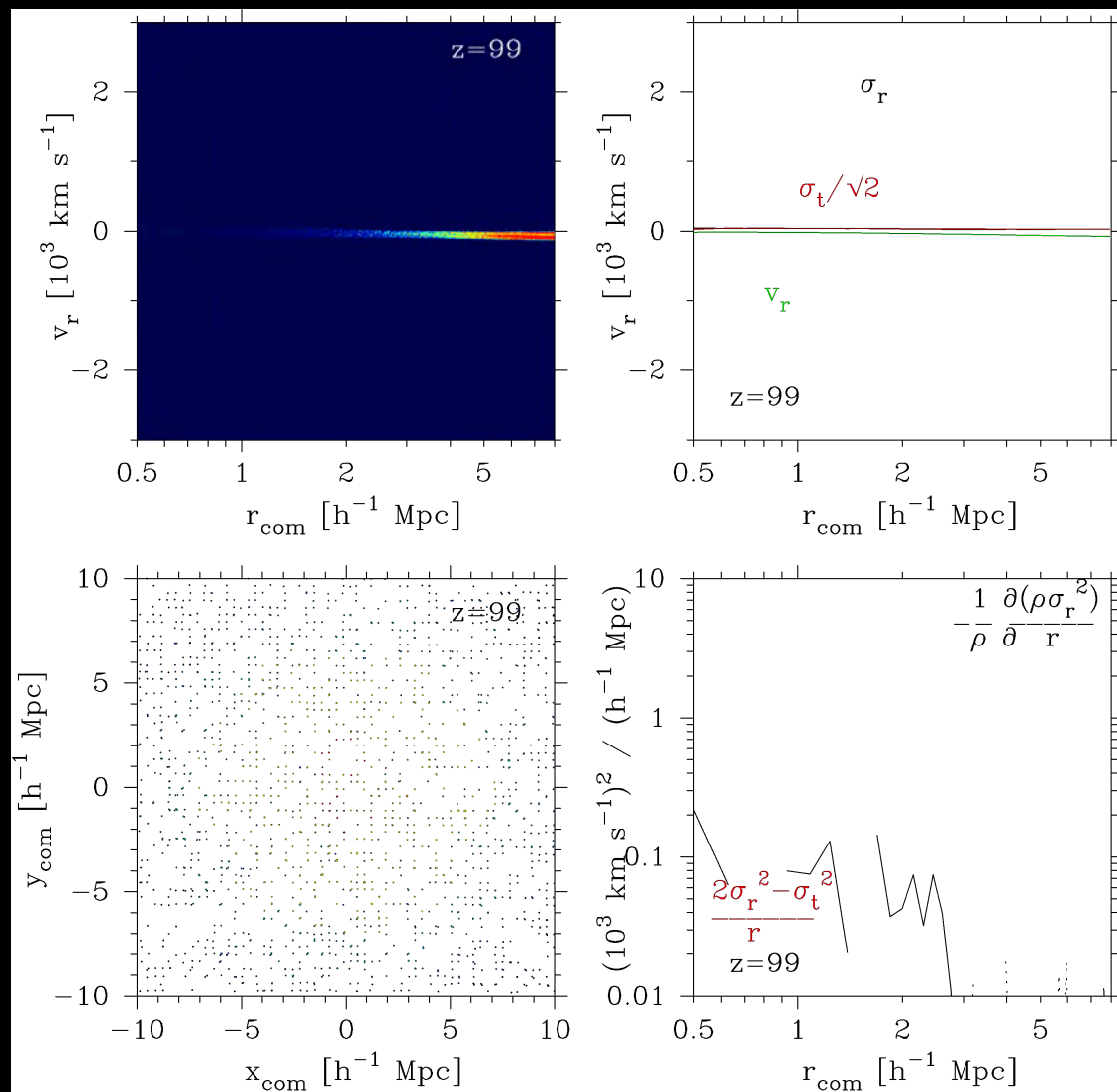
Black: non-FOF particles



# Generic trends from 100 simulated halos

- **Very good quantitative agreement until the turn-around epoch**
  - may be reasonable but not trivial at all, given the small-scale clumping, subhalo mergers inside, and/or the filamentary structure across the entire region
- **Systematic difference relative to SDC predictions after the turn-around epoch**
  - Delay of the turn-around epoch
  - Larger turn-around radius
  - Larger “virialized” radius

# Evolution of a halo ( $M=1.66 \times 10^{15} M_{\odot}$ ) in phase space (comoving coordinate)



# Effect of velocity dispersions

- Jeans equation for spherical collisionless system
  - radial velocity dispersion  $\sigma_r^2$
  - tangential velocity dispersion  $\sigma_t^2$

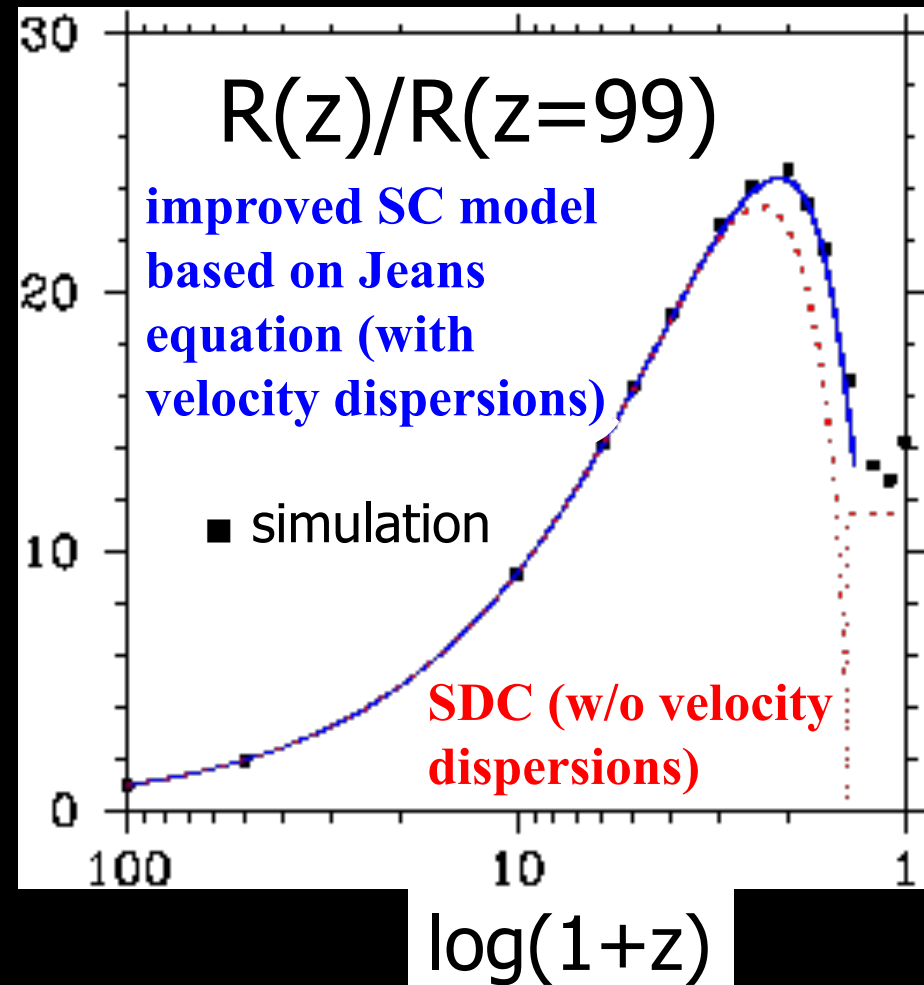
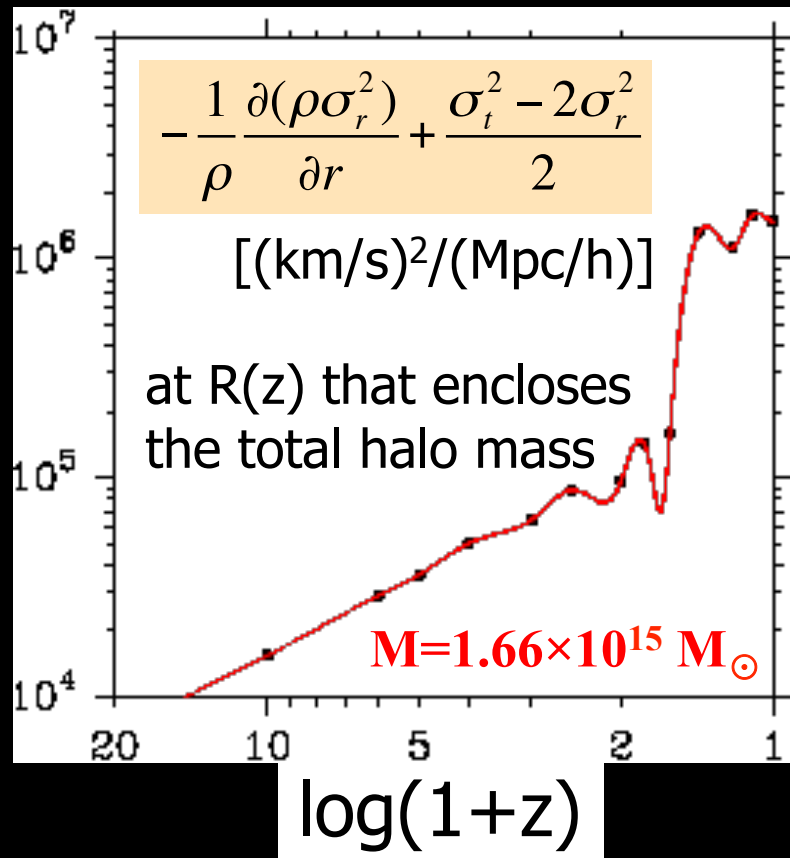
$$\frac{Dv_r}{Dt} = -\frac{1}{\rho} \frac{\partial(\rho\sigma_r^2)}{\partial r} + \frac{\sigma_t^2 - 2\sigma_r^2}{2} - \frac{GM}{r^2}$$

- SDC assumes an initially top-hat (homogeneous) sphere
  - neglects small-scale inhomogeneities, shell-crossing before turn-around, and thus no  $\sigma_r^2$  or  $\sigma_t^2$
- Larger  $t_{\text{turn-around}}$  and  $R_{\text{virial}}$  than predicted by SDC



# Improvement with velocity dispersions

- Evaluate the velocity dispersions from simulation data and solve the Jean equation
- Greatly improved !



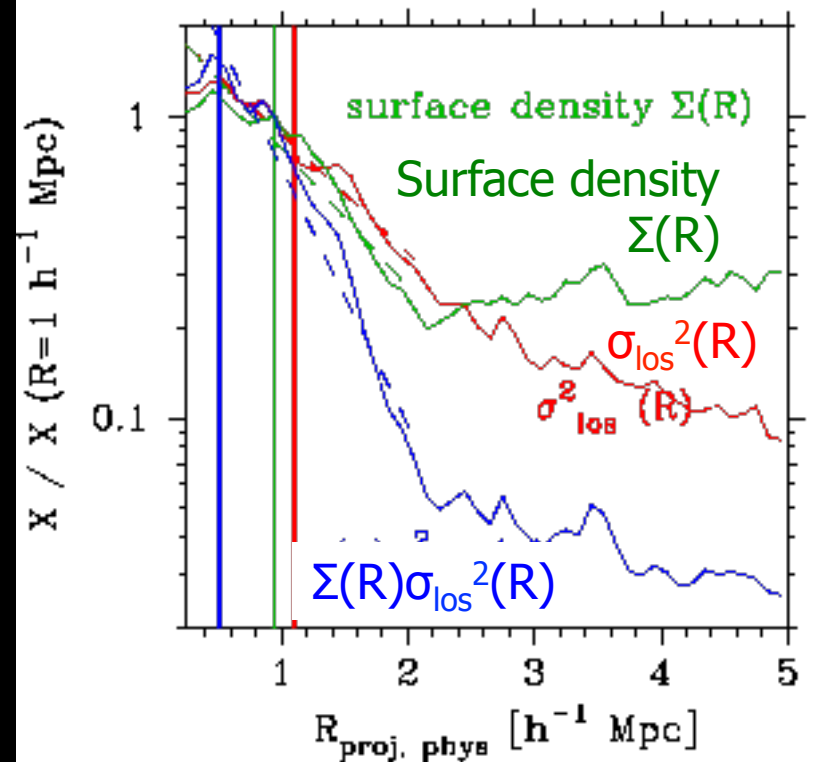
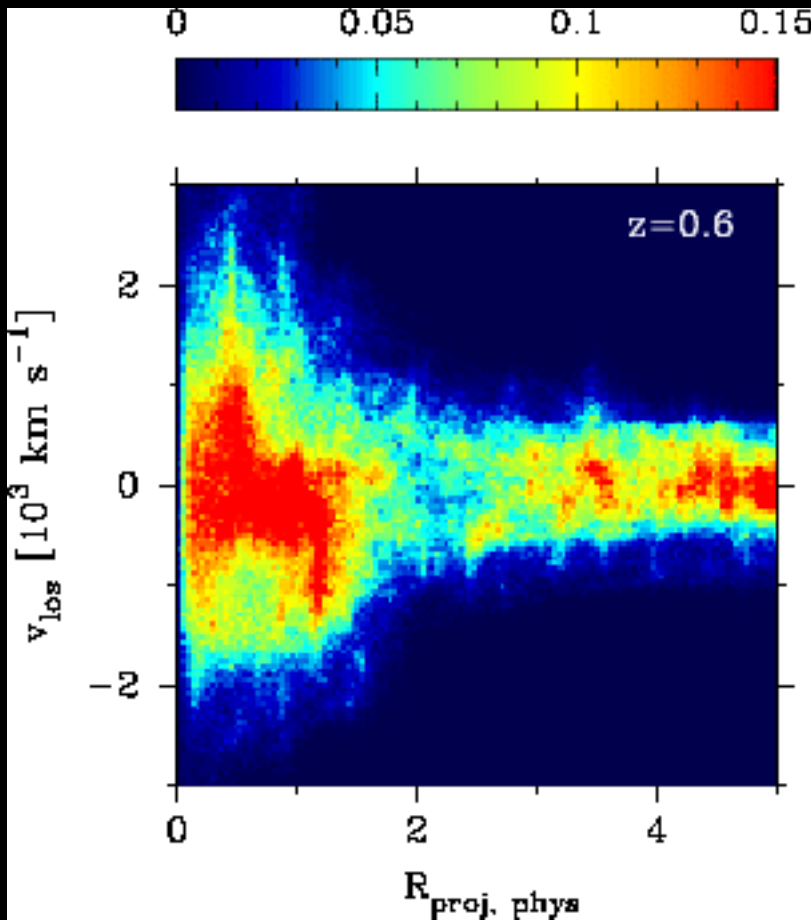
# Phase-space distribution in redshift space: line-of-sight velocity vs. projected radius

$M = 1.66 \times 10^{15} M_{\odot}$     $z = 0.6$

$$\Sigma(R) \propto \exp(-R / R_{\Sigma})$$

$$\sigma^2(R) \propto \exp(-R / R_{\sigma^2})$$

$$\Sigma(R)\sigma^2(R) \propto \exp(-R / R_{\Sigma\sigma^2})$$



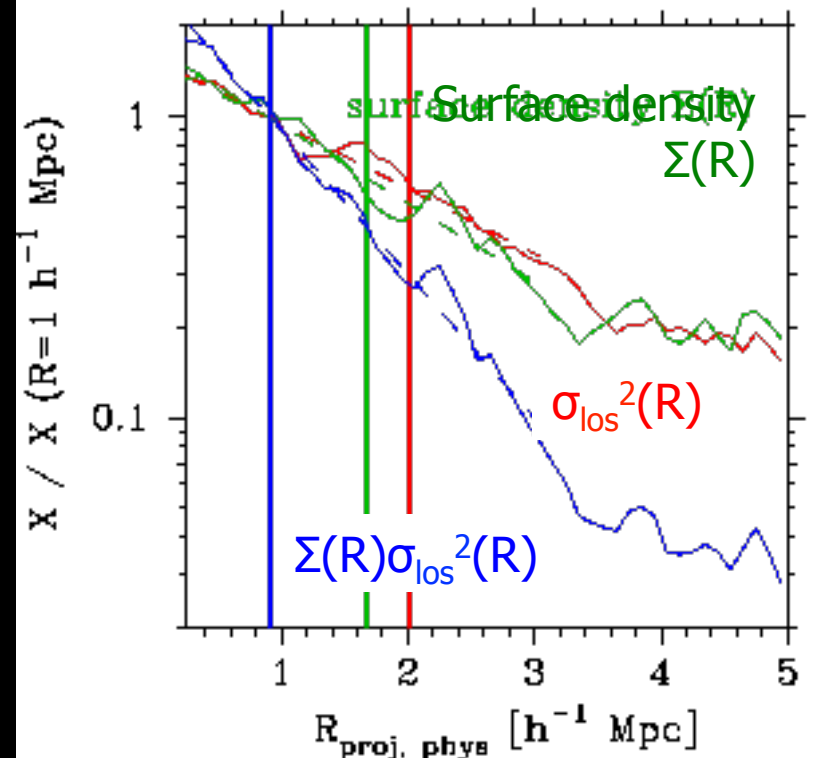
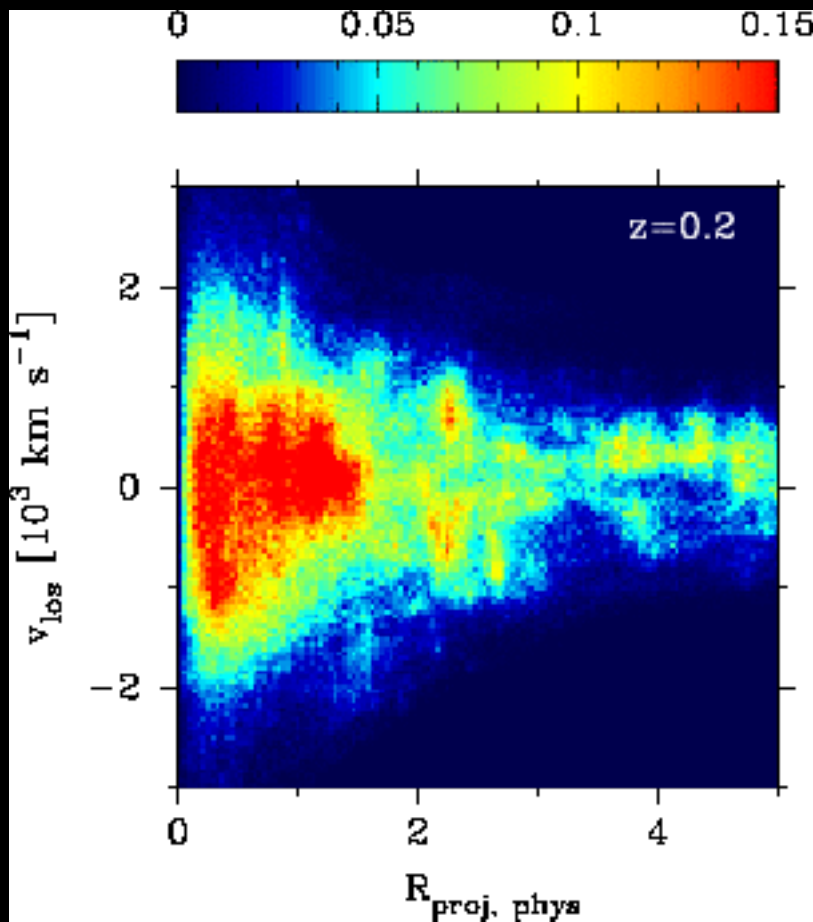
# Phase-space distribution in redshift space: line-of-sight velocity vs. projected radius

$M = 1.66 \times 10^{15} M_{\odot}$     $z = 0.2$

$$\Sigma(R) \propto \exp(-R / R_{\Sigma})$$

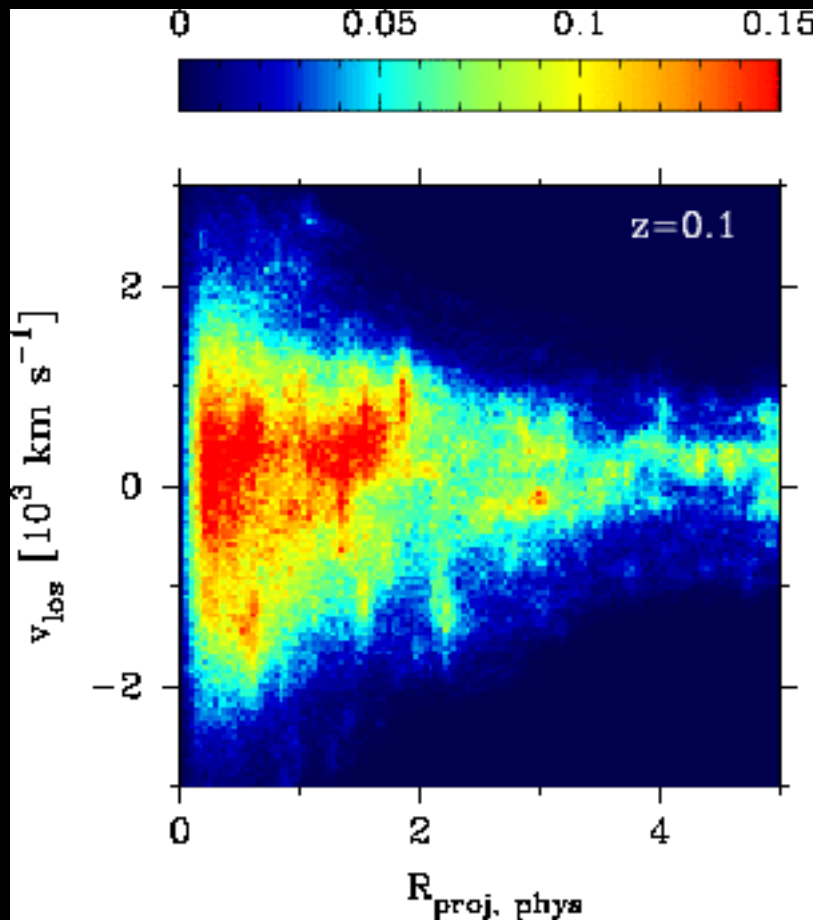
$$\sigma^2(R) \propto \exp(-R / R_{\sigma^2})$$

$$\Sigma(R)\sigma^2(R) \propto \exp(-R / R_{\Sigma\sigma^2})$$



# Phase-space distribution in redshift space: line-of-sight velocity vs. projected radius

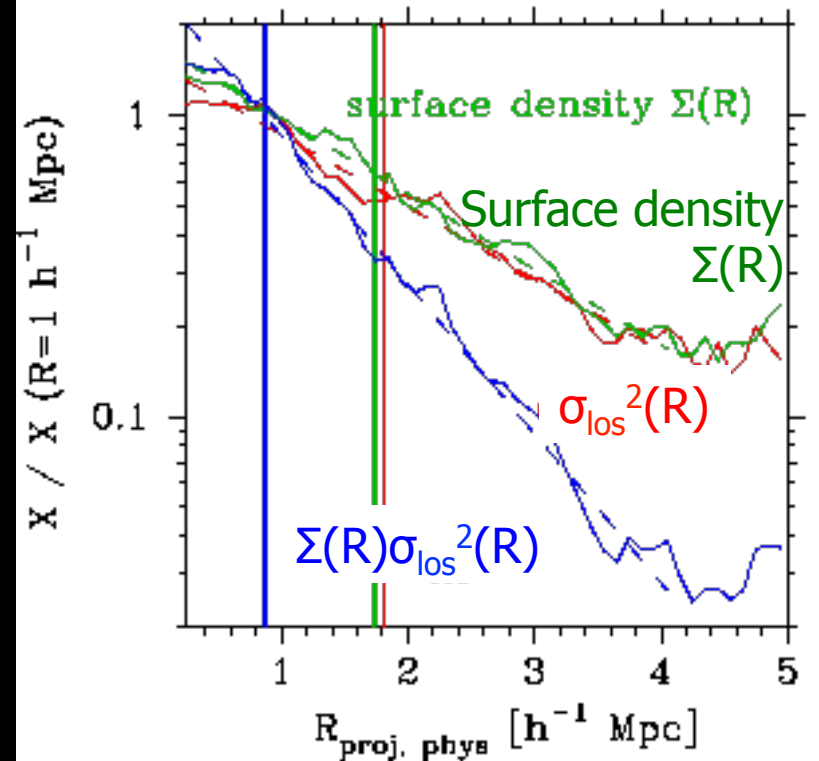
$M = 1.66 \times 10^{15} M_{\odot}$     $z = 0.1$



$$\Sigma(R) \propto \exp(-R / R_{\Sigma})$$

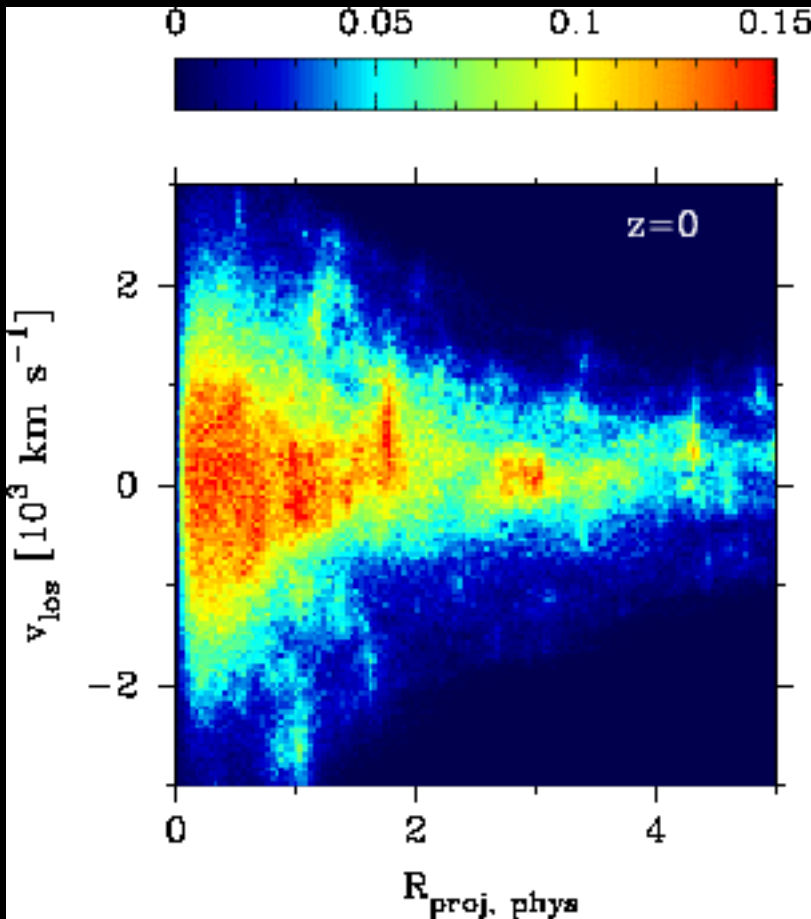
$$\sigma^2(R) \propto \exp(-R / R_{\sigma^2})$$

$$\Sigma(R)\sigma^2(R) \propto \exp(-R / R_{\Sigma\sigma^2})$$



# Phase-space distribution in redshift space: line-of-sight velocity vs. projected radius

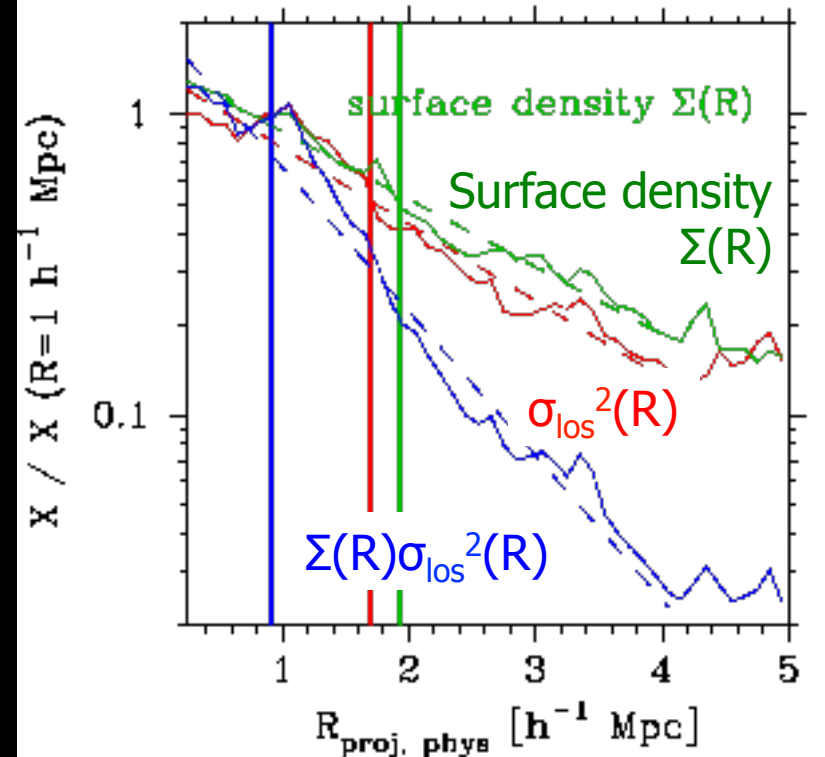
$M = 1.66 \times 10^{15} M_{\odot}$   $z=0$



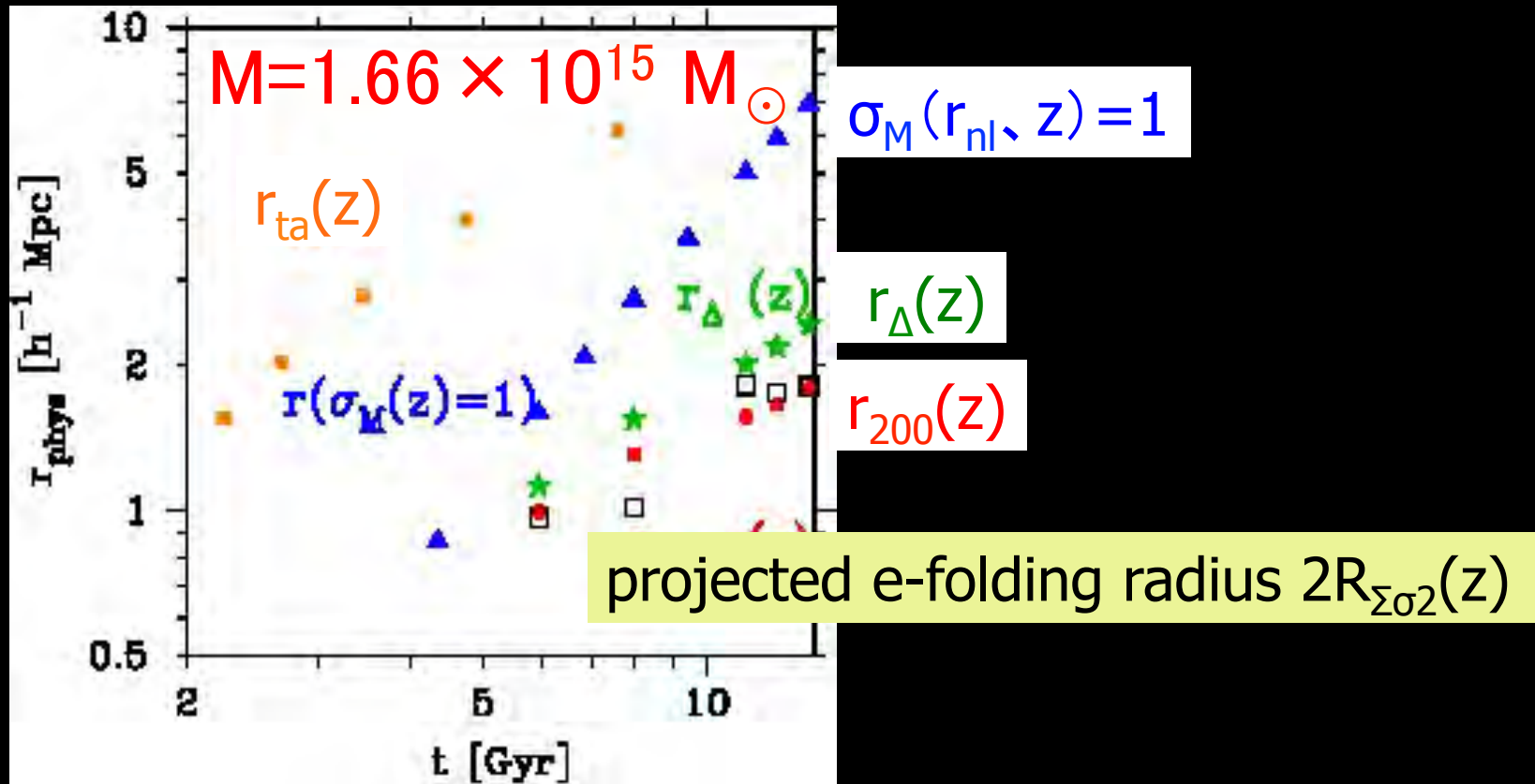
$$\Sigma(R) \propto \exp(-R / R_{\Sigma})$$

$$\sigma^2(R) \propto \exp(-R / R_{\sigma^2})$$

$$\Sigma(R)\sigma^2(R) \propto \exp(-R / R_{\Sigma\sigma^2})$$

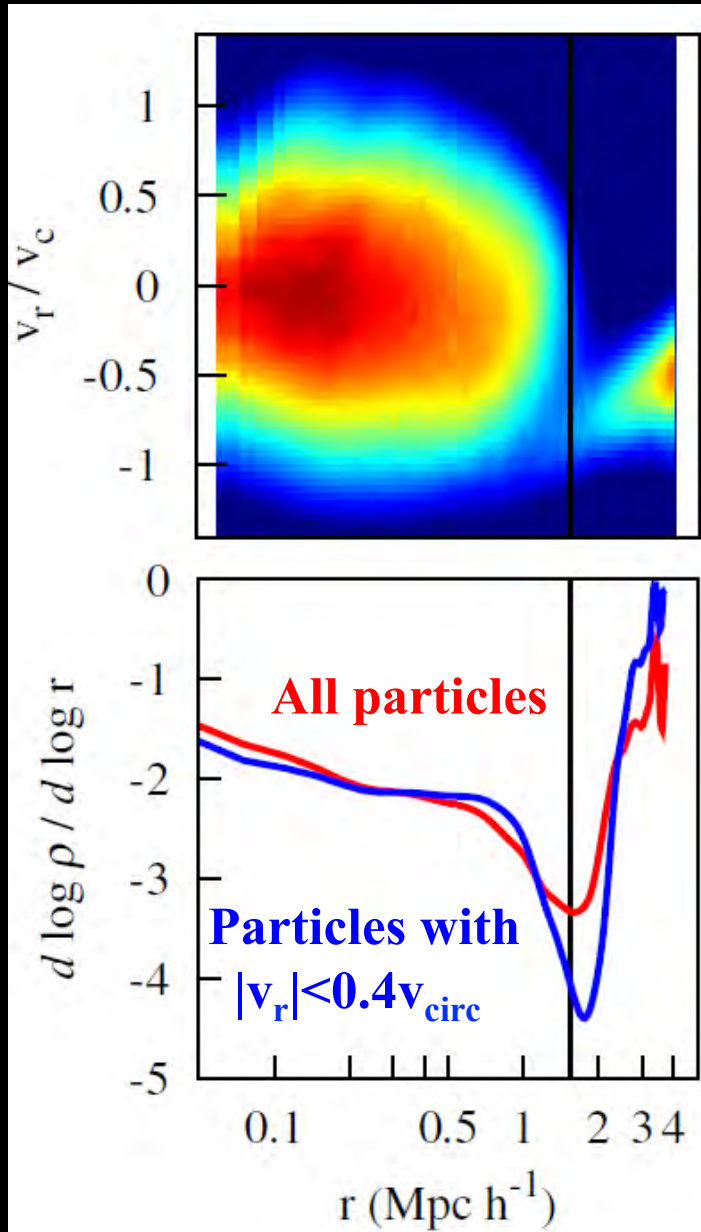


# Comparison among characteristic scales



- Projected e-folding scales  $2R_{\Sigma\sigma 2}(z) \sim R_{\Sigma}(z) \sim R_{\sigma 2}(z)$  are close to conventional “virial” radii ( $r_{200}$  or  $r_{\Delta}$ )
  - $r_{2nd \text{ apocenter}} \sim 0.367 r_{ta}$  and  $r_{3rd \text{ apocenter}} \sim 0.236 r_{ta}$  in Bertschinger’s solution (1985, ApJS, 58, 39)

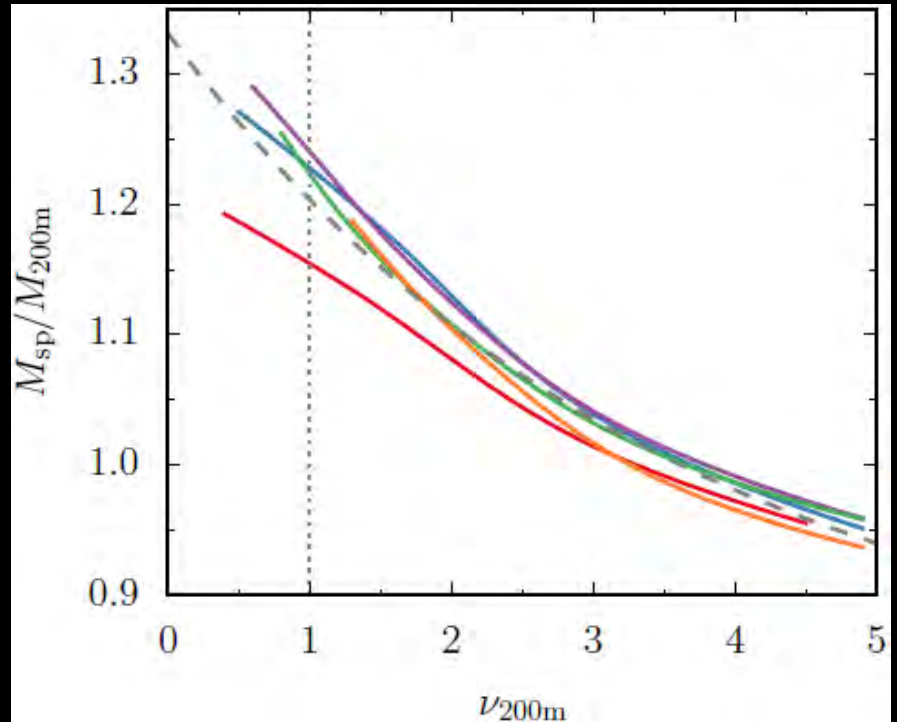
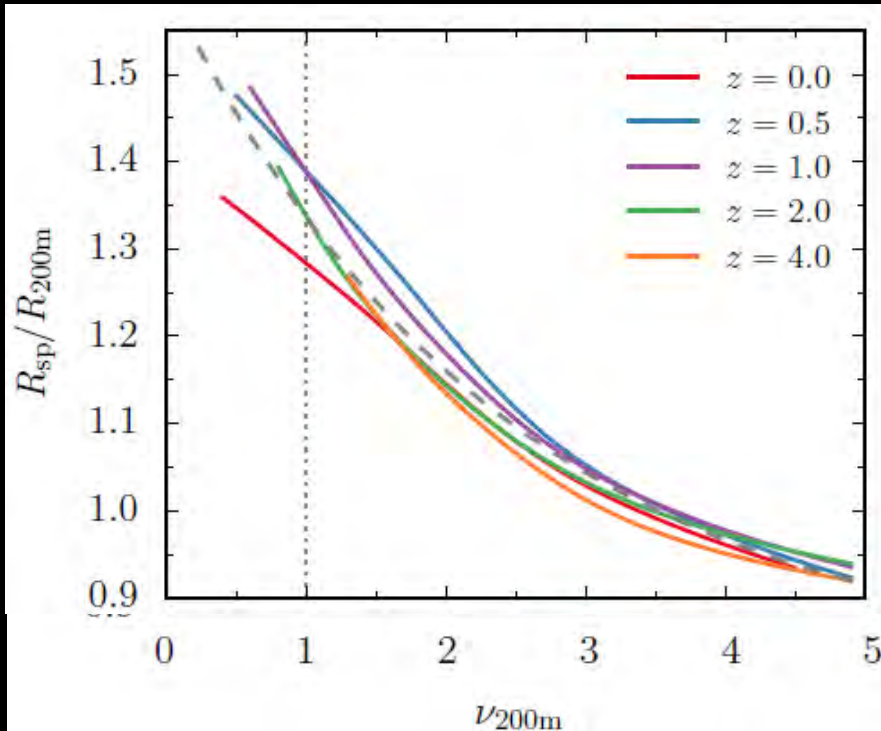
# Splashback in accreting dark halos



S. Adhikari, N. Dalal & R. T. Chamberlain, arXiv:1409.4482

- Physical definition of halo size = splashback radius ?
  - first apoapse after collapse
- Ensemble of halos from cosmological simulation
  - Clear signature in density profile of ensemble of halos
  - Not easy to determine the splashback radius from a single halo (at least observationally)

# The splashback radius as a physical halo boundary and the growth of halo mass

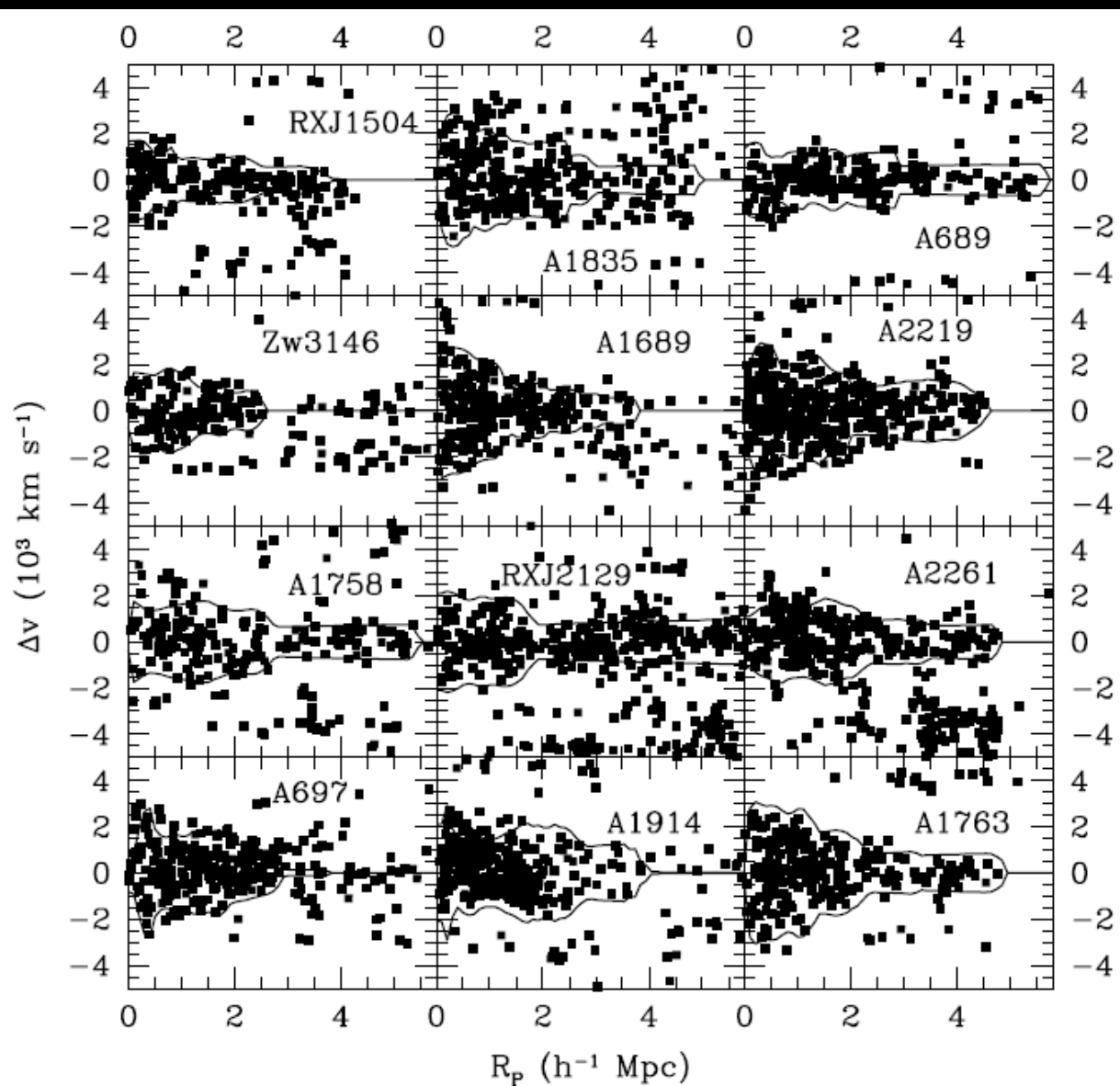


$$\frac{R_{\text{sp}}}{R_{200\text{m}}} = 0.81 \left( 1 + 0.97 e^{-v/2.44} \right)$$
$$\frac{M_{\text{sp}}}{M_{200\text{m}}} = 0.82 \left( 1 + 0.63 e^{-v/3.52} \right)$$

S. More, B. Diemer & A. V. Kravtsov,  
arXiv:1504.05591



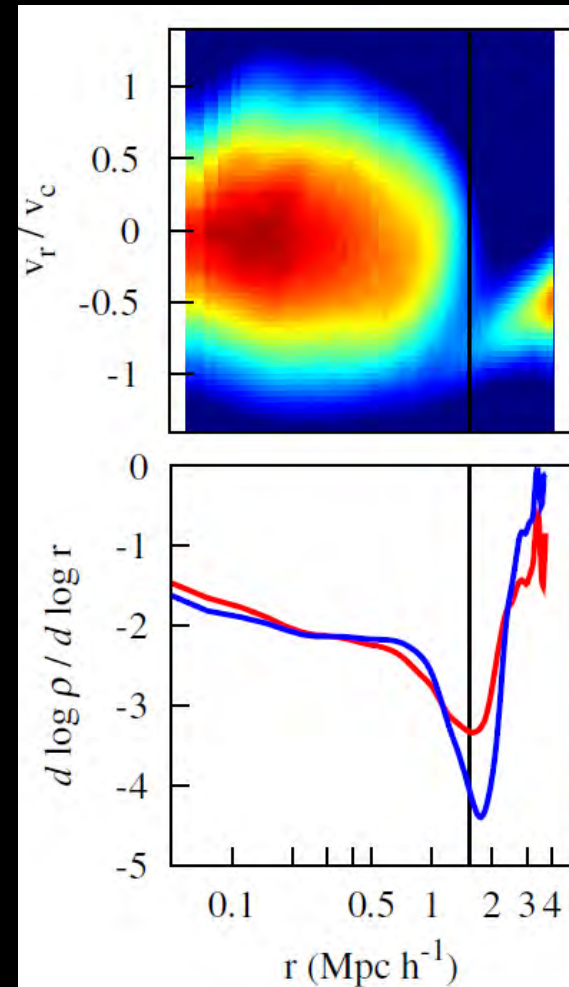
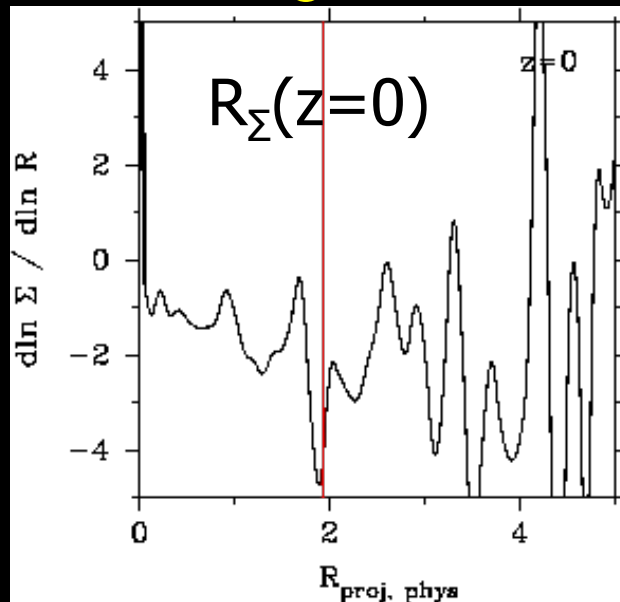
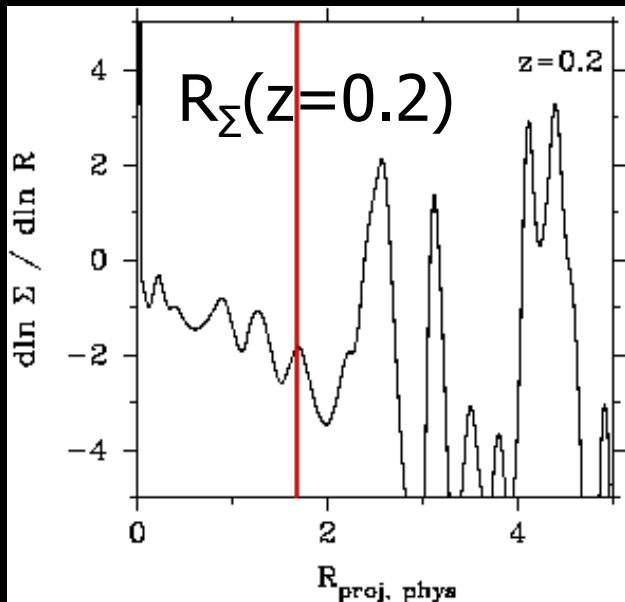
# Signature of the splashback radius ?



- Line-of-sight velocity of galaxies in X-ray selected clusters
- Rines et al. (2013)

# Relation to Splashback radius ?

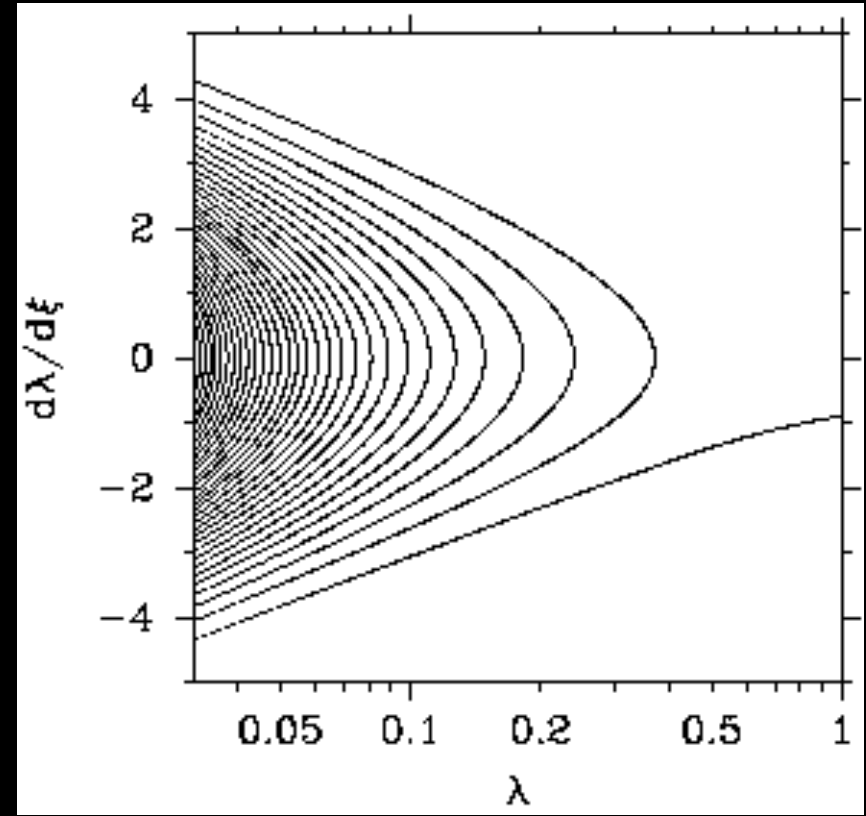
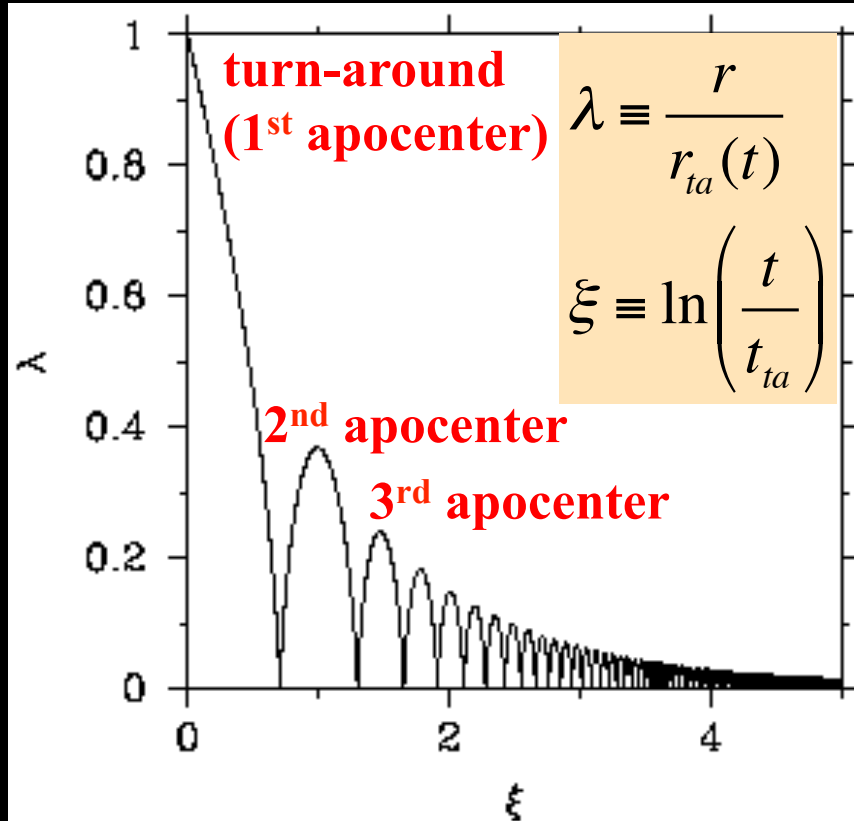
$$M = 1.66 \times 10^{15} M_{\odot}$$



- Qualitative agreement between our projected e-folding radius  $R$  and the splashback radius
- Our definition (e-folding scale in redshift space) is more relevant from an observational viewpoint

S. Adhikari, N. Dalal & R. T. Chamberlain, arXiv:1409.4482

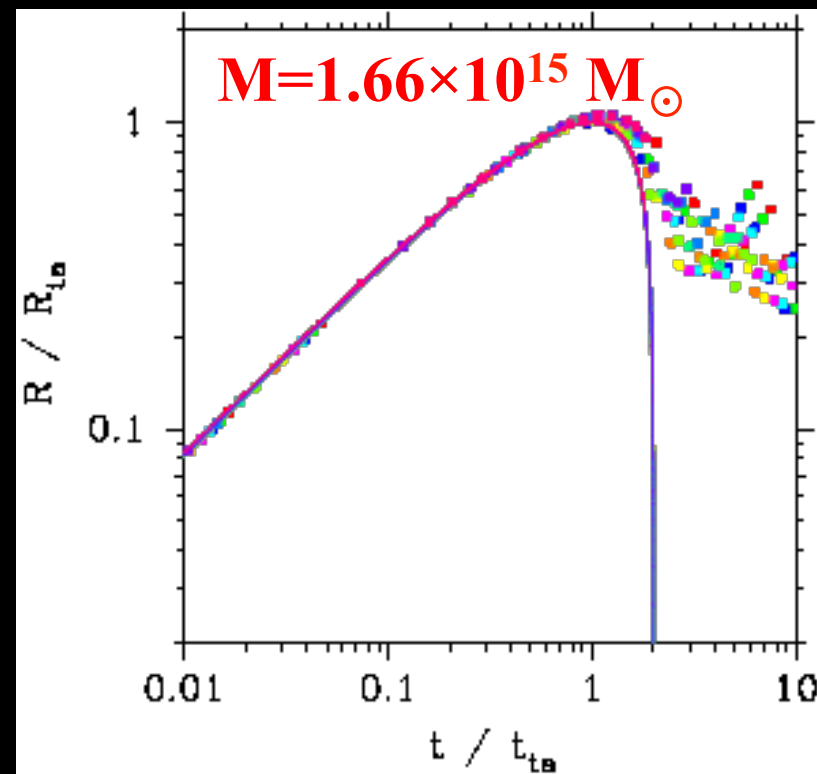
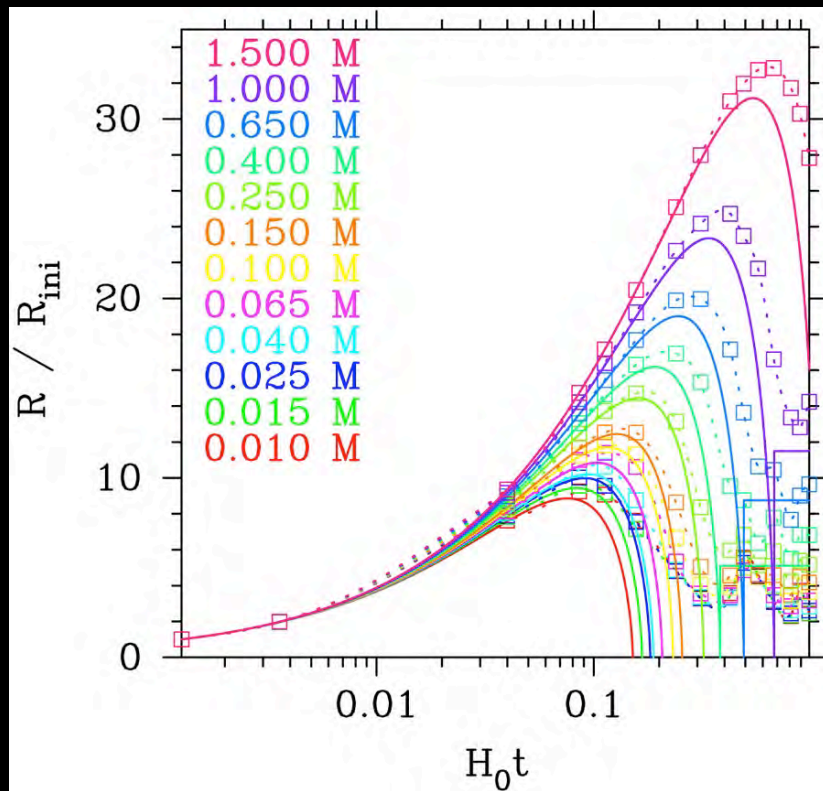
# Bertschinger's self-similar solution



- Self-similar shell crossing of collisionless spherical secondary infall

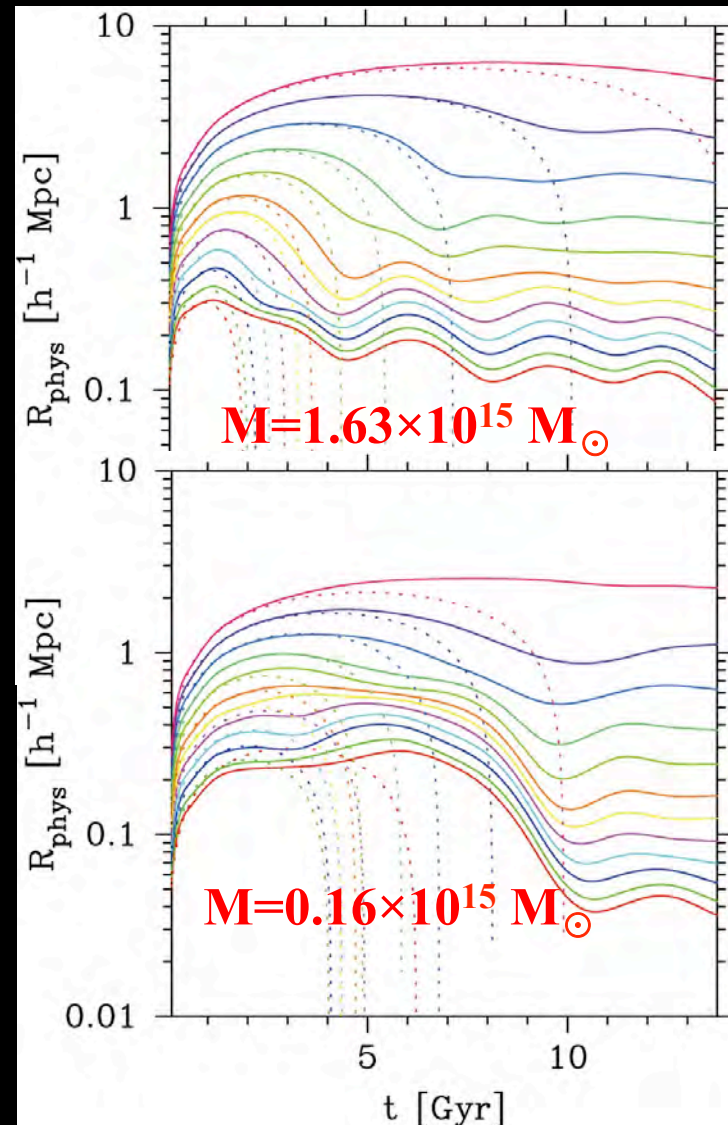
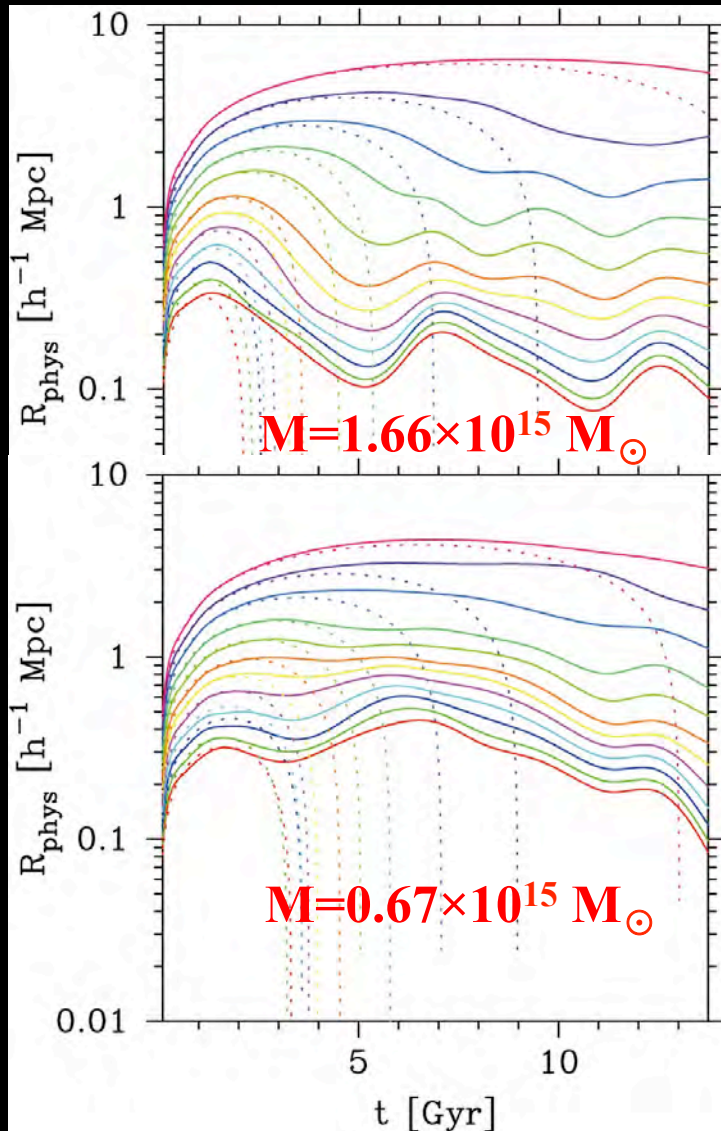
Bertschinger 1985, ApJS, 58, 39

# Scaled evolution of constant mass shells: $r_M(z)$ with $M(<r_M)=\text{const.}$ in the halo

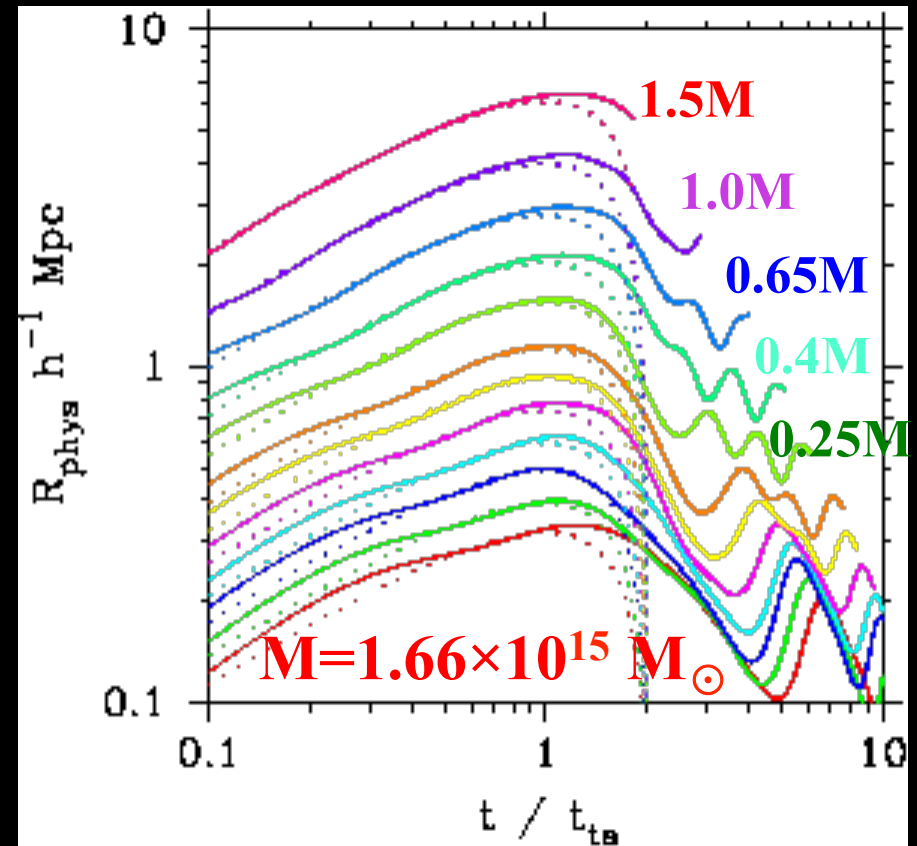
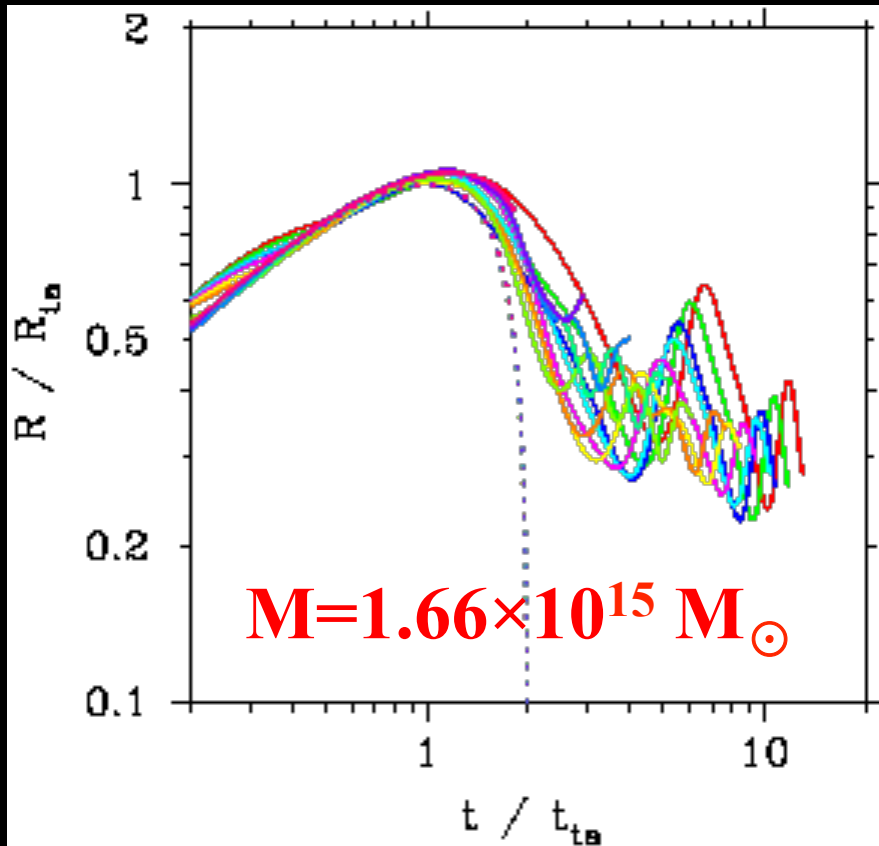


- usual assumption ( $r_{\text{vir}} = r_{\text{ta}}/2$ ) is not accurate
- Mixing of different mass shells is not complete

# Motion of constant mass shells for 4 different halos

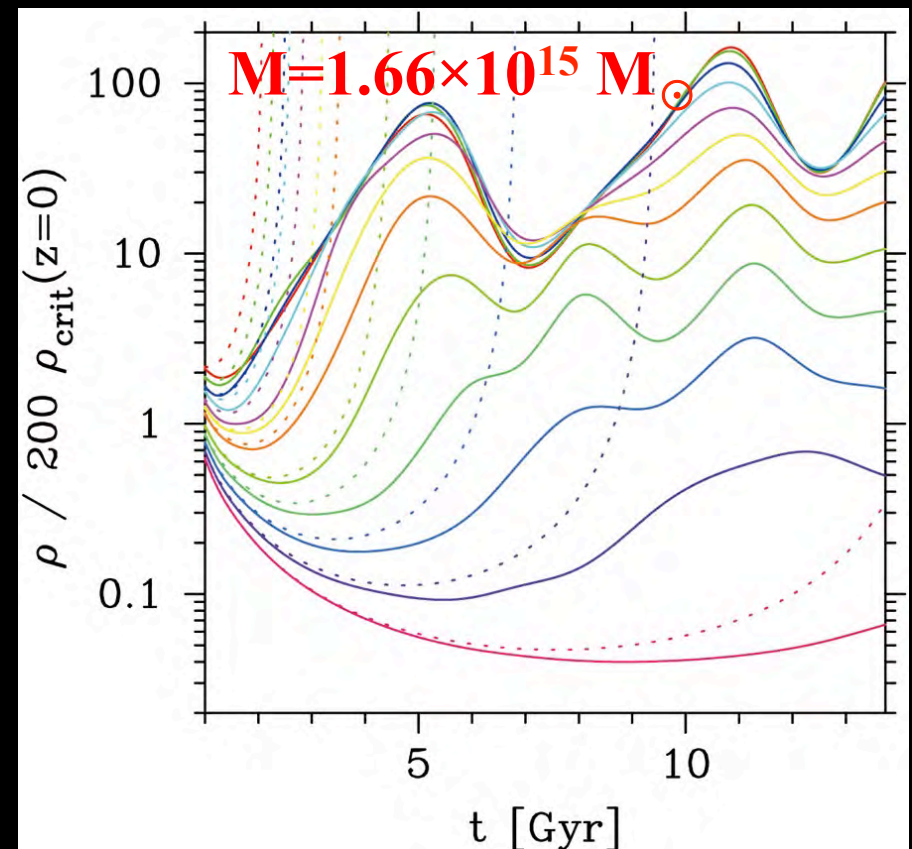
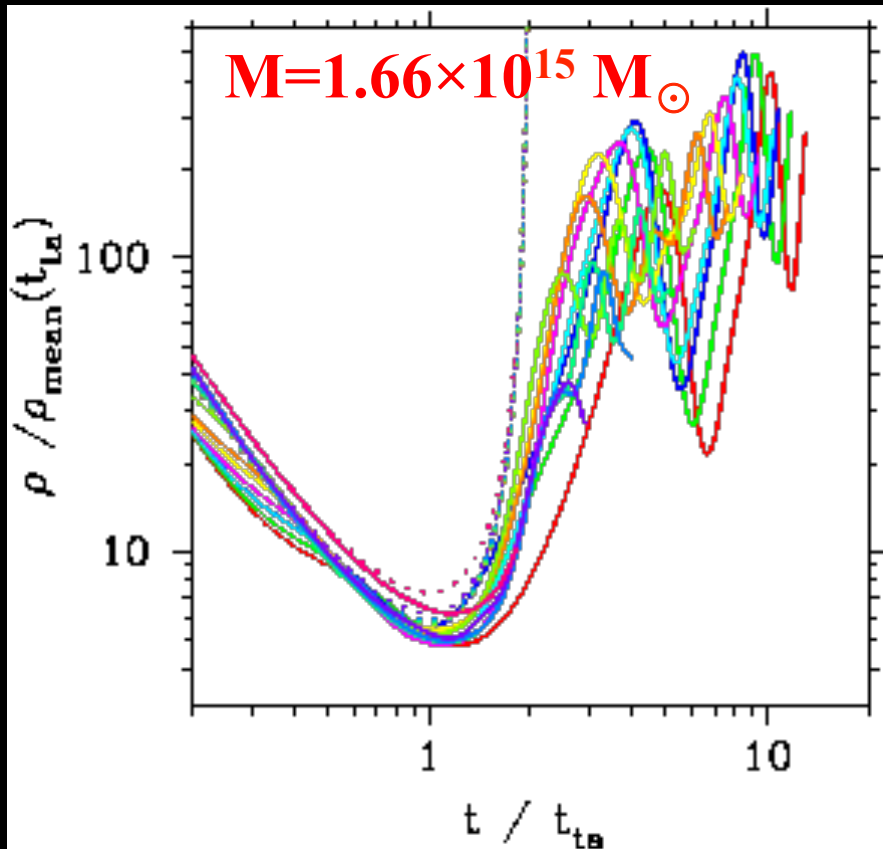


# “Virialized” mass shells change: not constant but oscillating



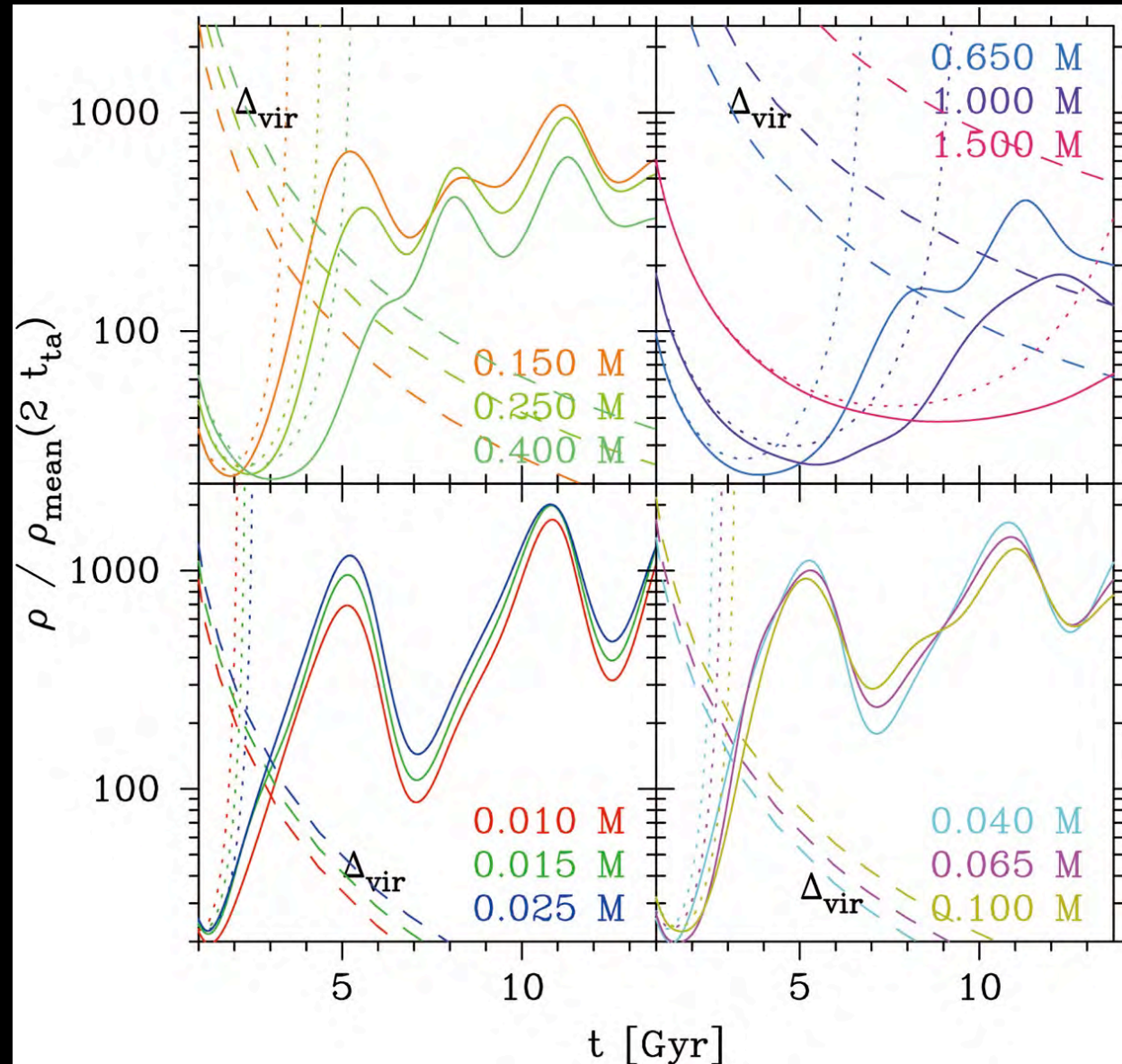
- Each mass shell continues to oscillate within the halo; halos are **not** static **but** dynamical

# “Virialized” mass shells change: not constant but oscillating



- Each mass shell continues to oscillate within the halo; halos are **not** static **but** dynamical

# “virialized” densities within different mass shells



$$M = 1.66 \times 10^{15} M_{\odot}$$

- Not constant
- Large coherent modulation



# Summary

- Spherical dust collapse model for dark matter halos is oversimplified
  - Better model is required for cluster cosmology, and indeed already overdue
- **We focus on the effect of velocity dispersions**
  - Delays the turn-around epoch, increases the virial radius
  - Confirmed by solving the Jeans equation
- Interesting scaling behavior of halo collapse
  - **“Virialized” halos are not static but dynamical, and indeed oscillating !**

# Future outlook

- Observational signature of the oscillating feature ?
  - affects the gas dynamics as well ?
  - Hydro-dynamical simulations to check ?
  - X-ray vs. weak lensing ?
- Toy model to describe the oscillation ?
  - Empirical scaling model ?
  - Modeling velocity dispersions ?