Enrico Trincherini
(SISSA, Trieste)

# *The Galileon*

Nicolis, Creminelli, ET, *Galilean Genesis: an alternative to inflation*, JCAP (2010)
Rattazzi, Nicolis, ET, *Energy's and amplitudes' positivity*, JHEP (2010)
Rattazzi, Nicolis, ET, *The Galileon as a local modification of gravity*, PRD (2009)

# Plan of the talk

**Part 1** Theory of a scalar field
invariant under field's gradient shift by a constant

A classical and a quantum peculiarity

$$\boxed{\partial_\mu \pi \to \partial_\mu \pi + b_\mu}$$

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**Part 2** It can modify gravity in the IR

Accelerating universe without a cosmological constant

It is not in conflict with experiments on smaller scales
(Vainshtein screening)

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Accelerating universe without a cosmological constant

It is not in conflict with experiments on smaller scales (Vainshtein screening)

**Part 3** It can violate the Null Energy Condition without instabilities

$$\rho + p \geq 0 \qquad \dot{\rho} = -3H(\rho + p)$$

Galilean Genesis

No Big Bang. Universe asymptotically Minkowski
then expand with increasing energy density till reheating and RD epoch

# General Relativity

$$(\partial h_c)^2 + \frac{h_c}{M_{\rm Pl}}(\partial h_c)^2 + \frac{h_c^2}{M_{\rm Pl}^2}(\partial h_c)^2 + \ldots + \frac{1}{M_{\rm Pl}^2}(\partial^2 h_c)^2 + \frac{h_c}{M_{\rm Pl}^3}(\partial^2 h_c)^2 + \ldots + \frac{1}{M_{\rm Pl}} h_c T$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{h^c_{\mu\nu}}{M_{\rm Pl}}$$

# General Relativity

$M_{\mathrm{Pl}}^2 \mathcal{R}$

$\mathcal{R}^2,\ \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}, \ldots$

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$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{h^c_{\mu\nu}}{M_{\mathrm{Pl}}}$$

$$\rho = M\delta^3(r) \qquad h_c \sim \frac{M}{M_{\mathrm{Pl}}}\frac{1}{r}$$


Non-linearities become important at a scale $r_S$ where $\frac{h_c}{M_{\mathrm{Pl}}} \sim 1 \Rightarrow r_S = \frac{M}{M_{\mathrm{Pl}}^2}$

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All the other terms are suppressed by extra powers of $\dfrac{\partial}{M_{Pl}}\sim\dfrac{1}{r\,M_{\rm Pl}}\ll 1$

We can compute classical non-linearities without knowing the UV compl.

# Scalar theory

$$(\partial\pi_{\rm c})^2 + \frac{1}{\Lambda^2}(\partial\pi_{\rm c})^3 + \frac{1}{\Lambda^4}(\partial\pi_{\rm c})^4 + \ldots + \frac{1}{M_{\rm Pl}}\pi_{\rm c}T$$

Shift symmetry

$$\pi \to \pi + c$$

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Non-linearities become important at a scale $r_V$ where $\frac{\partial\pi_c}{\Lambda^2} \sim 1 \Rightarrow r_V \sim \left(\frac{M}{M_{\rm Pl}\Lambda^2}\right)^{\frac{1}{2}}$

An infinite number of terms with unknown coefficients contributes at the same scale

## Scalar theory with higher derivatives

Usually they describe new pathological ghost-like degrees of freedom

$$-(\partial\phi)^2 + \frac{1}{M^2}(\Box\phi)^2 \rightarrow -(\partial\phi)^2 + (\partial\chi)^2 + M^2\chi^2$$

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Is there a HD lagrangian that gives 2 derivatives EOM?

$$\frac{\delta\mathcal{L}_\pi}{\delta\pi} = F(\partial_\mu\partial_\nu\pi)$$ Avoids new ghost-like DOF

$$\boxed{\pi(x) \to \pi(x) + c + b_\mu x^\mu}$$ The Galileon

Nicolis, Rattazzi, ET 08

$$\mathcal{L}^{(n)} \sim \partial^{2n-2}\pi^n$$

$$\Big(x(t) \to x(t) + x_0 + v_0 t\Big)$$

There are D+1 operators in D dimensions

$$\mathcal{L}^{(1)} = \pi \qquad\qquad \mathcal{L}^{(2)} = (\partial\pi)^2$$

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$$\mathcal{L}^{(1)} = \pi \qquad \mathcal{L}^{(2)} = (\partial\pi)^2$$

$$\mathcal{L}^{(3)} = (\partial\pi)^2\Box\pi \qquad \mathcal{L}^{(4)} = (\partial\pi)^2[(\Box\pi)^2 - \partial_\mu\partial_\nu\pi\partial^\mu\partial^\nu\pi]$$

$$\mathcal{L}^{(5)} = (\partial\pi)^2[(\Box\pi)^3 - 3\Box\pi\partial_\mu\partial_\nu\pi\partial^\mu\partial^\nu\pi + 2\partial_\mu\partial_\nu\pi\partial^\nu\partial^\alpha\pi\partial_\alpha\partial^\mu\pi]$$

# The Galileon

$$\pi(x) \to \pi(x) + c + b_\mu x^\mu$$

## Non-renormalization theorem

Luty, Porrati, Rattazzi 03

Loops of quantum fields with interactions $\mathcal{L}^{(3)}, \mathcal{L}^{(4)}, \mathcal{L}^{(5)}$ generate terms involving at least 2 derivatives on the external legs.

In particular galilean terms are not renormalized

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Classical non-linearities important $\quad \dfrac{\partial^2\pi_c}{\Lambda^3} \sim 1 \Rightarrow r_V \sim \left(\dfrac{M}{M_{\rm Pl}\Lambda^3}\right)^{\frac{1}{3}}$

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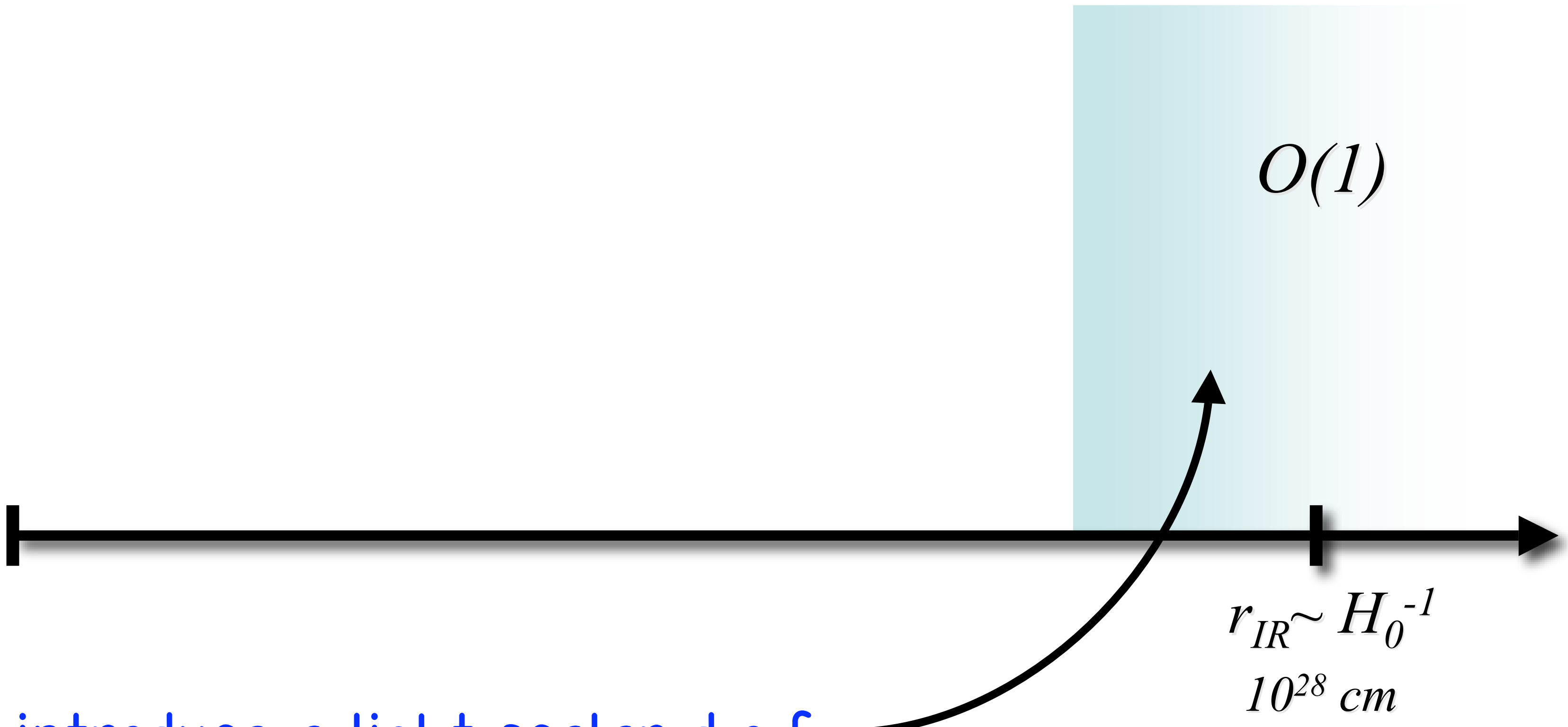
Part 2

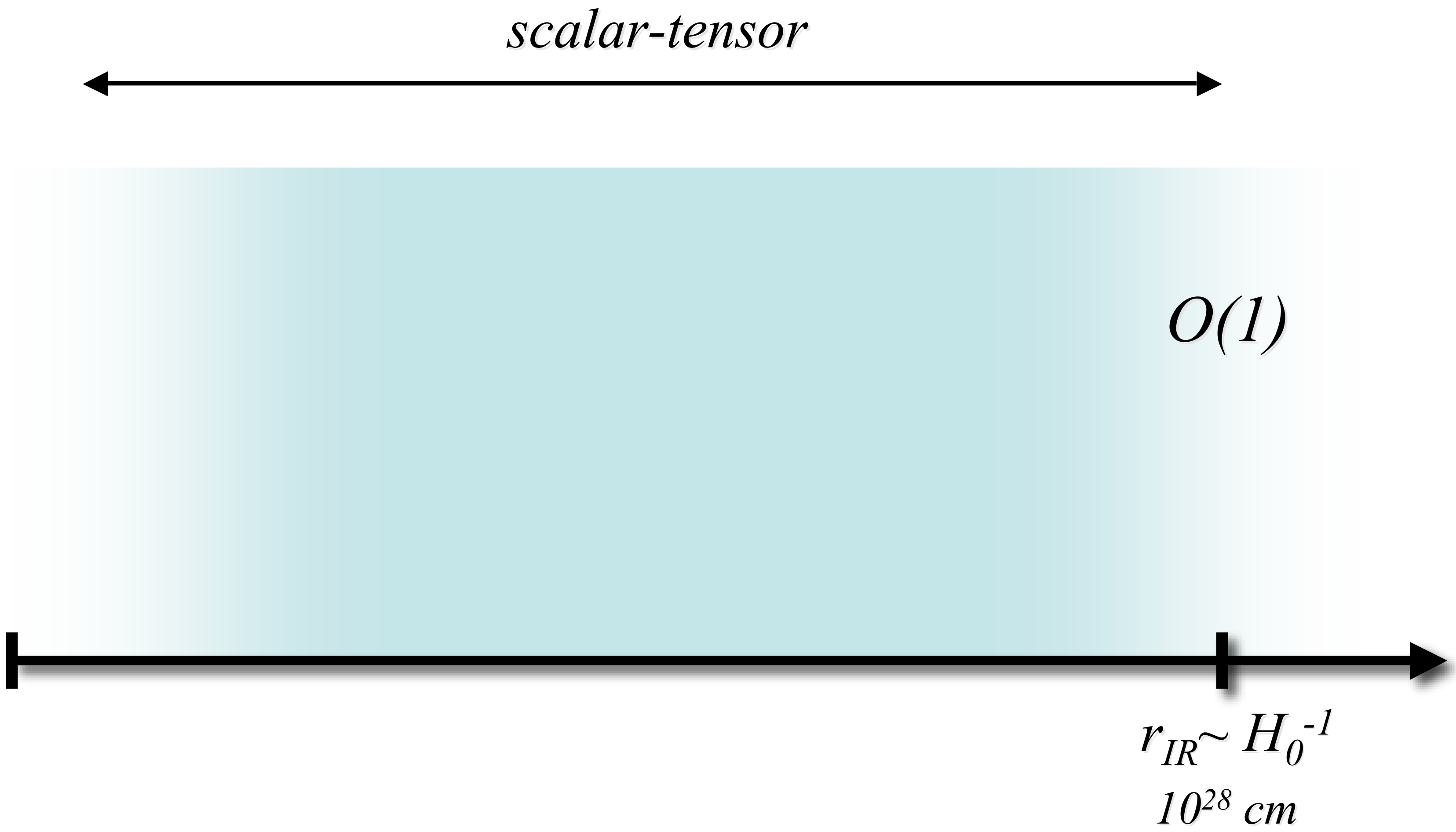
# *The Galileon as a local modification of gravity*

O(1)
$r_{IR}\sim H_0^{-1}$
$10^{28}$ cm



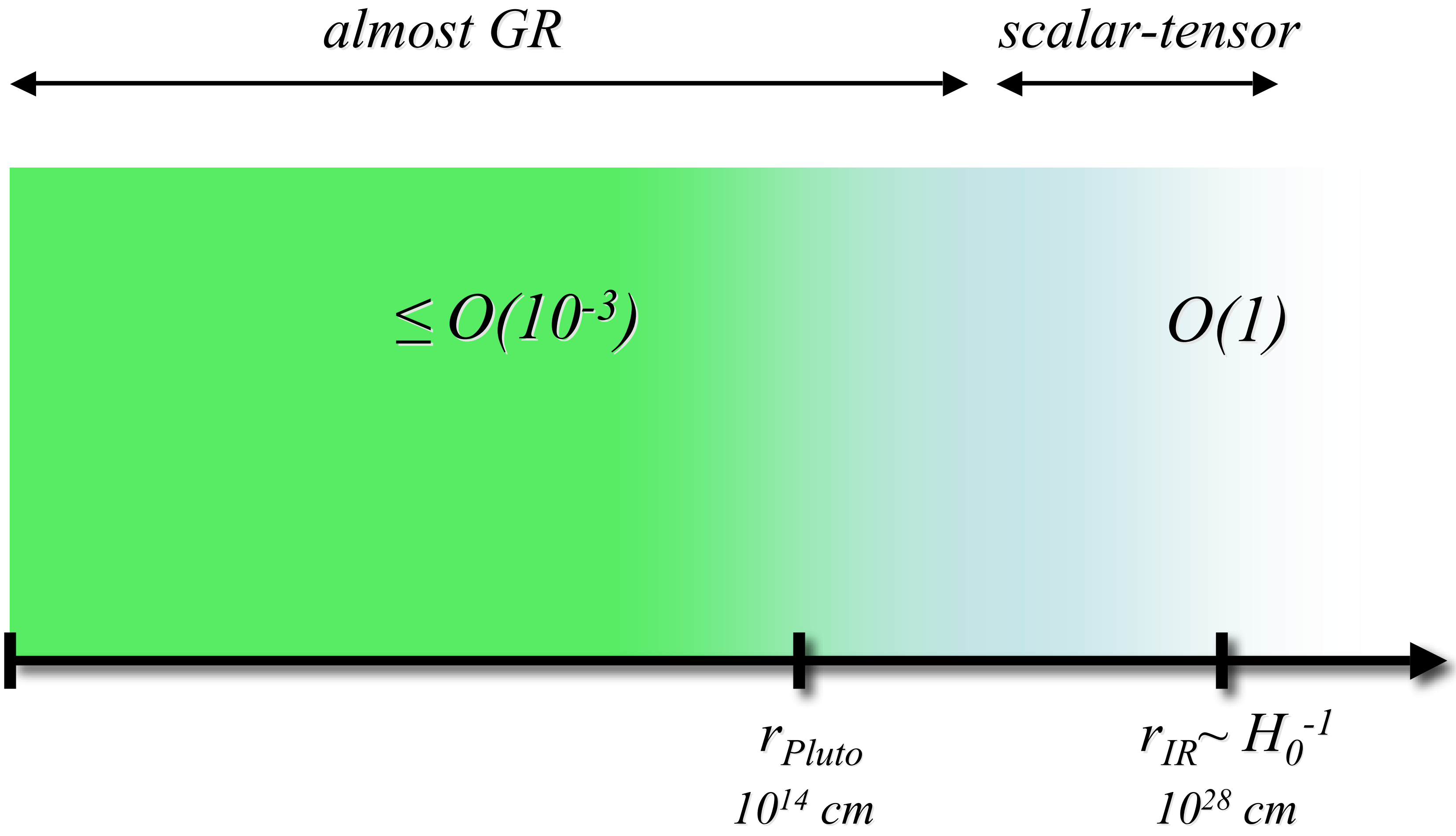

introduce a light scalar d.o.f

*f(R)*, Brans-Dicke, Fierz-Pauli massive gravity,
DGP, Lorentz-violating massive gravity,...

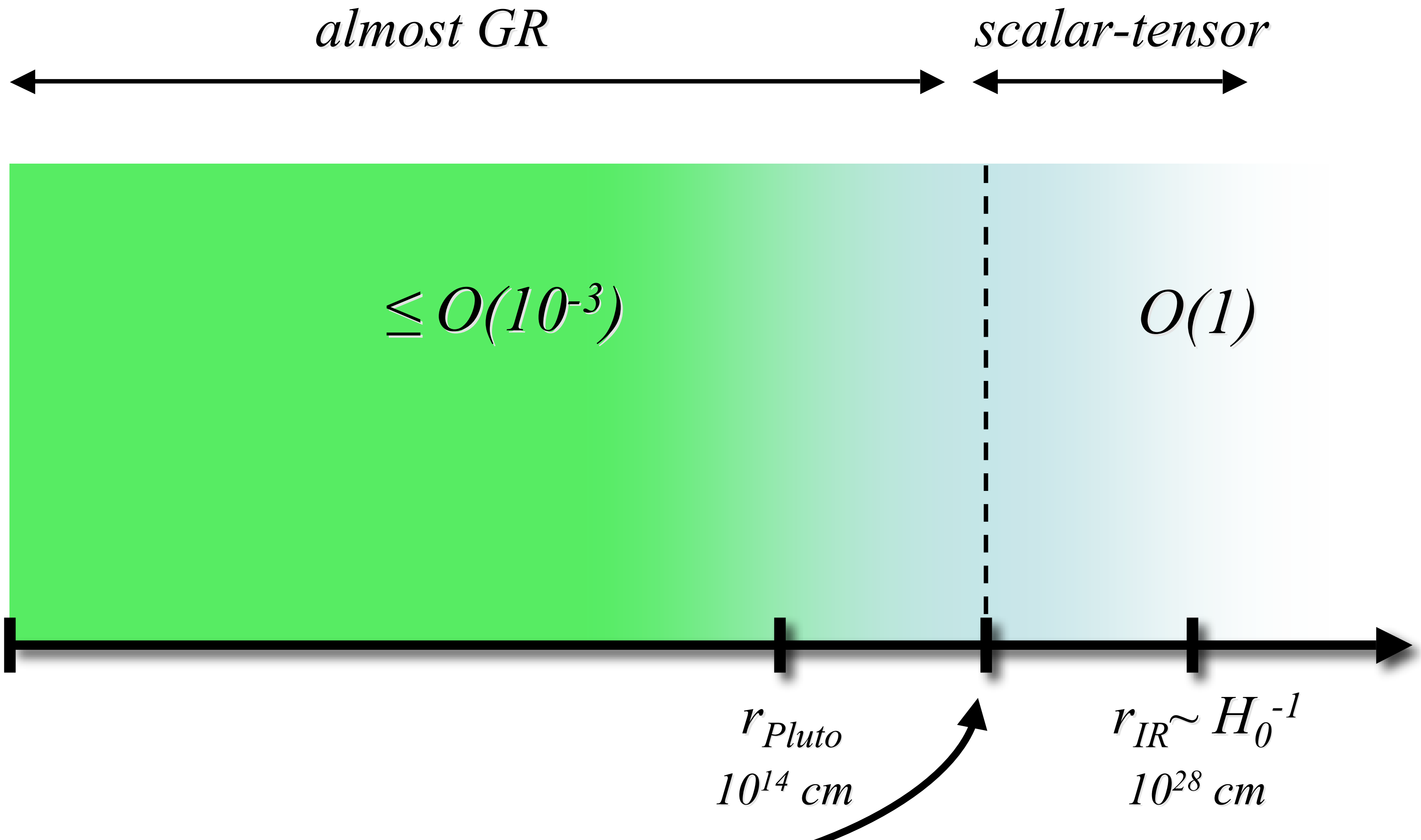



vDVZ discontinuity

Van Dam, Veltman, Zacharov 70



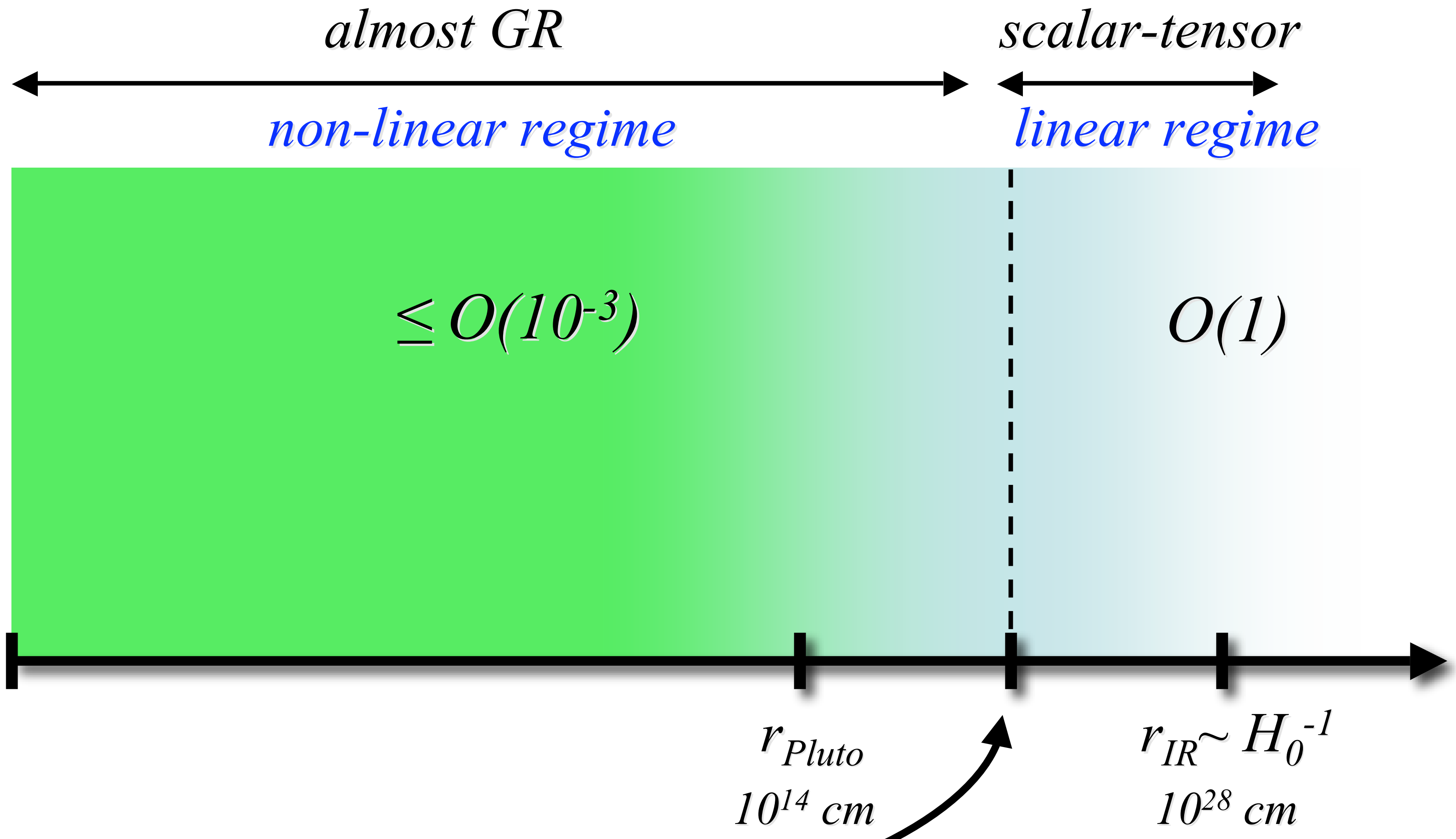
almost GR
scalar-tensor
$\leq O(10^{-3})$
$O(1)$
$r_{Pluto}$
$10^{14}$ cm
$r_{IR} \sim H_0^{-1}$
$10^{28}$ cm




screening mechanism

*f(R)*-like: chameleon mechanism  Khoury, Weltman 03

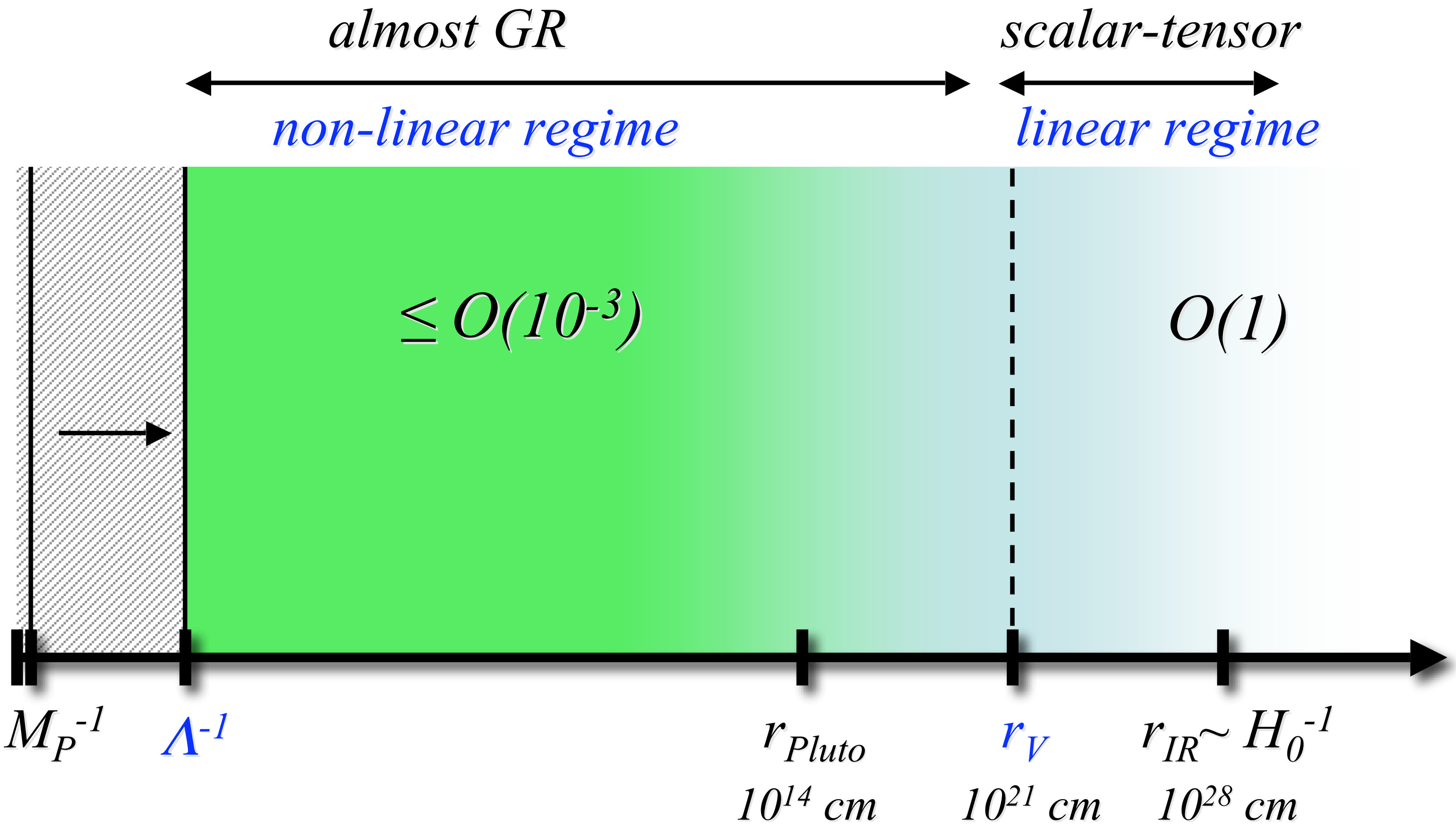
the scalar mass depends on the local density



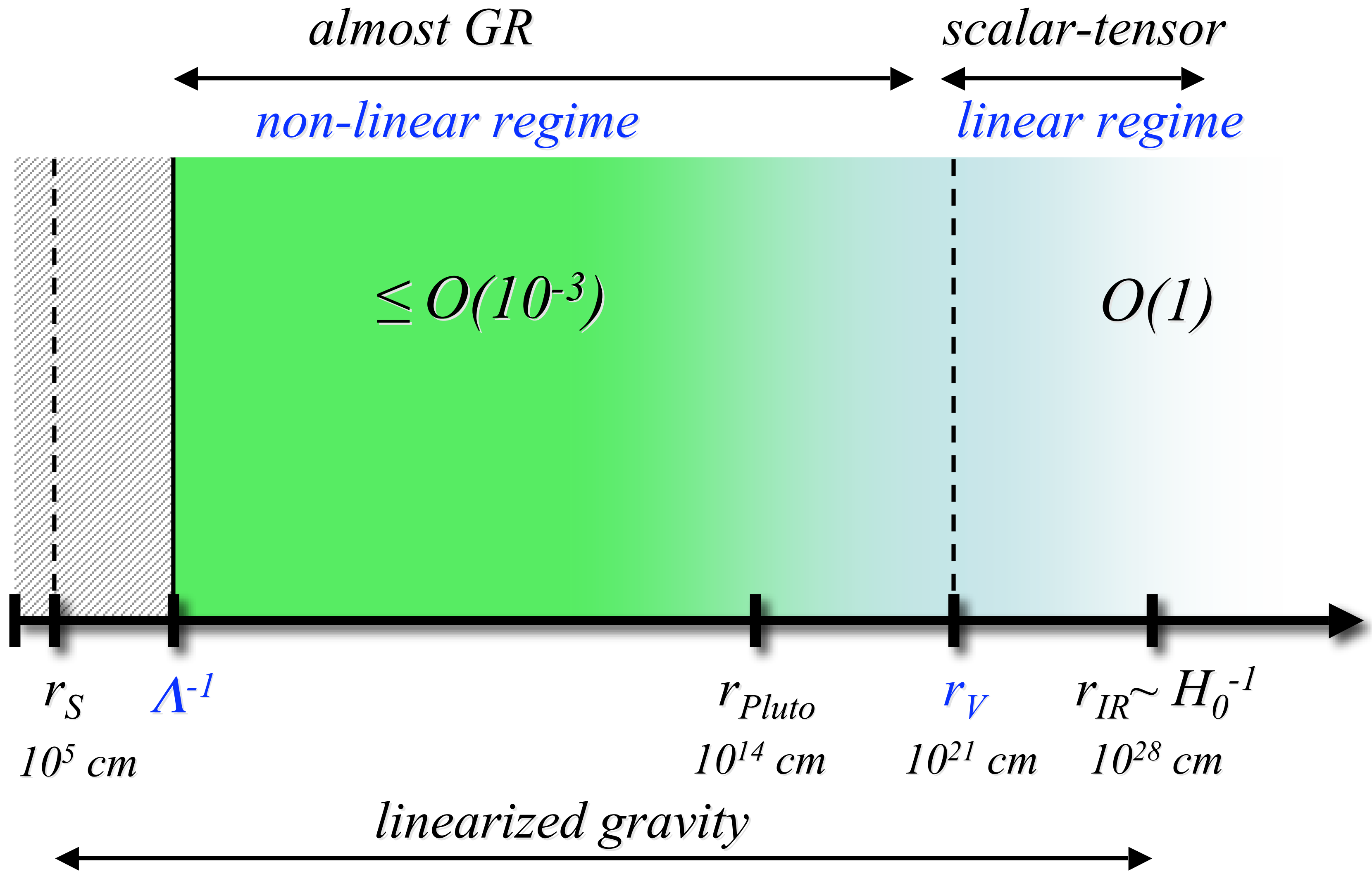

screening mechanism

Vainshtein effect: non-linear dynamics suppresses the scalar contribution

Vainshtein 72



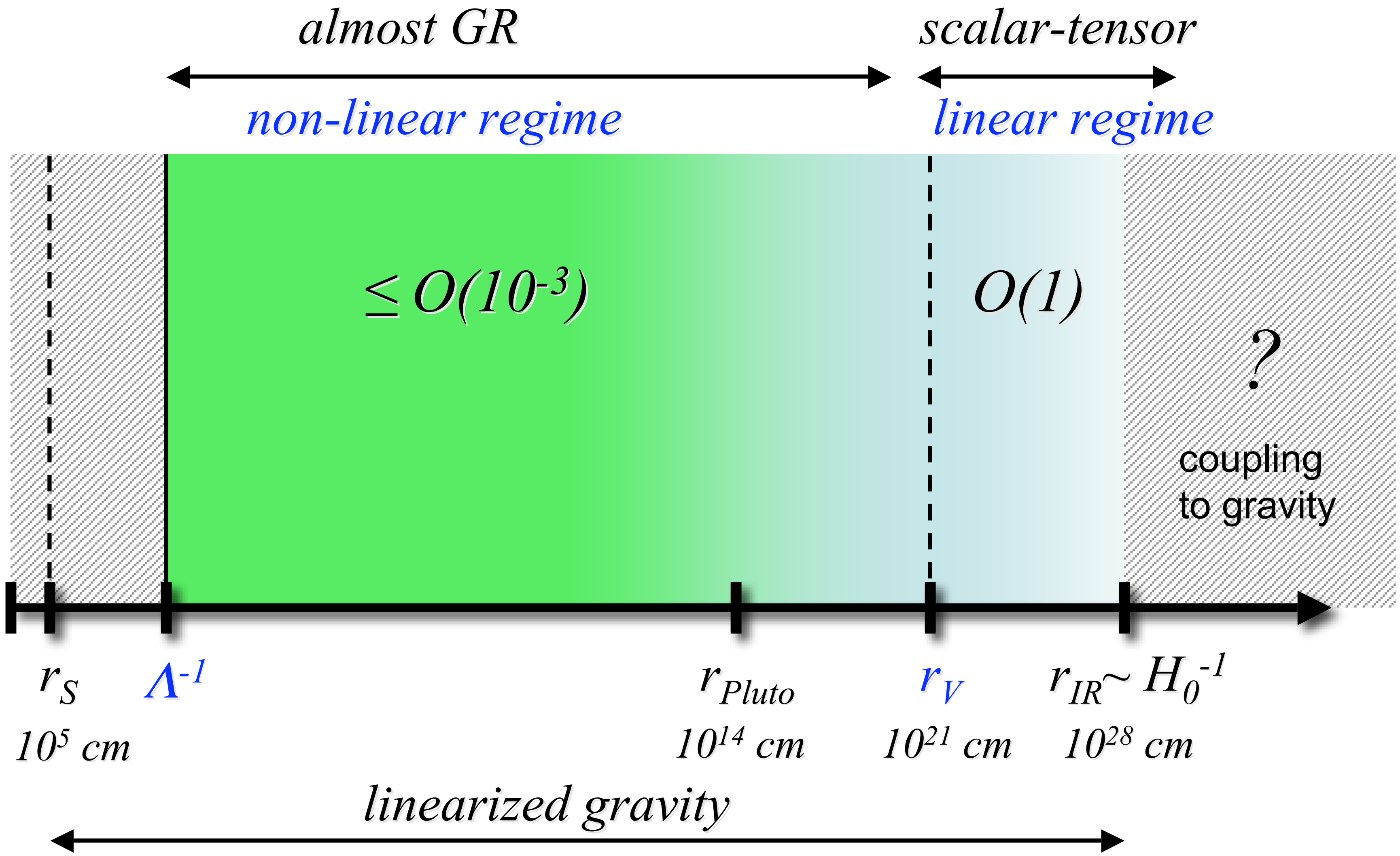
almost GR
scalar-tensor
non-linear regime
linear regime
$\leq O(10^{-3})$
$O(1)$
$M_P^{-1}$
$\Lambda^{-1}$
$r_{Pluto}$
$10^{14}$ cm
$r_V$
$10^{21}$ cm
$r_{IR} \sim H_0^{-1}$
$10^{28}$ cm




$$ds^2 \simeq \left[1-\frac{1}{2}H^2|\vec{x}|^2+\frac{1}{2}(2\dot{H}+H^2)t^2\right]\left(-dt^2+d\vec{x}^2\right)$$

FRW universe: conformal deformation of Minkowski

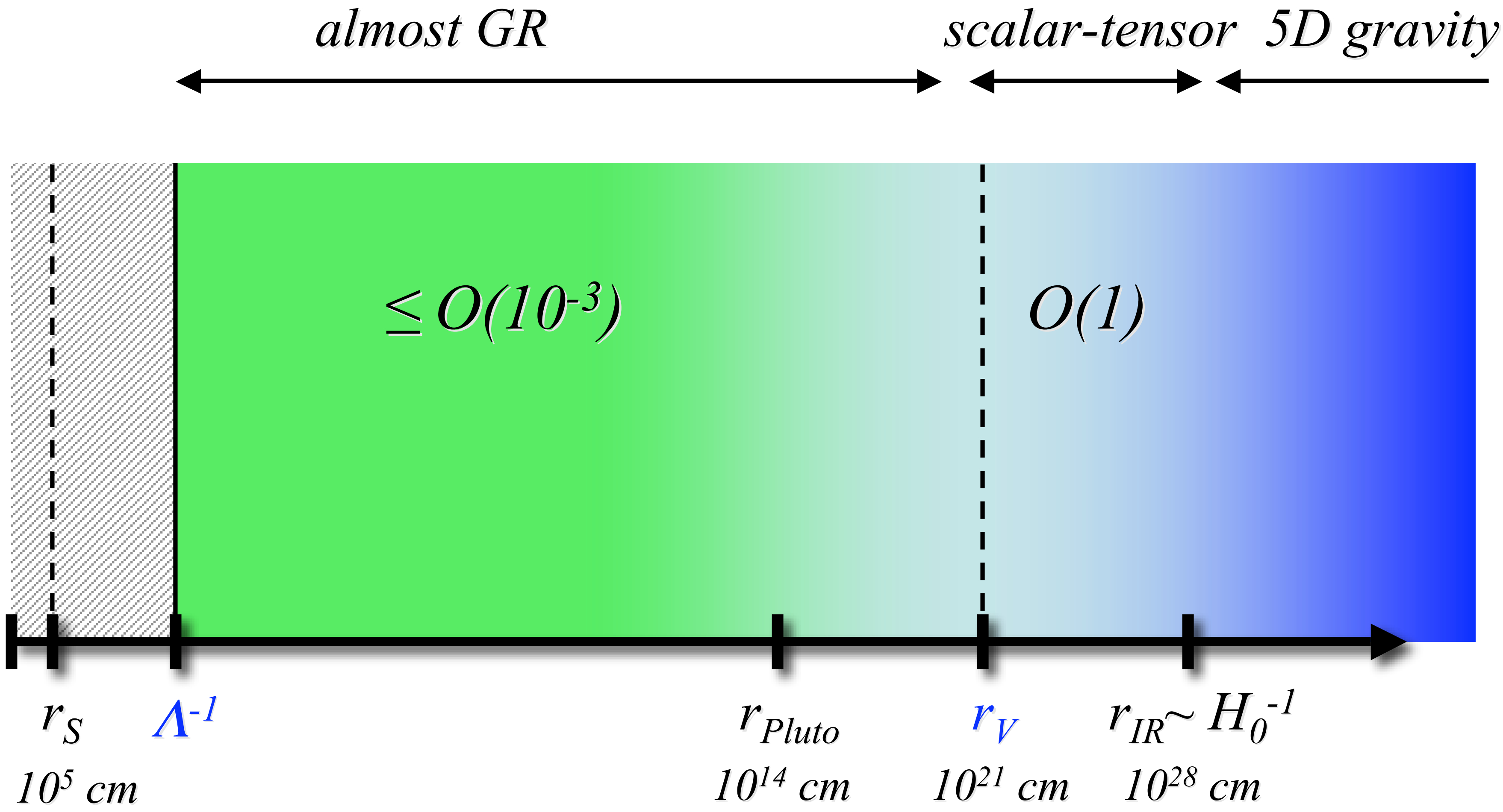
# the galileon: a local modification of gravity




$$ds^2 \simeq [1 - \frac{1}{2} H_0^2 x_\mu x^\mu](-dt^2 + d\vec{x}^2)$$ locally de Sitter

## DGP

Dvali, Gabadadze, Porrati 00




full non-linear 5D "self-accelerating" solution
plagued by a ghost instability

# Long-distance modification of GR

- locally and for weak gravitational fields due to a light scalar d.o.f. $\pi$
- $\pi$ kinetically mixed with the metric:

  replacing $\sqrt{-g}\,R$ with $\sqrt{-g}\,(1-2\pi)R$ plus whatever dynamics $\pi$ may have on its own;

- $\pi$ does not couple directly to matter (minimally coupled to $g_{\mu\nu}$ )

Demix $\pi$ and the metric by performing the Weyl rescaling: $h_{\mu\nu} = \hat{h}_{\mu\nu} + 2\pi\,\eta_{\mu\nu}$

$$S = \int d^4x \left[\frac{1}{2} M_{\rm Pl}^2 \sqrt{-\hat{g}}\hat{R} + \frac{1}{2}\hat{h}_{\mu\nu}T^{\mu\nu} + \mathcal{L}_\pi + \pi\, T^\mu{}_\mu\right]$$

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The approximate de Sitter space is realized in Jordan frame $g_{\mu\nu}$

The metric is nearly flat in Einstein frame

# Long-distance modification of GR

- locally and for weak gravitational fields due to a light scalar d.o.f. π
- π kinetically mixed with the metric:

  replacing $\sqrt{-g}\,R$ with $\sqrt{-g}\,(1-2\pi)R$ plus whatever dynamics $\pi$ may have on its own;

- π does not couple directly to matter (minimally coupled to $g_{\mu\nu}$ )

Demix π and the metric by performing the Weyl rescaling: $h_{\mu\nu} = \hat{h}_{\mu\nu} + 2\pi\,\eta_{\mu\nu}$

$$S = \int d^4x \left[\frac{1}{2} M_{\rm Pl}^2 \sqrt{-\hat{g}}\hat{R} + \frac{1}{2}\hat{h}_{\mu\nu}T^{\mu\nu} + \boxed{\mathcal{L}_\pi} + \pi\, T^\mu{}_\mu\right]$$

$$\pi_{\rm dS} = -\frac{1}{4}H_0^2 x_\mu x^\mu + \mathcal{O}(H_0^3 x^3)$$

$$M_{Pl}^2(\partial\pi)^2$$

$$\pi = \frac{\pi_c}{M_{Pl}}$$

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# The Galileon Lagrangian

$$\boxed{\mathcal{L}_\pi = \sum_{i=1}^{5} c_i \mathcal{L}_i} \qquad \mathcal{L} \sim (\partial\pi_c)^2 (\frac{\partial^2 \pi_c}{\Lambda^3})^n$$

$$\pi_c^{(\mathrm{dS})} = -\frac{1}{4} M_{\mathrm{Pl}} H_0^2 x^\mu x_\mu \qquad \frac{\partial^2 \pi_c^{(\mathrm{dS})}}{\Lambda^3} \sim \mathcal{O}(1) \Rightarrow \boxed{\Lambda \sim (M_{\mathrm{Pl}} H_0^2)^{1/3}} \sim (1000\ \mathrm{km})^{-1}$$

On top of this asymptotic solution one can study the effect of localized sources

$$\pi \longrightarrow \pi^{(\mathrm{dS})} + \pi$$

because of the symmetry of deSitter background the lagrangian for the perturbations is again galilean

$$\mathcal{L}_\pi = \sum_{i=2}^{5} d_i \mathcal{L}_i$$

$d_i$ are linear combinations of the $c_i$

# Spherically symmetric solution and the Vainshtein effect

localized source $\rho = M\delta^3(\vec{r})$

spherical symmetry $\pi = \pi_0(r)$

\+ shift and galilean symmetry

$\Rightarrow$ drastical simplification

e.o.m. is an algebraic equation for $\pi_0'(r)$

$$\frac{\delta \mathcal{L}_\pi}{\delta \pi} = \frac{1}{r^2}\frac{d}{dr} r^3 \left[ d_2 M_{\rm Pl}^2 (\pi_0'/r) + 2d_3 \frac{M_{\rm Pl}^2}{H_0^2} (\pi_0'/r)^2 + 2d_4 \frac{M_{\rm Pl}^2}{H_0^4} (\pi_0'/r)^3 \right] = M\delta^3(\vec{r})$$

$$d_2 M_{\rm Pl}^2 (\pi_0'/r) + 2d_3 \frac{M_{\rm Pl}^2}{H_0^2} (\pi_0'/r)^2 + 2d_4 \frac{M_{\rm Pl}^2}{H_0^4} (\pi_0'/r)^3 = \frac{M}{4\pi r^3}$$

# Spherically symmetric solution and the Vainshtein effect

rewrite the equation of motion as

$$d_2\left(\pi_0'/r\right)+2d_3\left(\pi_0'/r\right)\left(\pi_0'/rH_0^2\right)+2d_4\left(\pi_0'/r\right)\left(\pi_0'/rH_0^2\right)^2=\frac{M}{M_{\rm Pl}^2}\frac{1}{4\pi r^3}$$

in the linear regime, far from the source, the kinetic term dominates and we have the asymptotic behaviour

$$\pi_0\sim\frac{M}{M_{Pl}^2}\frac{1}{r} \qquad\qquad \frac{\pi_0}{\Phi_N}\sim\mathcal{O}(1)$$

the non-linear terms become relevant at the scale

$$\frac{\pi_0'}{H_0^2 r}\sim 1\Rightarrow r\sim\Big(\frac{M}{M_{Pl}^2}\frac{1}{H_0^2}\Big)^3\equiv r_V \quad (\sim 10^{21}\ \mathrm{cm\ for\ the\ sun})$$

$$\pi_0\sim\Big(\frac{M}{M_{Pl}^2}H^4\Big)^{1/3}r \quad \text{for } r<<r_V \qquad\qquad \frac{\pi_0}{\Phi_N}\sim\frac{r^2}{r_V^2}$$

# Results

The existence of only 5 possible terms makes possible a thorough analysis of very symmetric solutions. The sign of the coefficients $c_n$ can be chosen such that:

1. the galileon admits a de Sitter solution in the absence of a cosmological constant;

2. it is ghost free;

3. about this dS configuration there exist spherically symmetric solutions that describe the $\pi$ field generated by compact sources;

4. these configurations are also stable against small perturbations;

5. the Vainshtein effect is implemented.

Part 3

# *Galilean Genesis:*
# *an alternative to inflation*

# The standard picture

FRW universe

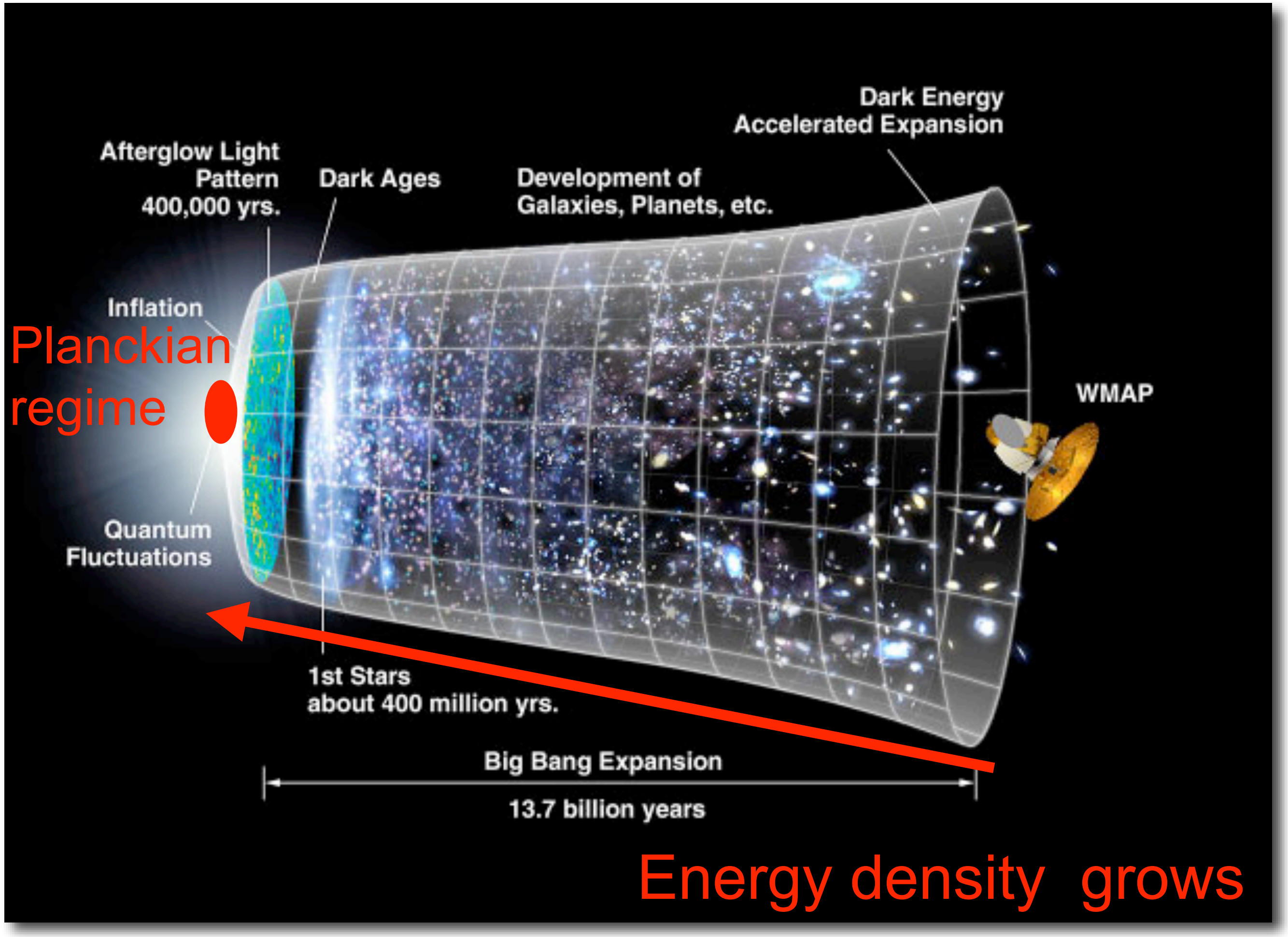
expanding

$$\dot{a} > 0$$

Going back in time

$$E \sim M_{\rm Pl}$$

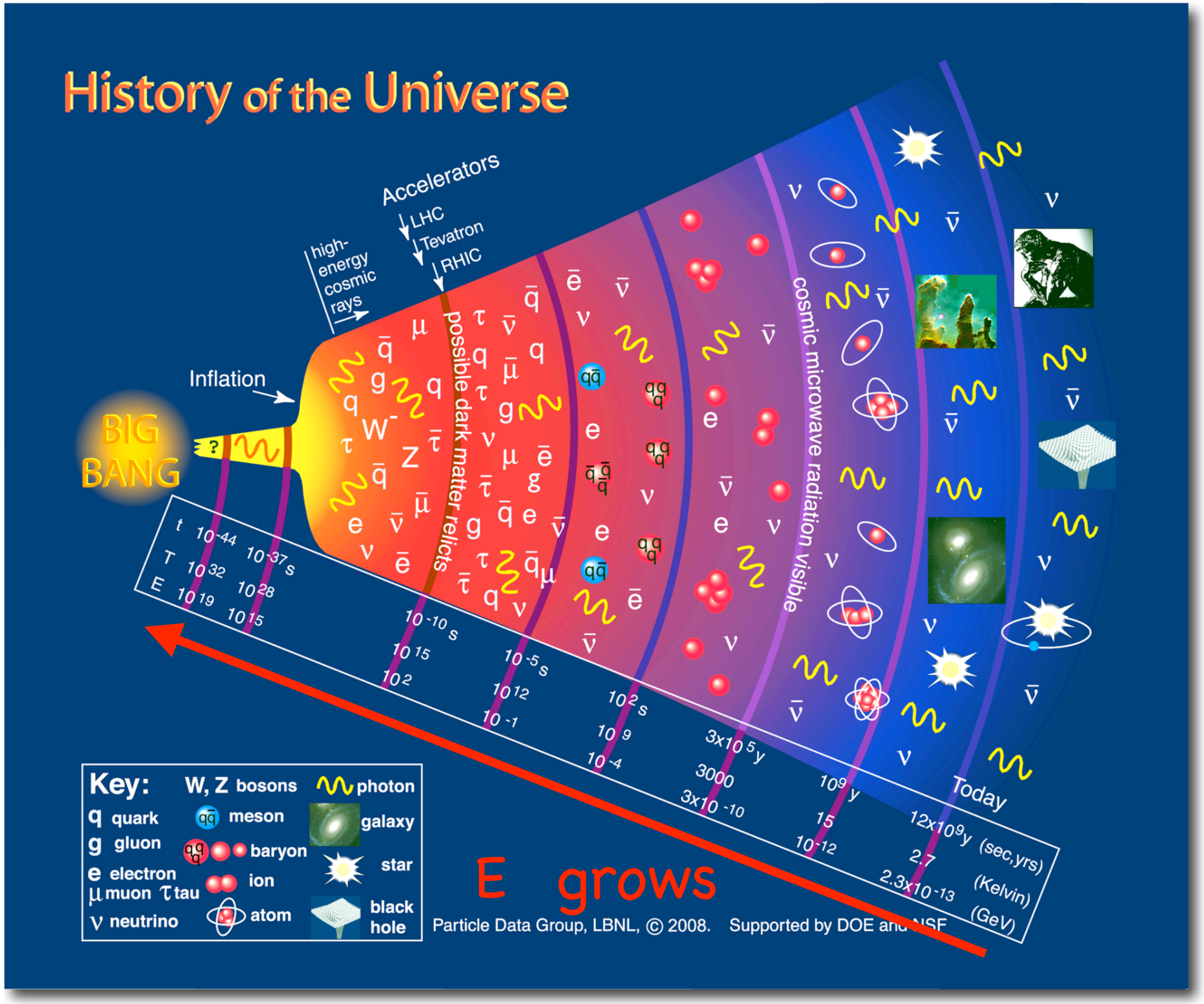
Big Bang



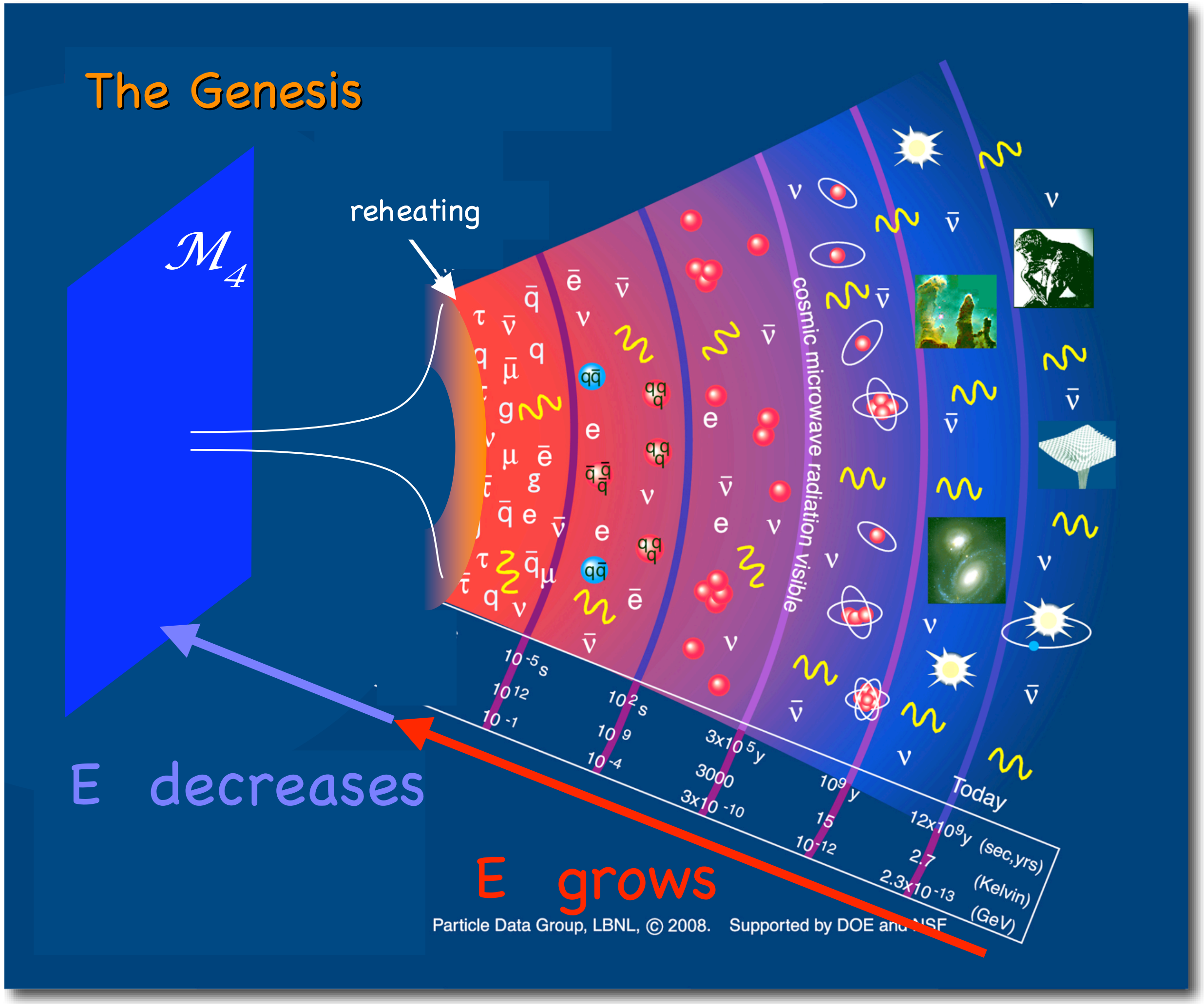

Quantum gravity effects become important. GR as an EFT breaks down

the origin of the expansion is inseparable from the UV completion of gravity

# The standard picture



# A different history: the Genesis



# How can it be possible?

The Big Bang paradigm assumes (at least) the null energy condition (NEC)

$T_{\mu\nu}n^\mu n^\nu \geq 0$ in FRW spacetime reduces to $\boxed{\rho + p \geq 0}$

$$\dot{H} = -4\pi G(\rho + p)$$

$$\dot{\rho} = -3H(\rho + p)$$

$$\boxed{\text{NEC} \implies \dot{H},\ \dot{\rho} \leq 0}$$

NEC satisfied by matter, radiation

NEC saturated by a cosmological constant

Is there a form of matter that violates it?

2 epochs of accelerated expansion $\ddot{a} > 0$

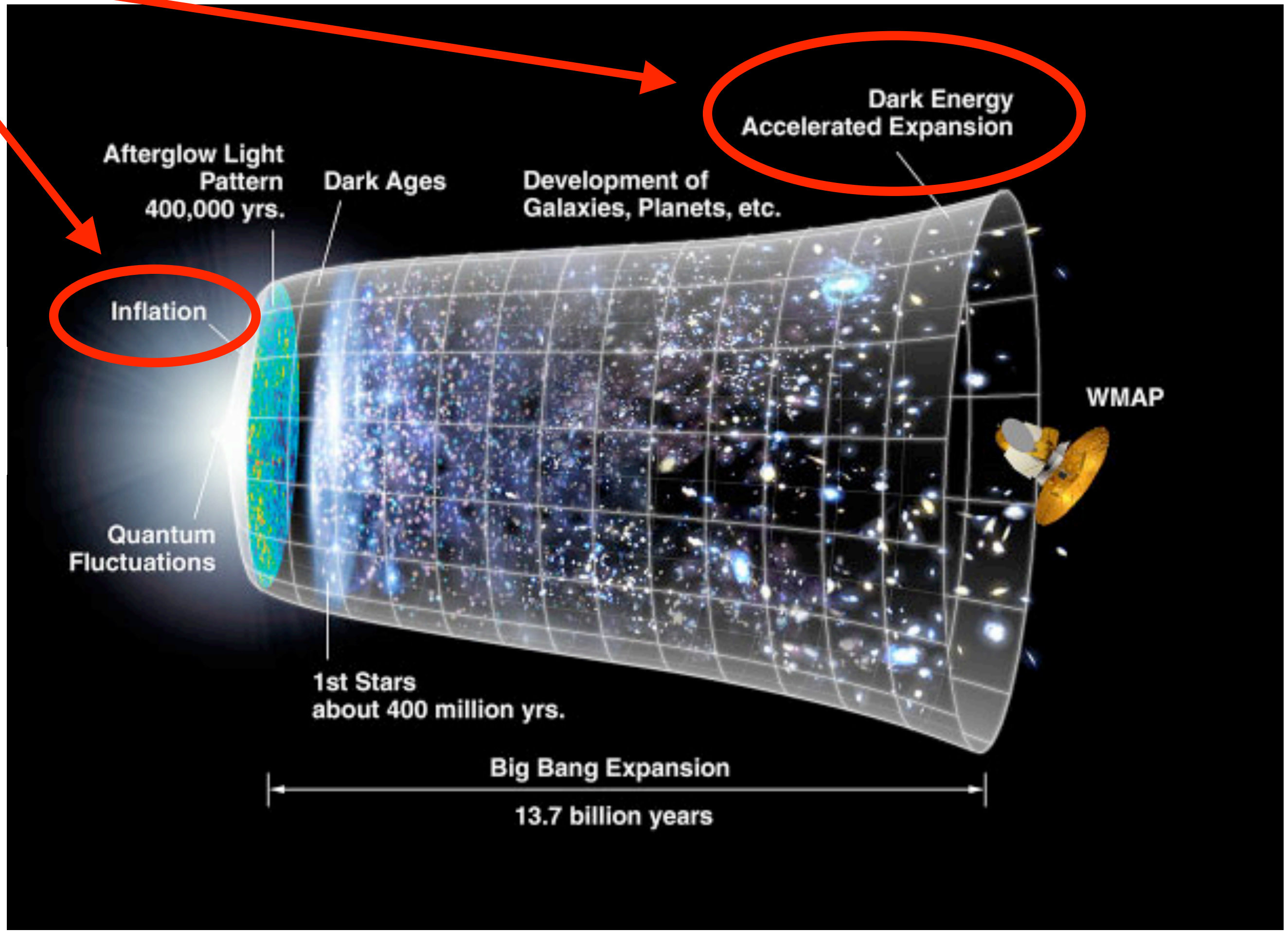
Strong energy condition (SEC)

$$(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T)\, \xi^\mu \xi^\nu \geq 0$$

$$\rho + 3p \geq 0$$

$$\Downarrow$$

$$\ddot{a} \leq 0$$




The two revolutions in cosmology in the last 25 years, inflation + present acceleration, are based on the violation of the SEC

# Can we violate the NEC?

Usually ~~NEC~~ are unstable:

signature $(-+++)$

$$\mathcal{L} = \pm\frac{1}{2}(\partial\phi)^2 - V(\phi)$$

$$\phi = \phi(t) \implies (\rho + p) = \mp\dot{\phi}^2$$

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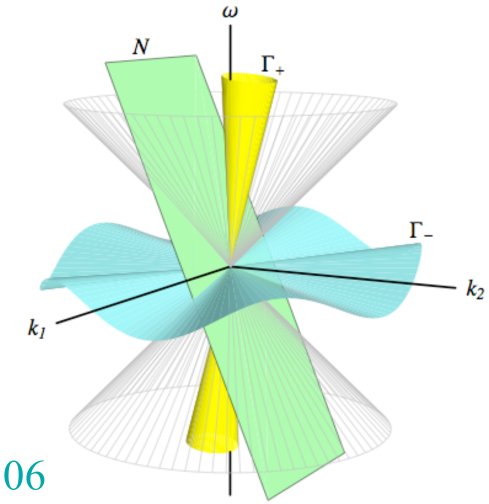
$$\mathcal{L} = \pm\frac{1}{2}(\partial\phi)^2 - V(\phi)$$

$$\phi = \phi(t) \implies (\rho + p) = \mp\dot{\phi}^2$$

## No-go theorem

$$\mathcal{L} = F(\phi_I, \partial\phi_I, \partial^2\phi_I, \ldots) \qquad I = 1, \ldots, N$$




There are no stable NEC-violating EFT
if we can neglect HD terms

Dubovsky, Gregoire, Nicolis, Rattazzi 06

- They are irrelevant at low energies. When they are important ~~EFT~~
- They describe new pathological ghost-like degrees of freedom

But there are exceptions…

## The ghost condensate

First exception: the ghost condensate Arkani-Hamed, Cheng, Luty, Mukohyama 03

Small deformation marginally violates the NEC $0 < \dot{H} \lesssim H^2$

Creminelli, Luty, Nicolis, Senatore 06

## The Galileon

HD lagrangian: the No-go theorem doesn't apply

It has 2 derivatives EOM: no ghost

$$\boxed{\pi(x) \to \pi(x) + c + b_\mu x^\mu}$$

galilean invariance

Can it violate the NEC?

# The conformal galileon

Nicolis, Rattazzi, ET 08

Promote galilean transformation + Poincaré to the conformal group SO(4,2)

$$\begin{aligned} \pi(x) &\rightarrow \pi(x) + c \\ \pi(x) &\rightarrow \pi(x) + b_\mu x^\mu \end{aligned}$$

# The conformal galileon

Nicolis, Rattazzi, ET 08

Promote galilean transformation + Poincaré to the conformal group SO(4,2)

$$\begin{aligned} \pi(x) &\rightarrow \pi(\lambda x) + \log \lambda \\ \pi(x) &\rightarrow \pi\big(x + (c\,x^2 - 2(c \cdot x)x)\big) - 2c_\mu x^\mu \end{aligned}$$

$\pi$ plays the role of the dilaton $\quad g_{\mu\nu} = e^{2\pi}\eta_{\mu\nu}$

$$\mathcal{L}^{(2)} \rightarrow e^{2\pi}(\partial\pi)^2$$

$$\mathcal{L}^{(3)} \rightarrow (\partial\pi)^2 \Box\pi + \frac{1}{2}(\partial\pi)^4$$

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$\pi$ plays the role of the dilaton $\quad g_{\mu\nu} = e^{2\pi}\eta_{\mu\nu}$

$$\mathcal{L}_\pi = f^2 e^{2\pi}(\partial\pi)^2 + \frac{f^3}{\Lambda^3}\Box\pi(\partial\pi)^2 + \frac{f^3}{2\Lambda^3}(\partial\pi)^4$$

$$\boxed{e^{\pi_{\rm dS}} = -\frac{1}{H_0 t}} \qquad -\infty < t < 0 \qquad H_0^2 = \frac{2\Lambda^3}{3f}$$

Spontaneously breaks SO(4,2) → SO(4,1) de Sitter group

Conservation+ scale invariance $\boxed{\begin{cases} \rho = 0 \\ p \propto -\frac{1}{t^4} \end{cases}}$ ~~NEC~~

$$\pi(x) = \pi_{\rm dS}(t) + \phi(x)$$

Stable luminal fluctuations

Nicolis, Rattazzi, ET 09

# Galilean Genesis

Creminelli, Nicolis, ET 10

$$\int d^4x\,\sqrt{-g}\left[f^2 e^{2\pi}(\partial\pi)^2 + \frac{f^3}{\Lambda^3}\Box\pi(\partial\pi)^2 + \frac{f^3}{2\Lambda^3}(\partial\pi)^4\right] + \mathcal{S}_{\rm EH}$$

Conformal galileon minimally coupled to gravity

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2 \qquad \pi = \pi(t)$$

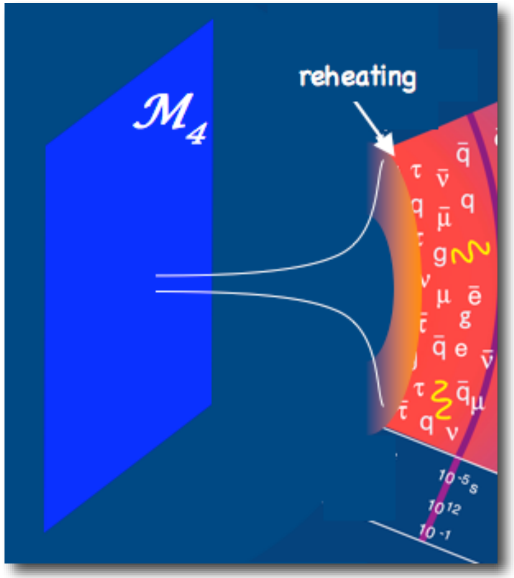
# Galilean Genesis

Creminelli, Nicolis, ET 10

$$\int d^4x\,\sqrt{-g}\left[f^2e^{2\pi}(\partial\pi)^2+\frac{f^3}{\Lambda^3}\Box\pi(\partial\pi)^2+\frac{f^3}{2\Lambda^3}(\partial\pi)^4\right]+S_{\rm EH}$$

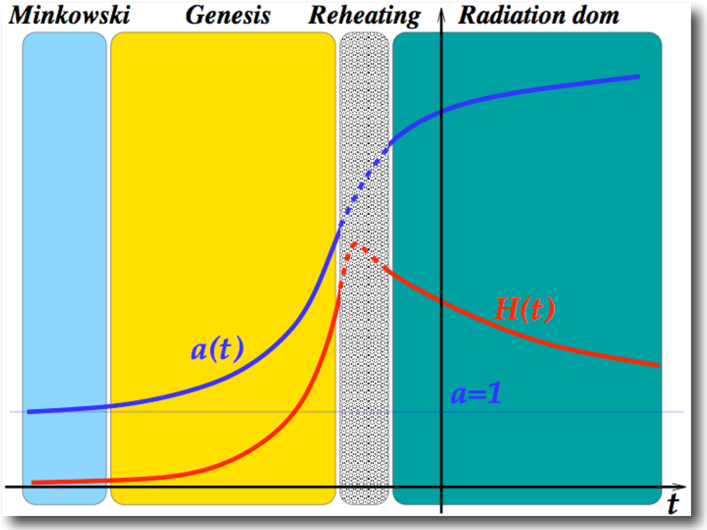
Conformal galileon minimally coupled to gravity

$$ds^2=-dt^2+a^2(t)d\vec{x}^2 \qquad \pi=\pi(t)$$




Solve Friedmann's equations for $H$ perturbatively

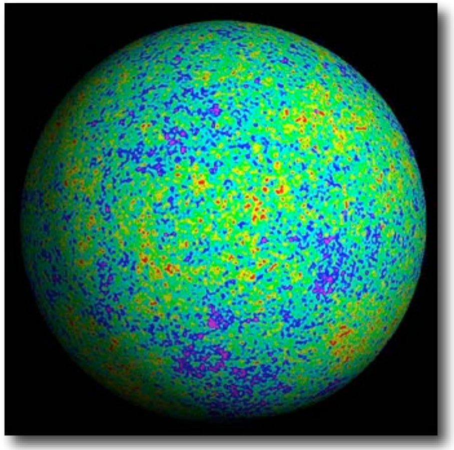
$$\dot{H}=-\frac{1}{2M_{\rm Pl}^2}(\rho+p)\sim\frac{f^2}{M_{\rm Pl}^2}\frac{1}{H_0^2t^4}$$




$$\boxed{H\simeq-\frac{1}{3}\frac{f^2}{M_{\rm Pl}^2}\frac{1}{H_0^2t^3}+c}$$

$$\pi\simeq\pi_{\rm dS}-\frac{1}{2}\frac{f^2}{M_{\rm Pl}^2}\cdot\frac{1}{H_0^2t^2}$$

# Scalar perturbations



Homogeneus attractor $t \to t + c$

$\pi$ perturbations are not scale invariant

$$\langle \zeta(t, \vec{k}) \zeta(t, \vec{k}') \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k}') \frac{1}{18} \frac{f^2}{M_{\rm Pl}^4 H_0^2} \frac{1}{2k} \frac{1}{t^2}$$

$\zeta$ action: $S_\zeta = \dfrac{9M_{\rm Pl}^4}{f^2} \displaystyle\int d^4x\, (H_0 t)^2 \left[\dot{\zeta}^2 - (\vec{\nabla}\zeta)^2\right]$ $\qquad \zeta \sim \text{const}, \quad \dfrac{1}{t}$

During the genesis, flow towards the "other" adiabatic mode

$$t \to t + \epsilon(t) \qquad x^i \to x^i(1 - \lambda) \qquad \Psi \to \Psi + H\epsilon - \lambda \qquad \Phi \to \Phi - \dot{\epsilon}$$

$$\epsilon(t) = \frac{\lambda}{a(t)} \int_0^t a(t')dt' + \frac{c}{a(t)}$$

Scalar perturbations of $\pi$ are always irrelevant at cosmological scales

# Scale invariant perturbations

Any coupling to $\pi$ has to go through the fictitious metric

$$g^{(\pi)}_{\mu\nu} = e^{2\pi(x)}\eta_{\mu\nu}$$

A spectator massless scalar field $\sigma$ behave as in de Sitter

Its spectrum is scale invariant because of the dS symmetry

Exp. small corrections from the evolution of the real metric for modes of cosmological interest

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Conversion of $\sigma$ fluctuations

Analogous to "second field" mechanism in inflation

Typical signatures

Large local non-Gaussianities

Low GWs: perturbations produced at low energy

Blue GWs: contraction or ~~NEC~~

$$\frac{d}{dt}H^2 = 2H\dot{H} > 0$$

# Superluminality

Perturbations are luminal about the fake de Sitter background

Any deformation will have superluminal perturbations

1) The cutoff is lower, it forbids superluminal propagation but also NEC-violating solution is outside the EFT

2) Both the solution and superluminal propagation are inside EFT

Not necessary an inconsistency

Ex: No closed time-like curves

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi 06

But the UV completion cannot be a Lorentz-invariant local QFT

# Summary

Interesting Effective Field Theory

Many questions need to be addressed

Swampland?