

Radiative Avalanche

1. radiative drag

(a) Four-momentum of a photon.

$$\hat{p}^{\mu} = \nu (1, \vec{\lambda}) \quad , \quad \hat{p}_\mu \hat{p}^\mu = \nu^2 (1 - \vec{\lambda}^2) = 0.$$

Four-velocity $u^\mu = (1, \vec{v})$ and express \hat{p}^μ in comoving frame.

$$\hat{p}^\mu = v_0 (1, \vec{l}_0)$$

$$\text{また. } u_\mu \hat{p}^\mu = \gamma \nu - \gamma \nu \frac{u \cdot l}{c} = v_0$$

$$\Rightarrow \begin{cases} v_0 = \nu \gamma \left(1 - \frac{u \cdot l}{c} \right) \\ \nu = v_0 \gamma \left(1 + \frac{u \cdot l_0}{c} \right) \end{cases}$$

$$\text{また. } \begin{cases} l_0 = \frac{v}{v_0} [l + (c \frac{\gamma-1}{u^2} u \cdot l - \gamma) \frac{u}{c}] \\ l = \frac{v_0}{\nu} [l_0 + (c \frac{\gamma-1}{u^2} u \cdot l_0 + \gamma) \frac{u}{c}] \end{cases}$$

$$\left. \begin{array}{l} \text{-般的 Lorentz 変換. } \vec{v} \sim (0, 0, v) \text{ とす. } \beta^{-2} \vec{\beta} \cdot \vec{x} = (0, 0, z) \\ \text{f.z. } [1 + (\gamma-1) \beta^2 \vec{\beta} \cdot \vec{x}] \cdot \vec{x} = (x, y, \gamma z) \\ \text{ただし. } \ell v = x, \gamma \beta = y \text{ とす. } \end{array} \right\}$$

$$\text{立体角の変換は. } d\Omega_0 = \left(\frac{v}{v_0} \right)^2 d\Omega = \left[\gamma \left(1 - \frac{u \cdot l}{c} \right) \right]^{-2} d\Omega$$

• E, F, P の定義

$$I_\nu = \frac{2\pi h^3}{c^2} n' \text{ photon occupation number} \quad I_\nu / \nu^3 \text{ は } \text{口-レーベル 不変.} \quad \frac{I_\nu}{\nu^3} = \frac{I_{\nu,0}}{\nu_0^3} \equiv f_\nu$$

$$\tilde{T}^{\mu\nu} = \frac{2\pi}{c} \int n l^\mu l^\nu \nu^3 d\nu d\Omega = \frac{1}{c} \int I_\nu l^\mu l^\nu d\nu d\Omega$$

$$\Rightarrow \tilde{T}^{\infty} = \frac{1}{c} \int I_\nu d\nu d\Omega \equiv E$$

$$\tilde{T}^{0i} = \frac{1}{c} \int I_\nu l^0 l^i d\nu d\Omega = \frac{1}{c} F^i$$

$$\tilde{T}^{i\bar{j}} = \frac{1}{c} \int I_\nu l^i l^{\bar{j}} d\nu d\Omega \equiv P^{i\bar{j}}$$

Transformation rules for E, F, P

$$\overset{I_{\infty} = f_0}{\curvearrowleft} I_0 = \left(\frac{v_0}{v} \right)^4 I = \left[\gamma \left(1 - \frac{v \cdot e}{c} \right) \right]^4 I$$

$\frac{I_{\infty}}{I} = f_0$
e.v.z 種/分. J, Z.

$$E_0 = \frac{1}{c} \int d\Omega I_0 = \frac{1}{c} \int \left(\gamma \left[1 - \frac{v \cdot e}{c} \right] \right)^{-2} d\Omega' \left[\gamma \left(1 - \frac{v \cdot e}{c} \right) \right]^4 I$$

$$= \frac{1}{c} \int d\Omega' \left| \gamma \left[1 - \frac{v \cdot e}{c} \right] \right|^2 I$$

$$= \gamma^2 \left(E - 2 \frac{v \cdot F}{c} + \frac{v_i v_k}{c^2} p^{\hat{i} \hat{k}} \right)$$

$$F_0^i = \frac{1}{c} \int d\Omega_0 I_0 l_0^i = \int d\Omega' \left| \gamma \left[1 - \frac{v \cdot e}{c} \right] \right|^2 I \cdot \left(\frac{v_0}{v} \right) \left[l^i + \left(c \frac{\gamma-1}{m^2} v \cdot e - \gamma \right) \frac{v^i}{c} \right]$$

$$= \int d\Omega' \left| \gamma \left[1 - \frac{v \cdot e}{c} \right] \right|^2 I \mid l^i + \left(c \frac{\gamma-1}{m^2} v \cdot e - \gamma \right) \frac{v^i}{c}$$

$$= \gamma \left\{ F^i - \frac{v^k p^{\hat{i} \hat{k}}}{c} + \frac{v^i}{c} \left(c \frac{\gamma-1}{m^2} v \cdot F - \gamma E \right) \right.$$

$$\left. - \frac{v^i}{c^2} \left(c \frac{\gamma-1}{m^2} v_i v_k p^{\hat{k} \hat{i}} - \gamma v \cdot F \right) \right\}$$

$$= \gamma \left\{ F^i + \left[\left(\gamma + \frac{\gamma-1}{v^2/c^2} \right) \frac{v \cdot F}{c^2} - \gamma E - \frac{\gamma-1}{v^2/c^2} \frac{v_i v_k}{c^2} p^{\hat{i} \hat{k}} \right] \frac{v^i}{c} \right\}$$

$$P_0^{\hat{i} \hat{j}} = \frac{1}{c} \int d\Omega_0 I_0 l_0^i l_0^j$$

$$= \int d\Omega' I \mid l^i + \left(c \frac{\gamma-1}{m^2} v \cdot e - \gamma \right) \frac{v^i}{c} \mid l^j + \left(c \frac{\gamma-1}{m^2} v \cdot e - \gamma \right) \frac{v^j}{c}$$

$$= P^{\hat{i} \hat{j}} + \frac{\gamma-1}{m^2/c^2} \left(\frac{v^i v_k}{c} p^{\hat{i} \hat{k}} + \frac{v^j v_m}{c} p^{\hat{j} \hat{m}} \right)$$

$$+ \left(\frac{\gamma-1}{m^2/c^2} \right) \frac{v^i v_j}{c} \frac{v_m v_n p^{\hat{n} \hat{m}}}{c} + \gamma^2 \frac{v^i v^j}{c^2} E$$

$$- \gamma \left(\frac{v^i F^j}{c^2} + \frac{v^j F^i}{c^2} \right) - 2 \gamma \frac{\gamma-1}{m^2/c^2} \frac{v^i v^j}{c^2} \frac{v \cdot F}{c^2}$$

$(\frac{m}{c})$ n-次

$$\Rightarrow E_0 = \gamma^2 \left(E - 2 \frac{v \cdot F}{c} \right) \sim E - 2 \frac{v \cdot F}{c}$$

$$F_0^i = \gamma \left(F^i - \gamma E v^i - v_k p^{\hat{i} \hat{k}} \right) \sim \cancel{\gamma} \left(F^i - E v^i - v_k p^{\hat{i} \hat{k}} \right)$$

$$P_0^{\hat{i} \hat{j}} = P^{\hat{i} \hat{j}} - \gamma \frac{1}{c^2} (v^i F^j + v^j F^i) \sim P^{\hat{i} \hat{j}} - \frac{1}{c^2} (v^i F^j + v^j F^i)$$

(b) Radiative Transfer Equation

$$\kappa^m \frac{\partial f}{\partial x^m} = \rho(\alpha - \beta f) - \rho K_{v0}^{sca} \int \phi_v(l', l) f(l) v dv' d\Omega' \\ + \rho K_{v0}^{abs} \int \phi_v(l, l') f(l') v' dv' d\Omega'$$

$$\cdot J_{v0} \equiv 4\pi v_0^3 \alpha, K_{v0}^{abs} \equiv \frac{\beta}{v_0}$$

• scattering redistribution function

$$\phi_v = \frac{3}{4} [1 + (l \cdot l')^2] \delta(v_0 - v_0') \frac{1}{4\pi} \\ \left(\begin{array}{l} \text{normalization} \\ \text{for } \int v \phi_v = 1 \end{array} \right)$$

$$\Rightarrow \frac{1}{c} \left[\frac{\partial f}{\partial t} + (\ell \cdot \nabla) f \right] = \rho \frac{J_{v0}}{4\pi v_0^3} - \rho v_0 K_{v0}^{abs} f - \rho v_0 K_{v0}^{sca} f \\ + \frac{3}{4} \rho K_{v0}^{sca} v_0 \int [1 + (l_0 \cdot l')^2] f(l') \frac{dl'}{4\pi}$$

replace f by I, E, F, P .

$$\Rightarrow \frac{1}{c} \frac{\partial I_v}{\partial t} + (\ell \cdot \nabla) I_v = \left(\frac{v}{v_0} \right)^2 \rho \left[\frac{J_{v0}}{4\pi} - (K_{v0}^{abs} + K_{v0}^{sca}) I_{v0} \right. \\ \left. + \frac{3}{4} K_{v0}^{sca} \frac{c}{4\pi} (E_{v0} + \log \log P_{v0}) \right]$$

• Moment eq. 由導出
freq integr

$$\Rightarrow \frac{1}{c} \frac{\partial I}{\partial t} + (\ell \cdot \nabla) I = \left(\frac{v}{v_0} \right)^3 \left[\frac{J_0}{4\pi} - (K_v^{abs} + K_v^{sca}) I_0 + \frac{3}{4} K_v^{sca} \frac{c}{4\pi} (E_v + \log \log P_v) \right]$$

0-th order (dLZ 積分)

$$\frac{\partial E}{\partial t} + \nabla \cdot F = \int d\Omega_0 \cdot r (1 + \frac{v \cdot \ell_0}{c}) \cdot \rho \quad [\dots] \\ = \rho r (j_0 - (K_v^{abs} E_0) - \rho r (K_v^{abs} + K_v^{sca}) \frac{v \cdot F_0}{c})$$

Lag moment

$$\begin{aligned}
 \frac{1}{c^2} \frac{\partial F^i}{\partial t} + \frac{\partial p^{ik}}{\partial x^k} &= \int d\omega_0 \left(1 + \frac{v \cdot \omega_0}{c} \right) \frac{v_0}{c} \left[\omega_0 + \left(c \frac{k-1}{m^2} v \cdot \omega_0 + r \right) \frac{v}{c} \right] \\
 &\quad \cancel{\rho \omega_0} \quad \rho \cdot [\dots] \\
 &= \int d\omega_0 \left[\omega_0 + \left(c \frac{k-1}{m^2} v \cdot \omega_0 + r \right) \frac{v}{c} \right] \rho \cdot [\dots] \\
 &= \rho r \frac{v^i}{c^2} (j_0 - c k_0^{abs} E_0) \\
 &\quad - \rho (k_0^{abs} + k_0^{sca}) \frac{k-1}{v^2} \frac{v^i}{c} (v \cdot F_0) \\
 &\quad - \frac{1}{c} \rho (k_0^{abs} + k_0^{sca}) F_0^i
 \end{aligned}$$

inertial frame $k \neq i$.

$$\begin{aligned}
 \frac{\partial E}{\partial t} + \nabla \cdot F &= \rho r \left(j_0 - c k_0^{abs} r^2 \left(E - 2 \frac{v F}{c} + \frac{v_i v_k}{c^2} p^{ik} \right) \right) \\
 &\quad - \rho r (k_0^{abs} + k_0^{sca}) \frac{v}{c} \cdot r \left\{ F^i + \left[\left(1 + \frac{k-1}{v^2/c} \right) \frac{v \cdot F}{c} - r k \right. \right. \\
 &\quad \left. \left. - \frac{k-1}{v^2/c} \frac{v_i v_k}{c^2} p^{ik} \right] v^i - v^i p^{ik} \right\} \\
 &= \rho r (j_0 - c k_0^{abs} E + k_0^{abs} v \cdot E) \\
 &\quad + \rho r^3 k_0^{sca} \left[\frac{v^i}{c} E + \frac{v_i v_k}{c} p^{ik} - \left(1 + \frac{v^i}{c} \right) \frac{v \cdot F}{c} \right]
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{c^2} \frac{\partial F^i}{\partial t} + \frac{\partial p^{ik}}{\partial x^k} &= \frac{\rho r}{c} \left(\frac{v^i}{c} j_0 - k_0^{abs} F^i + k_0^{abs} v_k p^{ik} \right) \\
 &\quad - \frac{\rho r}{c} k_0^{sca} \left[F^i - r^2 E v^i - v_k p^{ik} + r^2 v^i \left(\frac{2 v \cdot F}{c} - \frac{v_i v_k}{c^2} p^{ik} \right) \right]
 \end{aligned}$$

② EOM.

相対論的力学の EOM は、

$$(T_i^{\mu\nu} + \tilde{T}_i^{\mu\nu})_{;i\mu} = 0 \quad i = 1, 2, 3$$

これを $(\frac{d}{dt})$ の後にまで取ること。

$$\begin{aligned} \frac{\partial u^{\mu}}{\partial t} + (v \cdot \nabla) u^{\mu} &= -\nabla \psi - \frac{1}{\rho} \nabla P \\ &\quad - \frac{1}{\rho} \left(\frac{1}{c^2} \frac{\partial F^i}{\partial t} + \frac{\partial P^{ik}}{\partial x^k} \right) \\ &\quad - \frac{\dot{x}^2 v_i}{\rho c^2} \left[-\left(\frac{\partial E}{\partial t} + \frac{\partial \dot{x}^k}{\partial x^k} \right) + v_k \left(\frac{1}{c^2} \frac{\partial \dot{x}^j}{\partial x^j} + \frac{\partial \dot{P}^{jk}}{\partial x^k} \right) \right] \\ &= -\nabla \psi - \frac{1}{\rho} \nabla P + \frac{k_e^{abs} + k_e^{sca}}{c} (F^i - E v^i - v_k P^{ik}) \end{aligned}$$

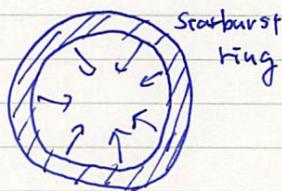
$$c^2 F^i \frac{du^i}{dt} + v_k \frac{du^i}{dr} - \frac{v^2}{r} = - \frac{1}{\Sigma} \frac{\partial \Pi}{\partial r} - \frac{GM}{r^2} + \frac{k_e \sigma_T}{mc} (F^i - E v^i - v_k P^{ik})$$

$$\frac{\partial v_k}{\partial t} + v_k \frac{du^i}{dr} + \frac{v_k v_i}{r} = - \frac{k_e \sigma_T}{mc} (F^i + E v^i + v_k P^{ik})$$

Radiative Avalanche: Starburst-induced Fueling to AGN.

- Star burst & AGN activity の 相関を 説明するモデル
- angular momentum loss by radiation drag
(Poynting-Robertson effect)

Q モデル



point: radiation drag は energy density (= 比例) と
Flux が比例して作用を得る。

- までは rotation なしの時を考える。

$$E_0(r < R) = \int_0^{2\pi} \left(\Sigma_L R \Delta R / 4\pi c l^2 \right) d\phi$$

$$\text{where } l \equiv (r^2 + R^2 - 2rR \cos\theta)^{\frac{1}{2}}$$

$$= \frac{\Sigma_L R \Delta R}{4\pi c} \times 2\pi \times \frac{1}{R-r}$$

$$= \frac{L_*}{4\pi c (R^2 - r^2)} \quad (L_* := 2\pi R \Delta R \Sigma_L)$$

$$F_0^r(r) = \int_0^{2\pi} \frac{\Sigma_L R \Delta R}{4\pi c l^2} \cos\theta \, d\phi$$

$$\begin{aligned} \text{where } \cos\theta &= (k_r \cos\phi - r)/l \\ \sin\theta &= R \sin\phi / l \end{aligned}$$

approx solution

$$\rightarrow -\frac{L_*}{4\pi R^2} \left[\frac{r}{2R} + \frac{2r^3}{\pi R^2 (R-r)} \right]$$

$(p_r^r, p_\theta^\theta, p_\phi^\phi)$

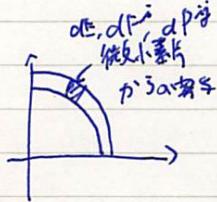
$$= \int_0^{2\pi} \frac{\Sigma_L R \Delta R}{4\pi c l^2} (\cos^2\theta, \sin\theta \cos\theta, \sin^2\theta) d\phi$$

$$= \left(\frac{E_0}{2}, 0, \frac{E_0}{2} \right)$$

- 次に Lorentz 变換して lab 系にうつす。
rotation velocity ∇ は $\frac{r}{c} \ll 1$ と考え。 $O(\frac{r}{c})$ まででよ

さて、

$$\left\{ \begin{array}{l} dE = dE_0 + 2\nabla^i dF_{,0}^i / c^2 \\ dF^{i\hat{i}} = dF_{,0}^i + dE_0 \nabla^{i\hat{i}} + \nabla^{\hat{i}} dP^{i\hat{i}} \\ dP^{i\hat{i}\hat{i}\hat{i}} = dP_{,0}^{i\hat{i}\hat{i}\hat{i}} + (\nabla^{i\hat{i}} dF_{,0}^{\hat{i}} + \nabla^{\hat{i}} dF_{,0}^{i\hat{i}}) / c^2 \end{array} \right.$$



積分すると…, $E = E_0$ (F の項は元通りに残す)

$$F^r = F_{,0}^r$$

$$F^\phi = F_{,0}^\phi + \int d\phi \frac{dE}{dr} \nabla \cos\phi + \int dP^{qr} \nabla_r + \int dP^{qr} \nabla_q$$

$$P^{rr} = P_{,0}^{rr}, \quad P^{qq} = P_{,0}^{qq}, \quad P^{rr} = P_{,0}^{rr} + \dots$$

$\Rightarrow F_\phi^r = 0$ となるに、 $F^\phi = \frac{r}{R}(E_0 + \frac{E_0}{2}) = \frac{3}{2}E_0 \frac{r}{R}$ (complicated)

- 速度 v の fluid element に作用する radiation の力は、

$$f_r = \chi (F^r - v_r E - v_r P^{rr} - v_\phi P^{r\phi}) / c$$

$$f_\phi = \chi (F^\phi - v_\phi E - v_r P^{q\phi} - v_r P^{r\phi}) / c$$

where $\chi = \frac{k + n_e \sigma_T}{\rho}$

radial : P^{rr} の項は $O(v^2/c^2)$ となるで落ちてす。

flux の項は disk を小さくする方向に働くから、角運動量には影響ない。

また、drag の項と比べて $O(\frac{v}{c})$ で大きい。

よって drag が速い、time scale² quasi-rotation balance に至る考え方。
($n \ll n_\phi$)

AM : 角運動量輸送の式は

$$\frac{1}{r} \frac{d(rn_\phi)}{dt} = \frac{3\chi E}{2c} \left(\frac{r}{R} \nabla - v_\phi \right)$$

② $x \equiv r/R$, $\tilde{v}_\phi \equiv v_\phi / D$ とする。

$$\frac{d(x\tilde{v}_\phi)}{dt} = x(x - \tilde{v}_\phi)/(1-x^2) \quad \text{where } \tilde{t} \equiv t/t_*$$

$$t_* \equiv 8\pi c^2 R^2 / 3 \propto L_*$$

$\tilde{v}_\phi \equiv x^n$ を仮定する。 $(n = -\frac{1}{2}: \text{Kepler}, n = 0: \text{M=strel}, \dots)$

$$\frac{dx}{dt} = (1-x^{n+1})/(n+1)x^{n-2}(1-x^2)$$

$$-1 < n < 1 \text{ かつ } \begin{aligned} \text{rhs} &\stackrel{x \rightarrow 0}{\approx} \frac{-x^{n+1}}{(n+1)x^{n-2}(1-x^2)} = -\frac{1}{n+1} \frac{x}{1-x^2} \sim \frac{x}{n+1} \end{aligned}$$

$$\therefore \frac{r}{R} \propto \exp[-t/(1+n)t_*]$$

t_{acc} を見出す。

一方で $n = 1$ のときは、~~全て~~ 全角運動量が輸送されない。

$$x \rightarrow 1 \text{ のときは, } \delta \equiv 1-x \Rightarrow x^{n-1} = (1-\delta)^{n-1} \sim 1 - (n-1)\delta$$

$$\Rightarrow x' = \frac{(n-1)\delta}{(n+1)\{1-(n-2)\delta\}} + 2$$

$$\rightarrow \frac{n-1}{2(n+1)}$$

$$\text{したがって, } t_{\text{acc}} = 2t_* \frac{(1+n)}{(1-n)}$$

③ perfectly ionized gas

drag force \Rightarrow compton drag : $\chi = \frac{\sigma_T}{m_p}$

$$\tau : \frac{M_p G}{\pi R_*^2 m_p} \sim 26 \left(\frac{M_p}{10^9 M_\odot} \right) \left(\frac{R_*}{100 \text{ pc}} \right)^{-2}$$

ここで M を以下のように評価する。

$$\begin{aligned} M(r) &\sim -2\pi r \sum_i n_i v_r / c(r) \\ &= \frac{3L_*}{8c^2} \frac{1-n}{1+n} \left(\frac{r}{R} \right)^2 \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} -v(r) \sim \frac{r}{2L_* (1+n)(1-n)}$$

断面積等にようつて式が得られる。

また、 $I(r) \propto r^2 + \text{const}$ と様子が着火点で起った。

$$r \sim R_* \text{ は } I \propto \max r^2. \quad \boxed{M \sim \frac{L_*}{C^2}}$$

o dust が ある場合 ...

$$\tau \sim 5000 \cdot \left(\frac{h_2}{10^{10} h_0} \right) \left(\frac{R_d}{100 \text{pc}} \right)^{-2} \left(\frac{f_{\text{ag}}}{0.001} \right) \quad \leftarrow a_d = 0.1 \mu\text{m}$$

Mはて1は1だと1がt_rは依存する。

$$n_d \sigma_d \gg n_e \sigma_T \quad (\cancel{f_{\text{ag}} \leftarrow 1 - \frac{1}{1 + f_{\text{ag}}}})$$

$$\text{さて} \quad \chi \approx n_d \sigma_d / p_g \quad t \text{の} \tau^{\circ}.$$

$$\tau_r = 2 \cdot 4 \times 10^7 \text{yr} \left(\frac{L^*}{3 \times 10^{12} L_\odot} \right)^{-1} \left(\frac{R}{100 \text{pc}} \right)^2 \left(\frac{f_{\text{ag}}}{0.001} \right)^{-1} \left(\frac{a_d}{0.1 \mu\text{m}} \right)$$

-¹, viscous timescale

$$\tau_{\text{visc}} \sim \tilde{\tau} / 2(1-h) \alpha c_s^2 \quad \tilde{\tau} : \text{specific AM} \sim RD \sim (GMR)^{\frac{1}{2}}$$

$$\approx 3.2 \times 10^8 \text{yr} \left(\frac{1}{1-h} \right) \left(\frac{1}{\alpha} \right) \left(\frac{M}{10^{10} M_\odot} \right) \left(\frac{R}{100 \text{pc}} \right)^{\frac{1}{2}} \left(\frac{T_g}{10^4 \text{K}} \right)^{-1}$$

$$t \text{の} \tau^{\circ} \quad 10 \text{pc} < R < 100 \text{pc} \quad \text{511715.} \quad \tau_r < \tau_{\text{visc}} \tau^{\circ}$$

BH to Bulge Relations

① radiation density originating from bulge stars.

$$E \simeq L_* / c R^2$$

$\Rightarrow \propto r^{-2}$, radiation drag $\propto r^2$ と重力に逆比例する。

$$\frac{d \ln J}{dt} \simeq -x E/c \quad (\text{今 } x = \kappa/\rho)$$

$$\begin{cases} \text{optically thin} \\ \rightarrow = -\frac{\kappa \rho K}{c^2} \cdot \frac{L_*}{c R^2} \sim -\frac{\tau L_*}{c^2 M_g} \\ \text{thick} \\ \rightarrow = -\frac{L_*}{c^2 M_g} \end{cases}$$

$$\text{合計 } = -\frac{L_*}{c^2 M_g} (1 - e^{-\tau})$$

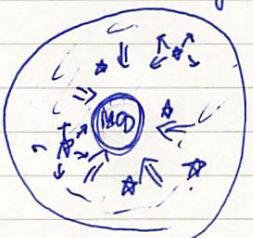
F. 2.

$$\dot{M} = -M_g \frac{d \ln J}{dt} \simeq \frac{L_*}{c^2} (1 - e^{-\tau}) \sim 0.1 \text{ Mo/yr} \left(\frac{L_*}{10^{12} L_\odot} \right)$$

(10^8 to 10^{10} Eddington units)

$$M_{\text{HOO}} = \int_0^t \dot{M} dt \simeq \int_0^t \frac{L_*}{c^2} dt$$

・次に Bulge の進化を考へ、 L_* を評価する。



$$\frac{df_g}{dt} = -S(t) + F(t)$$

(instantaneous recycling approx.)
(SN, wind etc.)

- instantaneous recycle approx.

$$F(t) = S(t) (1 - \alpha)$$

$$\Rightarrow \boxed{\frac{df_g}{dt} = -\alpha f_g} \quad \cdot \text{ Schmidt law}$$

$$S(t) = k f_g$$

$$\therefore f_g = e^{-\alpha k t}, \quad M_* / M_b = S(t) = k e^{-\alpha k t}.$$

星の一生

- MS中に放出されるエネルギーは

$$0.14 \varepsilon \cdot m c^2 \quad \text{where } \varepsilon : \text{energy conversion eff of nuclear fusion}$$

$$= 0.007$$

$$\Rightarrow L_* = 0.14 \varepsilon M_b c^2 \cdot k e^{\alpha_{\text{eff}} t}$$

$$\therefore M_{\text{bulge}} = 0.14 \varepsilon \alpha^{-1} M_b (1 - e^{-\alpha_{\text{eff}} t})$$

Stellar mass in the bulge

よし

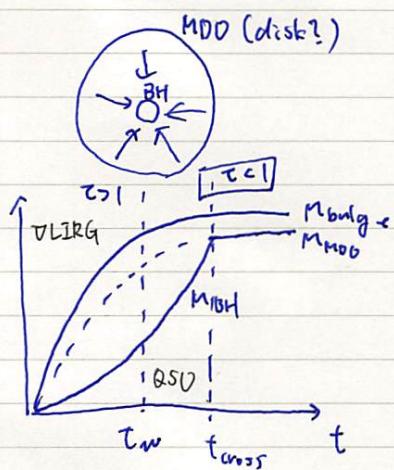
$$\cancel{M_{\text{bulge}} / M_{\text{bulge}}} = 0.3 \varepsilon \alpha_{0.5}^{-1}$$

$$= 0.002 \alpha_{0.5}^{-1}$$

(時刻や α に注意!!)

BH growth

上で議論した MDO が BH とは限らない。BH の降着が Eddington 限界に達するまで考慮して。



$$M_{\text{BH}} = M_0 e^{r_{\text{eff}} t_{\text{Edd}}}$$

\uparrow
seed mass

t_{Edd} optically thin になると
左図のようにシナリオ 1 に移る。