12/3/2010 seminar@RESCEU

Cosmic Microwave Background from Cosmic Strings/Cosmic Superstrings

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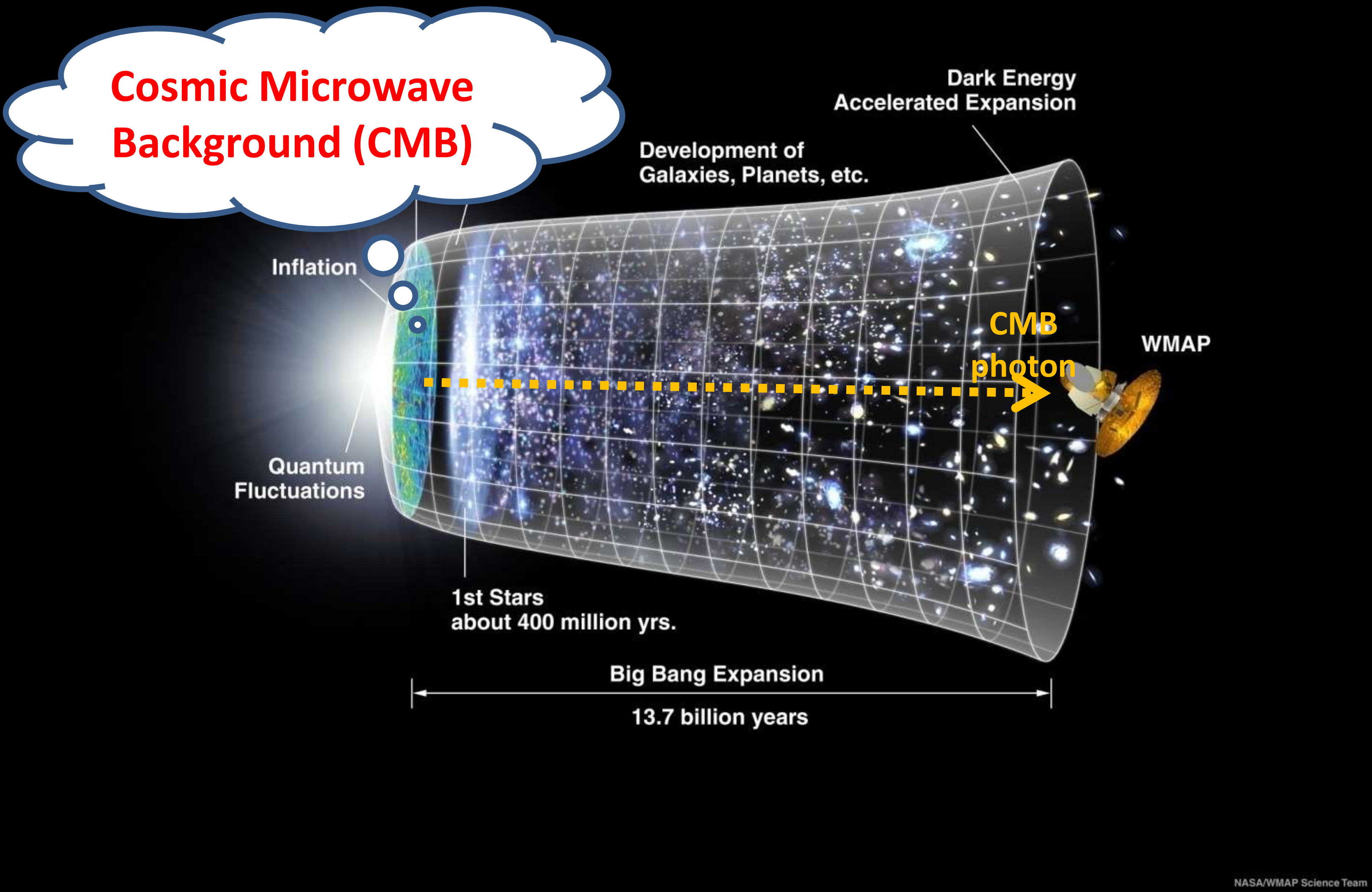
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

- in prep.
- PRD82, 063518 (2010), 1006.0687[astro-ph.CO]
- JCAP05,033 (2010), 1004.0600[astro-ph.CO]
- JCAP10,003 (2009), 0811.4698 [astro-ph]

0.1 : Standard cosmological model



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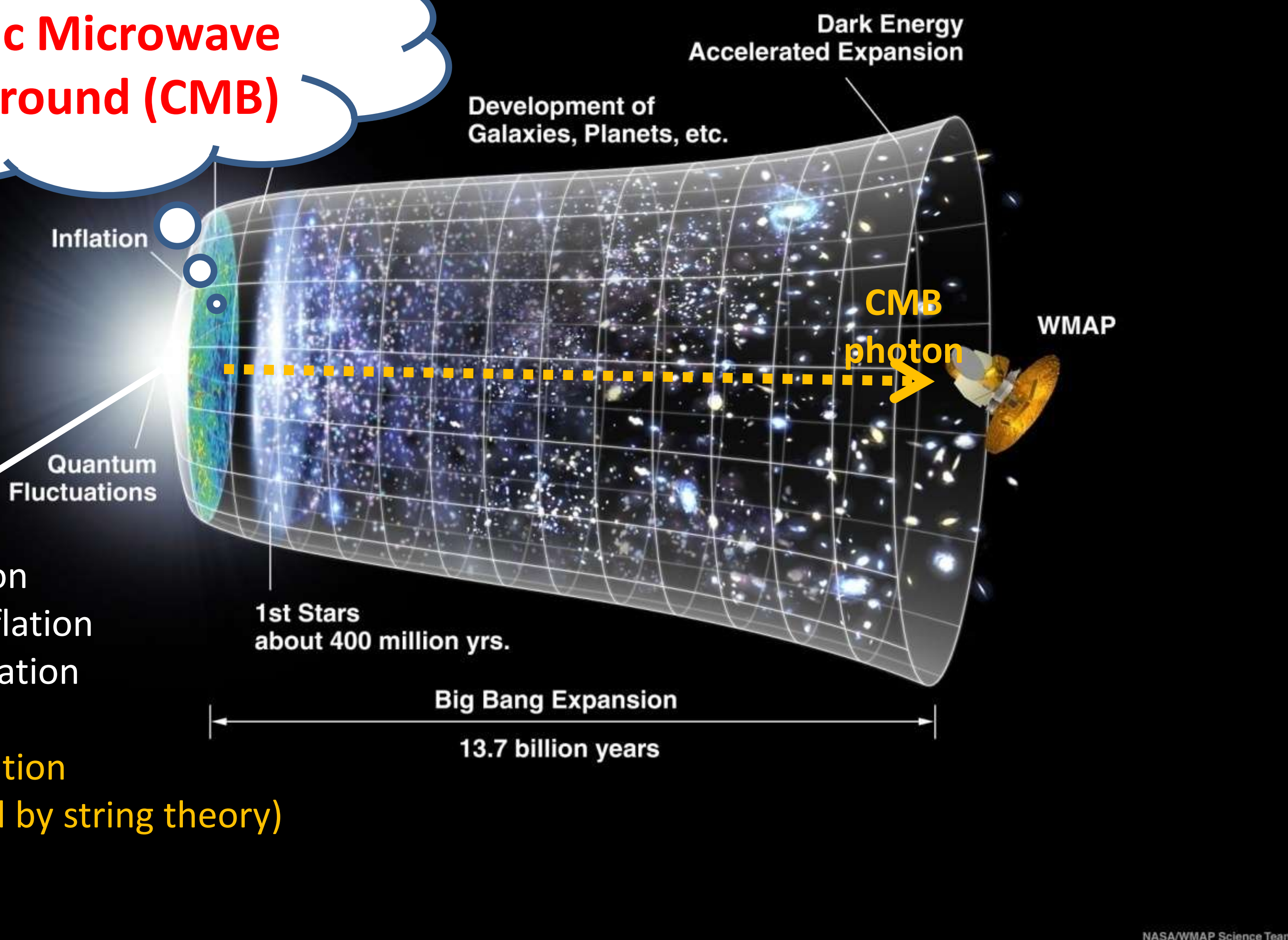
Cosmic Microwave Background (CMB)



Old inflation
Chaotic inflation
Hybrid inflation
…
Brane inflation
(motivated by string theory)

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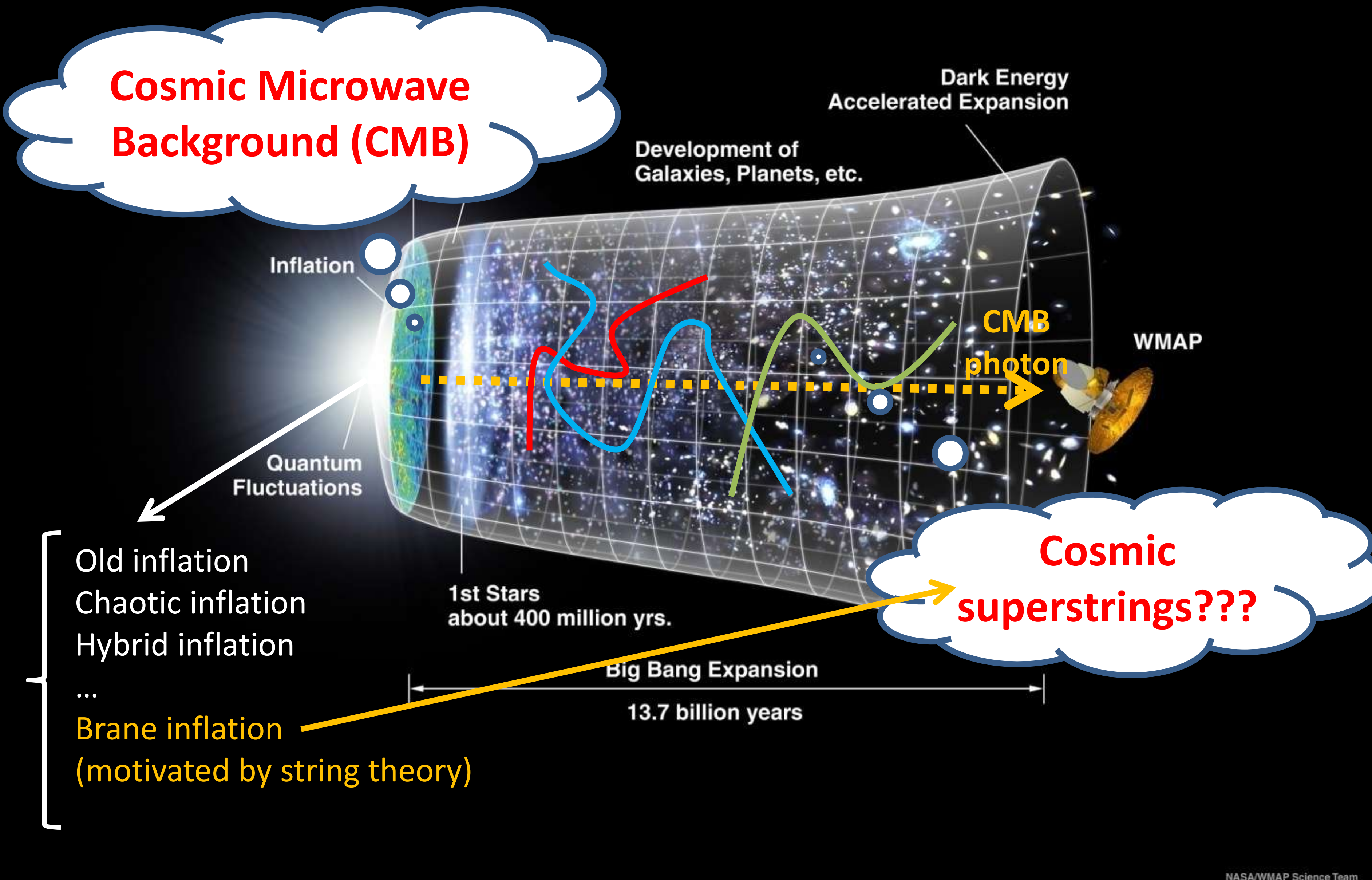

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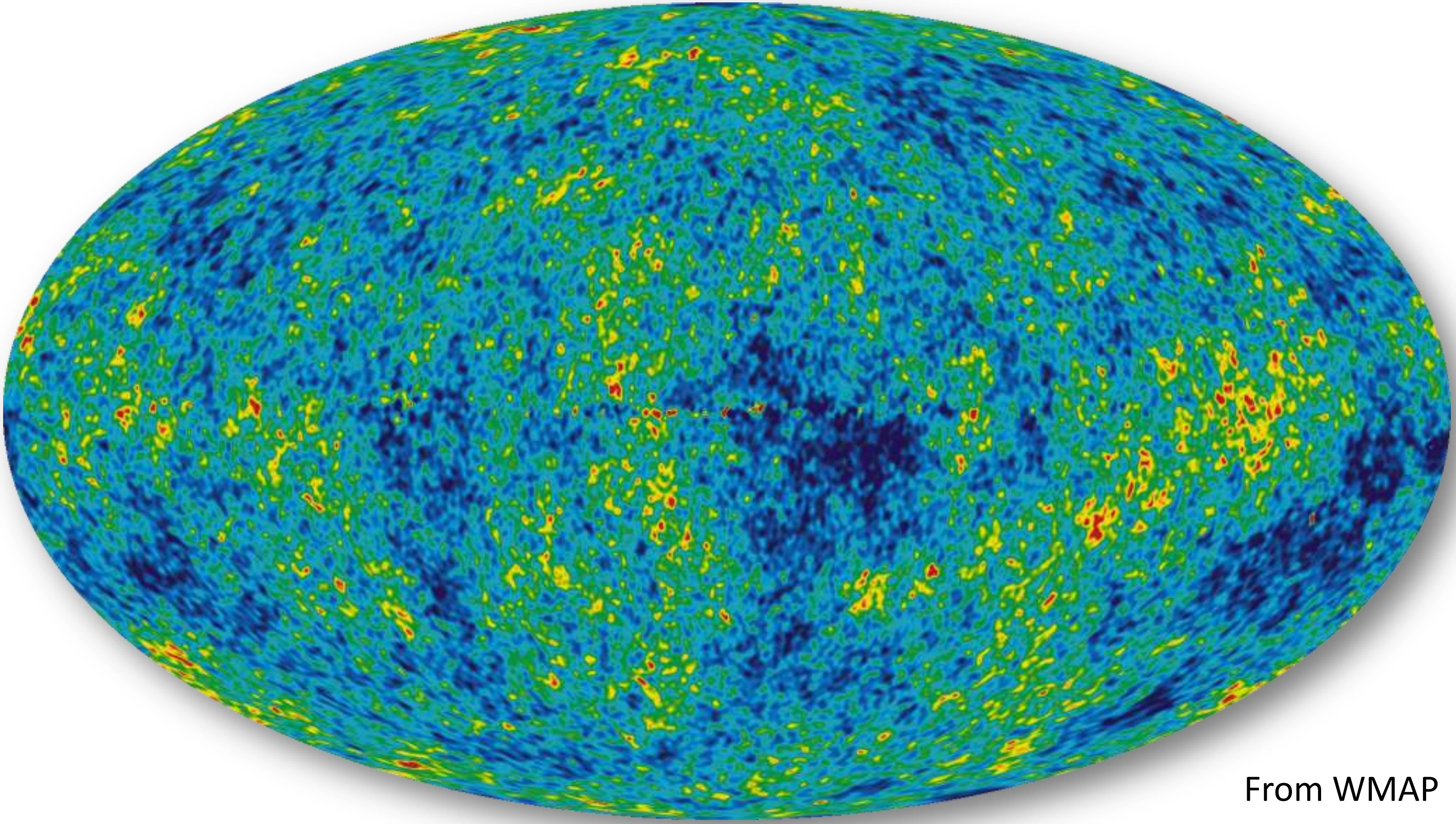
0.2 : CMB sky

Cosmic Microwave Background gives a unique window for understanding the early universe and high energy physics.



From WMAP

0.2 : CMB sky

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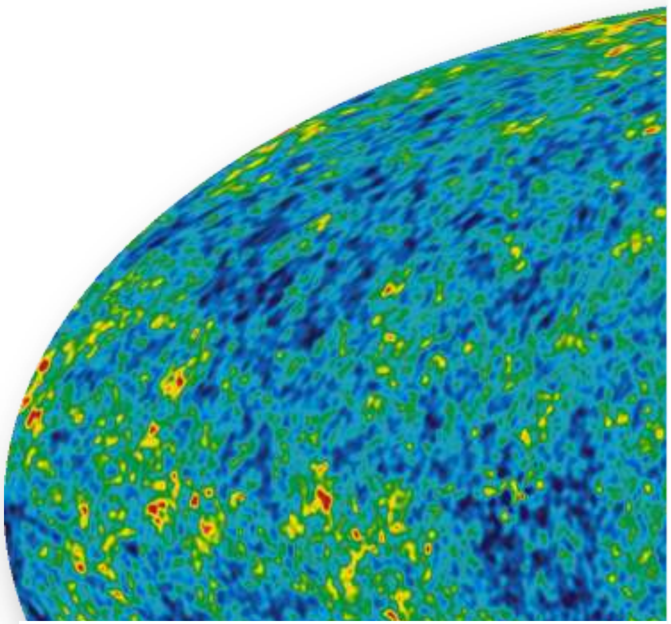
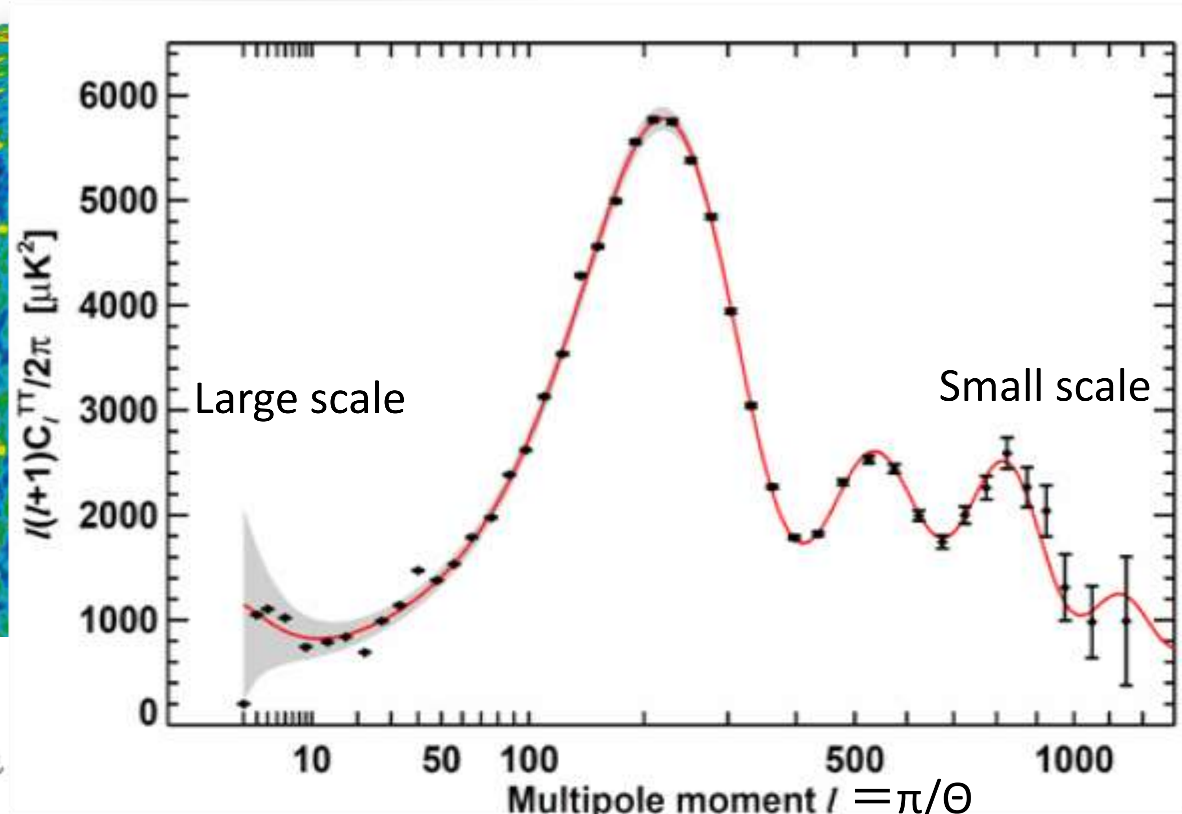
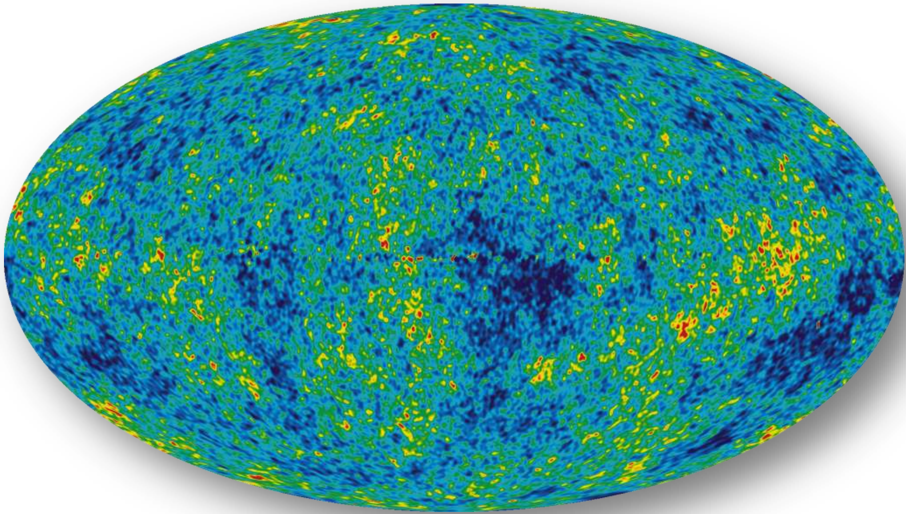




$$a_{\ell m} = \int d\mathbf{n} \frac{\Delta T}{T} Y_{\ell m}^{*}$$

$$C_\ell = \frac{1}{2\ell+1} \sum_m \langle a_{\ell m} a_{\ell m}^{*} \rangle$$

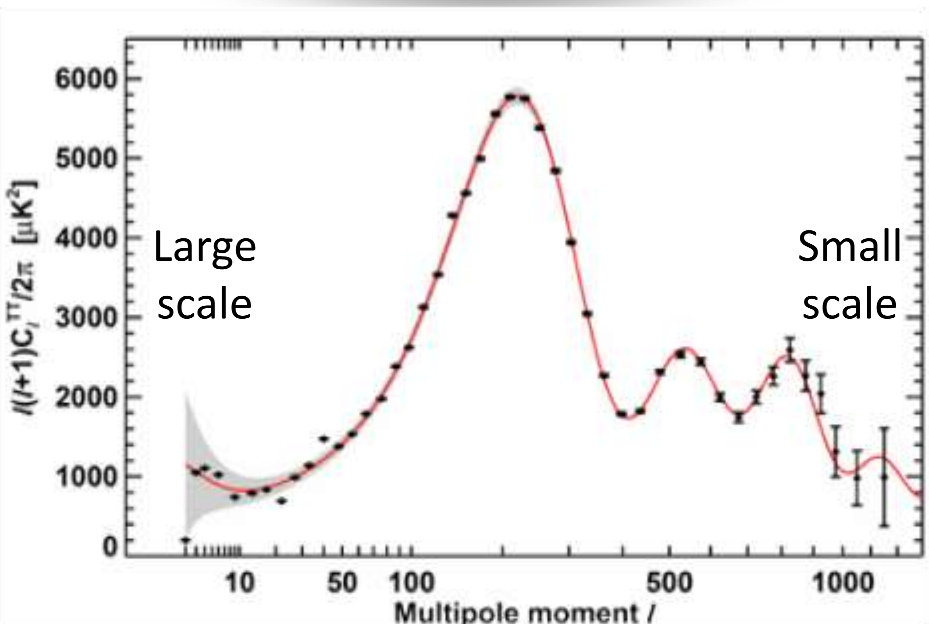
From WMAP



✓WMAP (l<1500)

[Komatsu et al. ('10)]

- ➢almost flat : $\Omega_{0,obs} \sim 1$
- ➢ almost scale invariant
- ➢almost(?) Gaussian fluctuations



✓ small scale observation
✓ polarization
✓ non-Gaussianity

Plan

Part 1 : cosmic "standard" strings and superstrings

- ✓What are cosmic strings / cosmic superstrings?
- ✓ Evolution of cosmic (super-)strings network

Part 2 : CMB from cosmic (super-)strings

- ✓ Weak lensing due to strings and CMB polarization
- ✓(Analytic formula for string TT angular power spectrum)

Part 1 : cosmic “standard” string and superstrings

Question:

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- Line-like topological defect.
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- A probe of phase transitions in the early universe.

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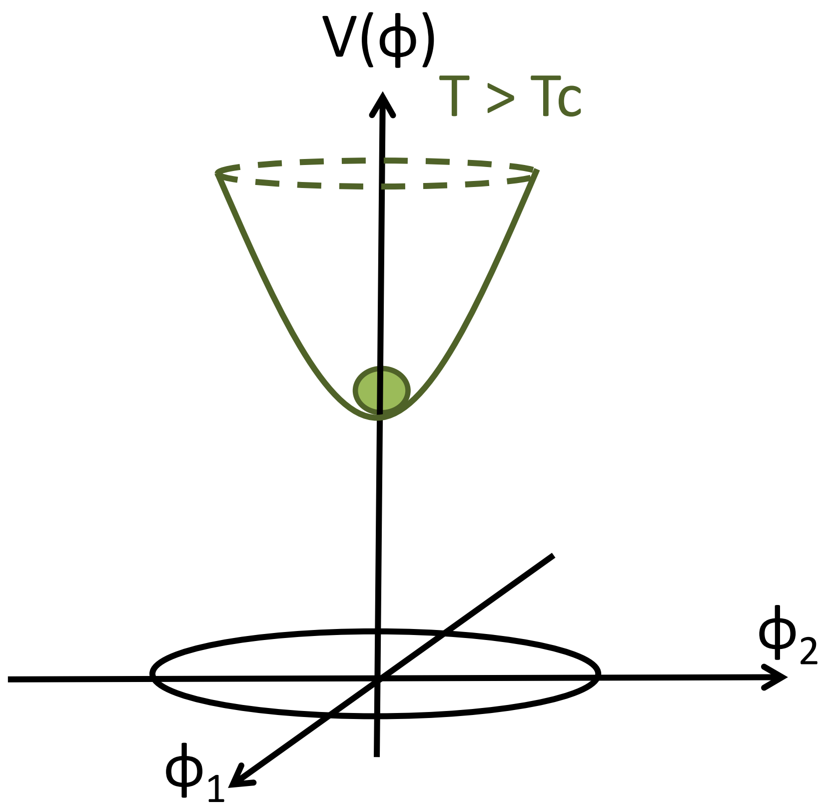
- Fundamental objects in string theories, such as F-strings or D-branes.
- Formed in the early universe through "collisions of D-branes".
- A new probe of very high energy physics, i.e. string cosmology !!!

1.1 : "Standard" field theoretic cosmic strings

→ The non-trivial phase mapping from the internal space to the physical space leads to the formation of a cosmic string. [Kibble (1976)]



①At T>Tc, the fluctuations around Φ=0.

②The expansion and cooling of the universe leads that U(1) sym is broken spontaneously.

③At each spacetime location, the phase θ must be randomly chosen and uncorrelated on the horizon scale.

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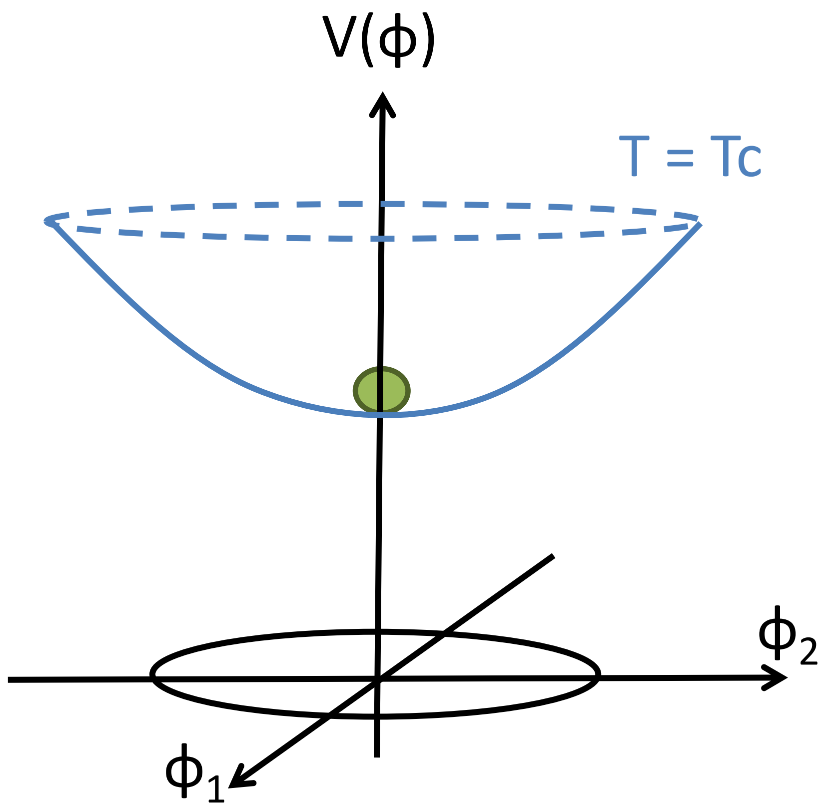

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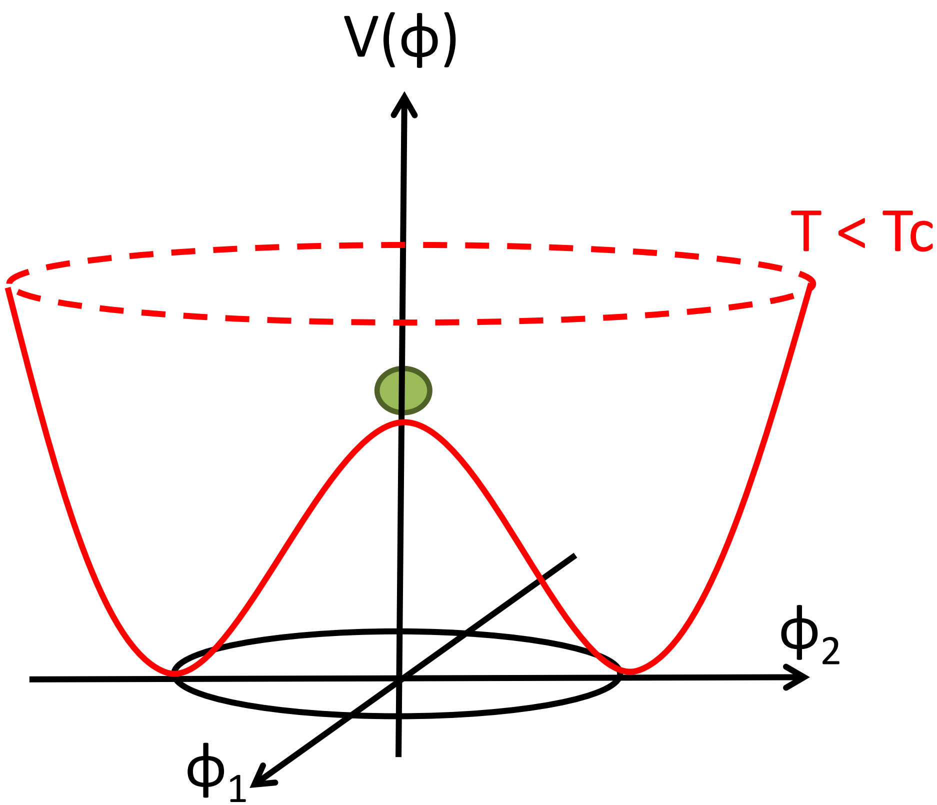

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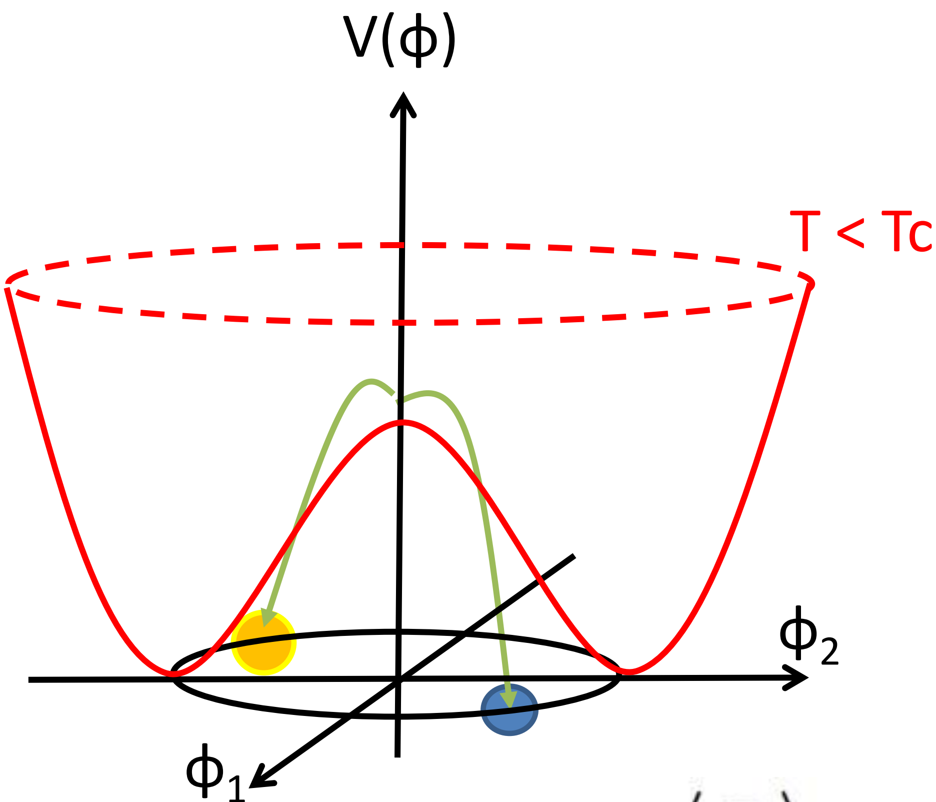

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$$\langle \Phi \rangle = \eta_V e^{i\vartheta}$$

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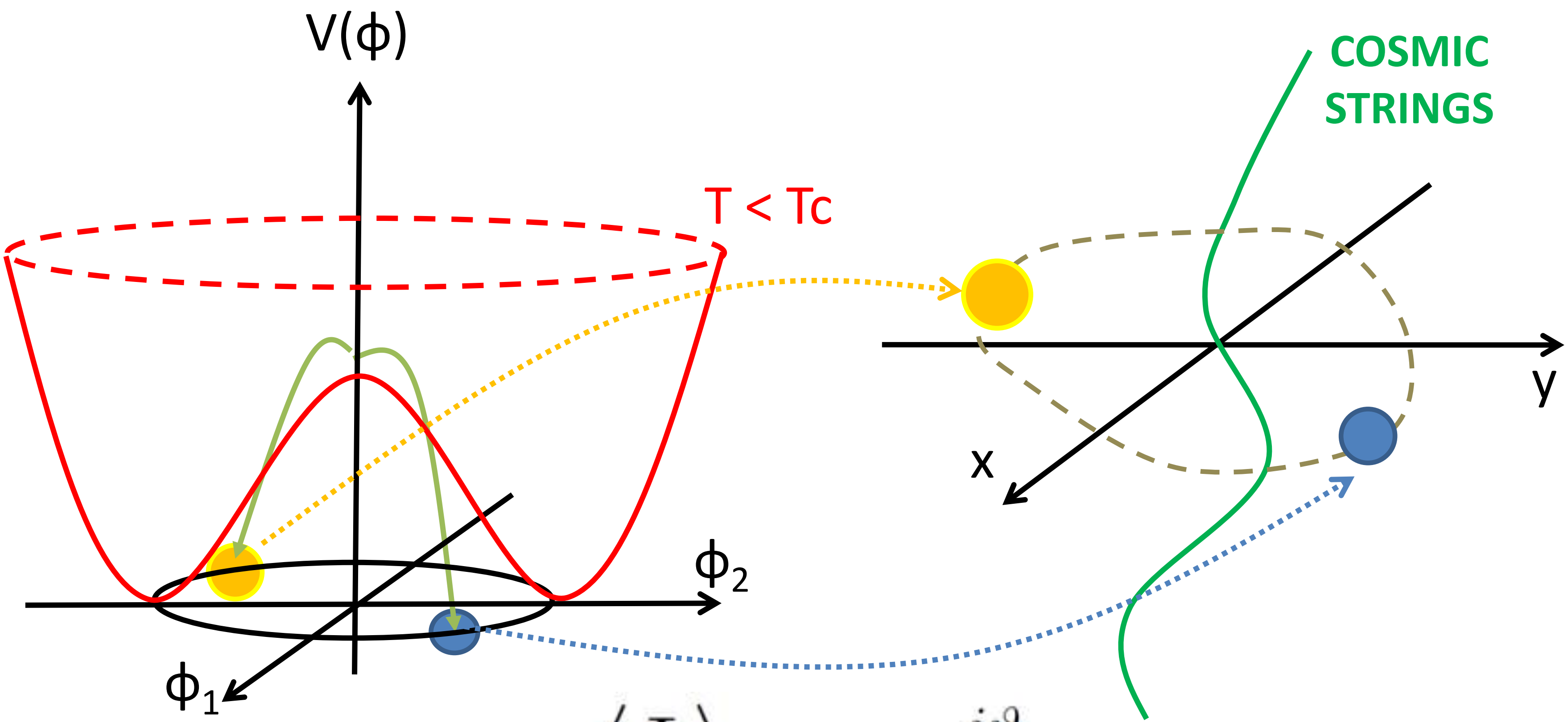
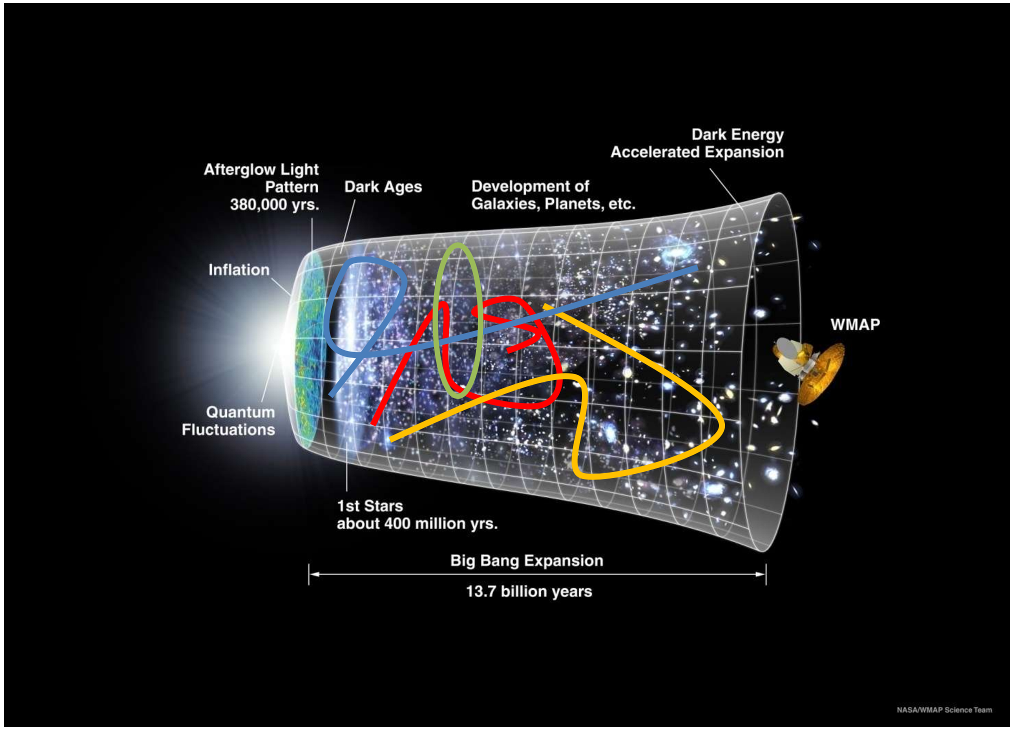


$$\langle \Phi \rangle = \eta_{\rm V} e^{i\vartheta}$$

There is no direct evidence for their existence. However, there are good theoretic reasons for believing that these exotic objects exists!

The “tension” of the string , “μ”, is directly related to the symmetry breaking energy scale :

$$\boxed{G\mu \approx \frac{M_{\rm PT}^2}{M_{\rm pl}^2},}$$

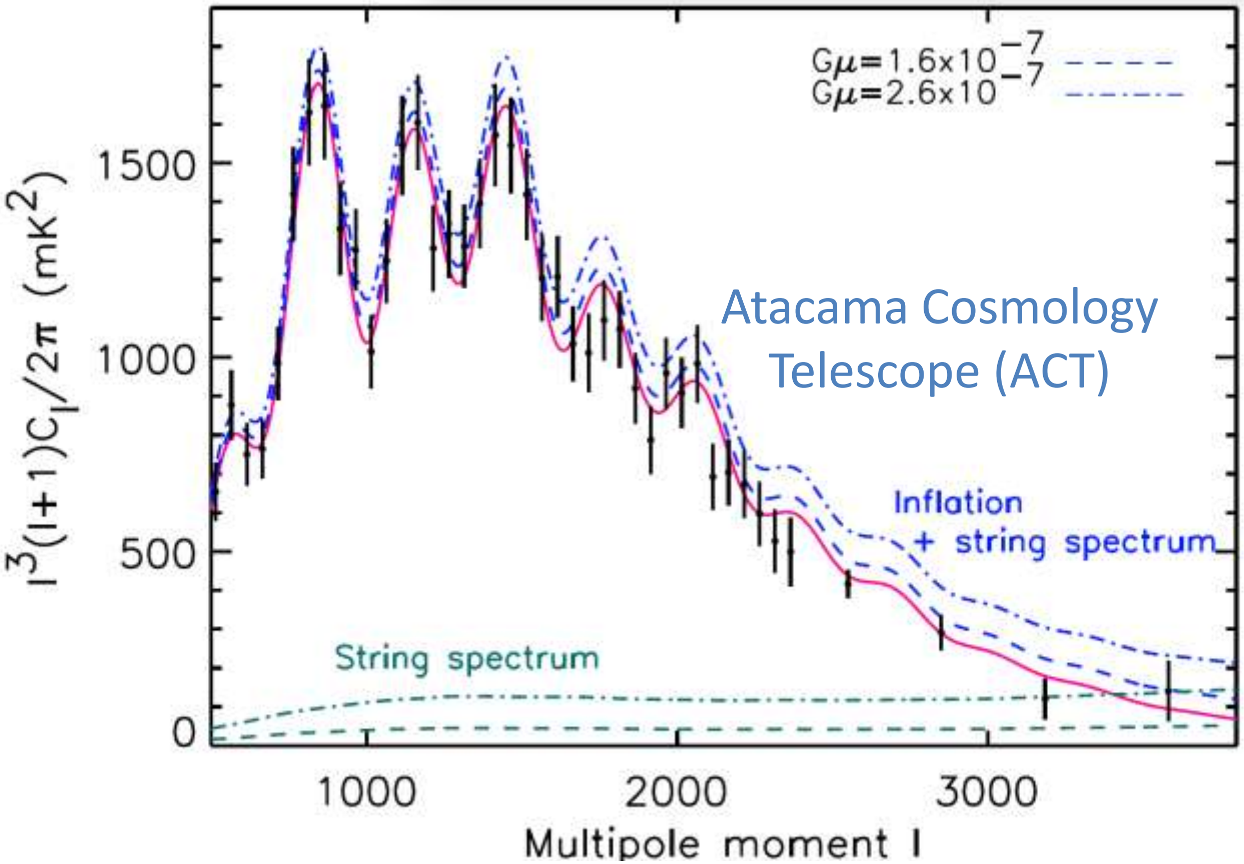


Observational verification of the existence of cosmic strings will have a profound implications to unified theory !

1.2 : CMB constraints for "standard" cosmic strings

- Unusual gravitational properties gives a characteristic stringy signature on CMB.

$$G\mu < 1.6\times 10^{-7}\ (95\%\mathrm{CL})$$



[Dunkley+ (ACT), 2010]

At small scale where the primary fluctuations damped, the signal due to cosmic strings could be observable!

$$\ell(\ell+1)C_\ell^{\Theta\Theta} \propto \ell^{-1}$$

[Hindmarsh(1994), Hindmarsh, Ringeval, Suyama (2009), DY+(2010b)]

[see also Bevis+ (2008),(2010), Pogosian+ (2009), Battye, Moss (2010),..]

1.3 : COSMIC SUPERSTRINGS

[review: Polchinski(2005), Davis+Kibble (2005), Copeland+Kibble (2009), Sakellardiadou(2009), Majumdar (2008)]

Witten [Witten(1985)] argued that cosmic strings are fundamental quantum strings and they could have been in the early universe and stretched to macroscopic scale with the expansion of the universe.

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①If stable, one would expect strings to be at a energy scale close to Planck scale !

$$G\mu_{\rm F} \approx \frac{m_{\rm s}^2}{M_{\rm pl}^2} \approx \mathcal{O}(1) \longrightarrow$$

These strings are naturally ruled out from the current observations.

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To make cosmic sized cosmic superstrings realistic objects, we need to introduce new idea.

① Warped geometry

➢ As is familiar from Randall-Sundrum, to make cosmic strings much lighter is to make 4-dimensional constants dependence on the extra dimensions.

[Copeland, Myers, Polchinski 2003]

$$ds^2 = e^{2A(y)} g^{(4)}_{\mu\nu}(x) dx^\mu dx^\nu + e^{-2A(y)} g^{(6)}_{mn}(y) dy^m dy^n$$

Warping gives the significant contributions to the quantities depending on the metric such as the stress-energy tensor:

$$\Rightarrow \quad T_{\mu\nu} = -\mu_s e^{2A(y)} g^{(4)}_{\mu\nu} \delta^8(x, y)$$

$$\boxed{\mu_{\text{eff}} = \mu e^{2A(y)}}$$

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Warping give … g
on the metri…

effective tension
= a probe of the internal space

$$\Rightarrow \quad T_{\mu\nu} = -\mu_s e^{2A(y)} g^{(4)}_{\mu\nu} \delta^8(x, y)$$

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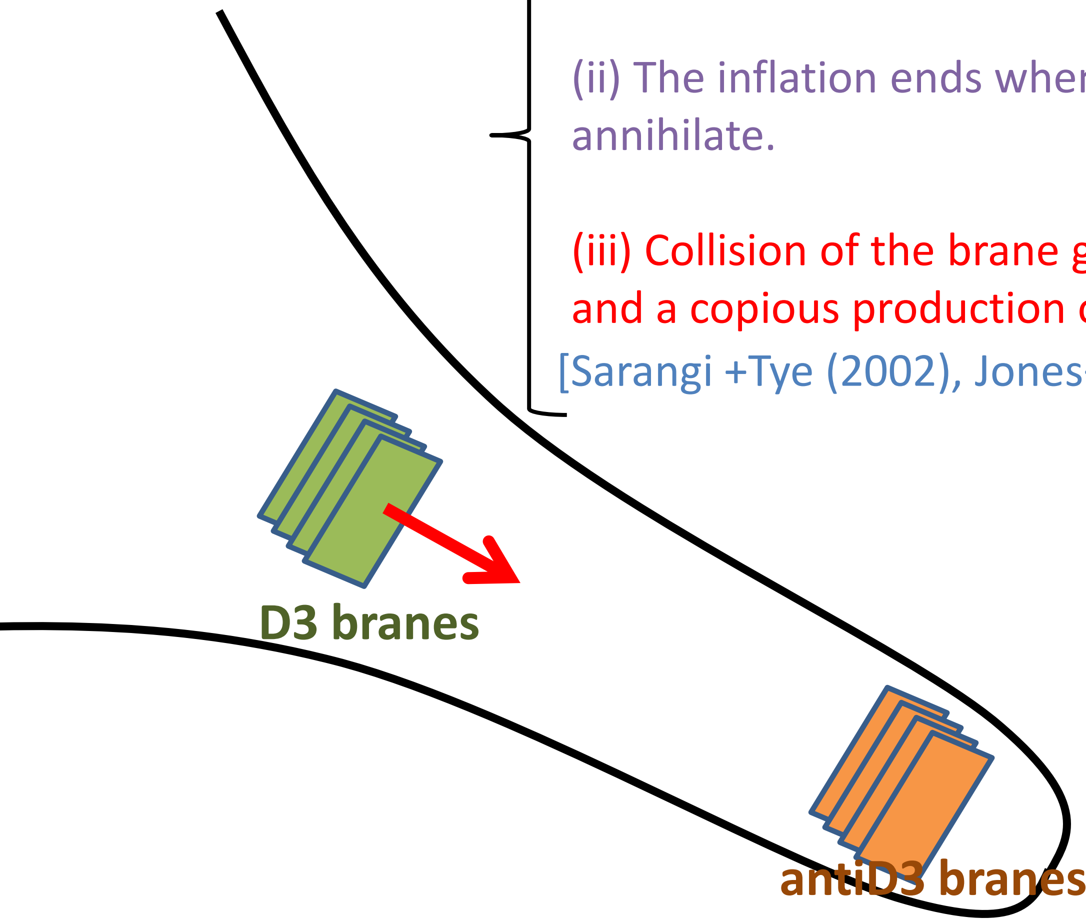
② Brane Inflation [Kachru+(2003), Dvali+Tye(1999), Burgess+(2001)]

provides a consistent scenario incorporating inflation, graceful exit, reheating and also possible production of cosmic superstrings.

(i) The inflation is driven by the attractive force between D branes and anti D branes.

(ii) The inflation ends when the brane collides and partially annihilate.

(iii) Collision of the brane gives a possible reheating process and a copious production of various lower dimensional objects.
[Sarangi +Tye (2002), Jones+Stoica+Tye(2003), Dvali+Vilenkin(2004)]



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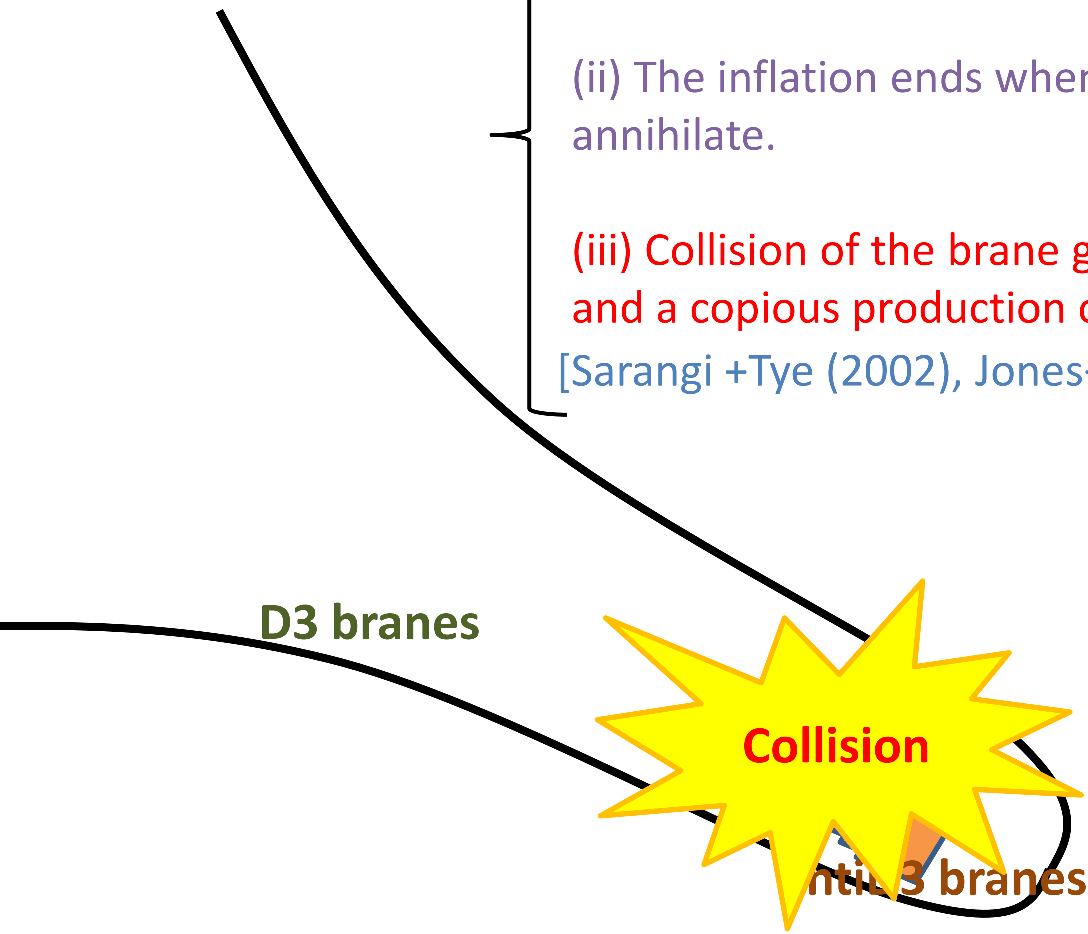

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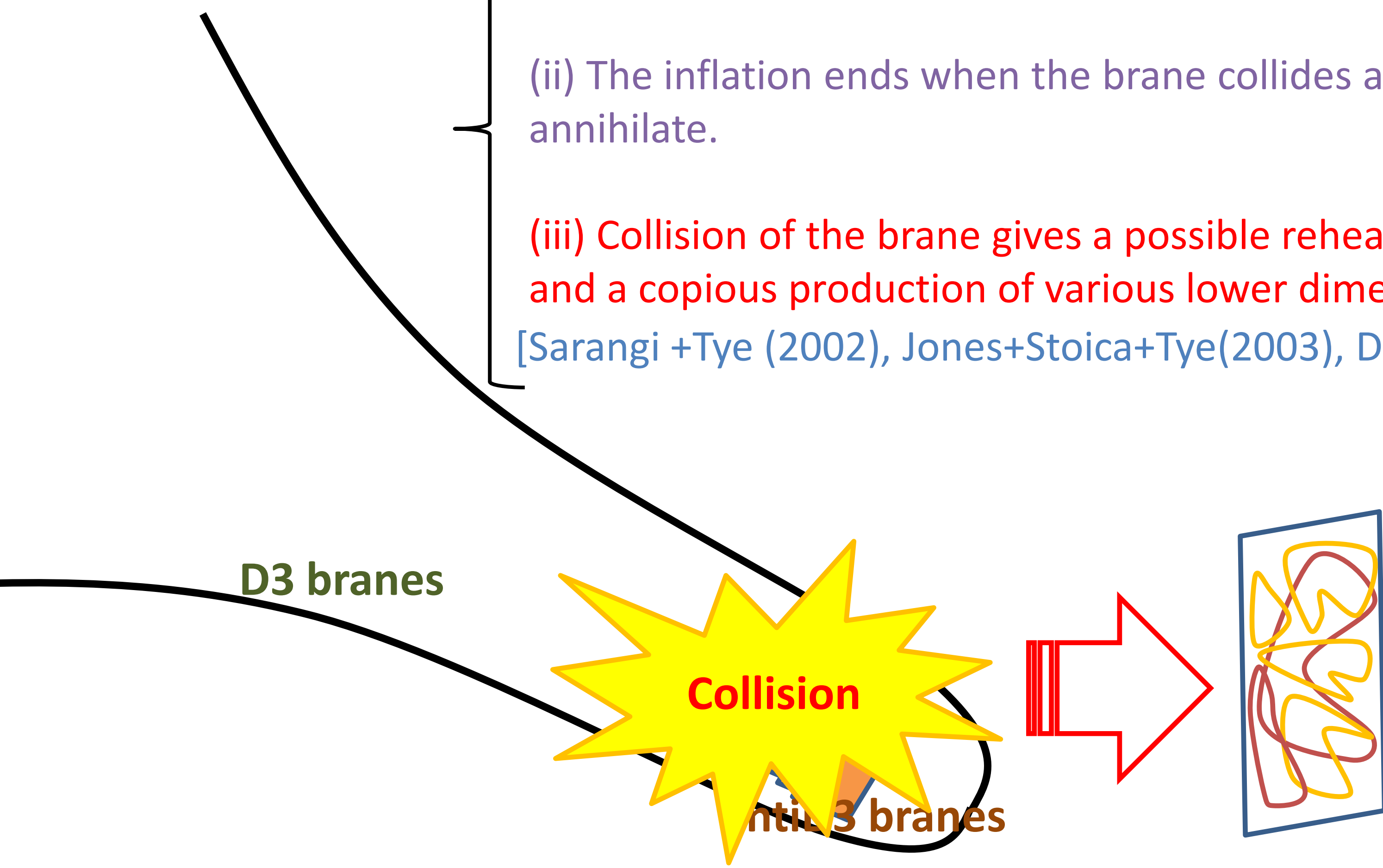
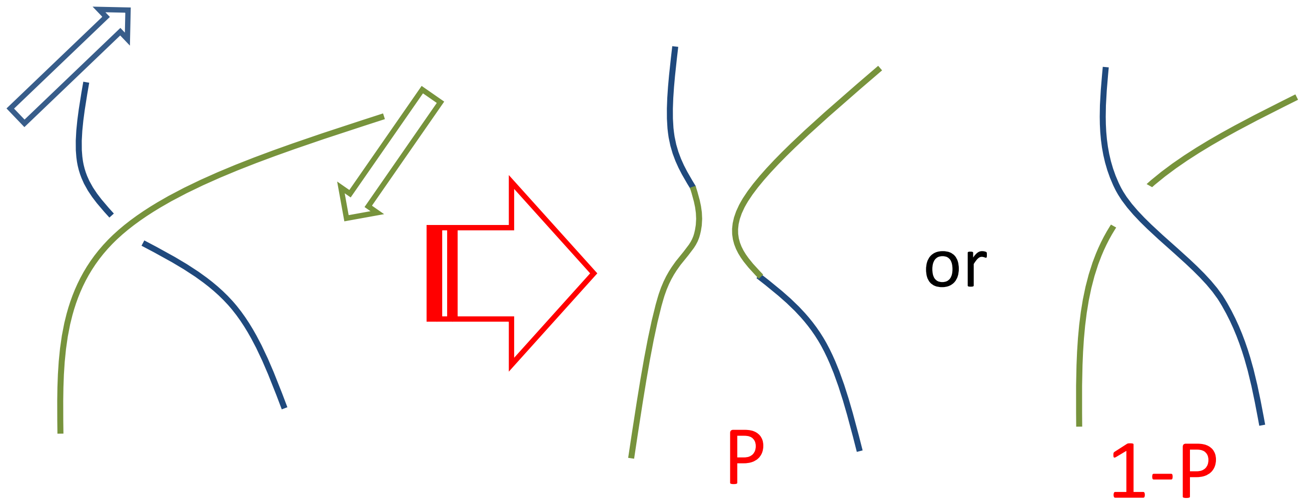


- D1-strings
- closed F-string

There are some good theoretical reason for believing realistic cosmic superstrings exists, but …

Question:
Can we distinguish COSMIC SUPERSTRINGS from CONVENTIONAL STRINGS in observations?

⇒ INTERCOMMUTING PROBABILITY "P"



- “standard” field theoretic strings (v<vc~0.98)

  : $P = 1$

[numerical: Shellard(1987), Matzner(1988), Moriarty+(1988),
Achucarro+de Putter(2006), Achucarro+Verbiest(2010)]
[analytic: Eto+ (2007), Hashimoto+Tong (2005), Hanany+Hashimoto(2005)]

- cosmic superstrings

  : $P \ll 1$

[Polchinski(1988),
Jackson, Jones, Polchinski (2005),
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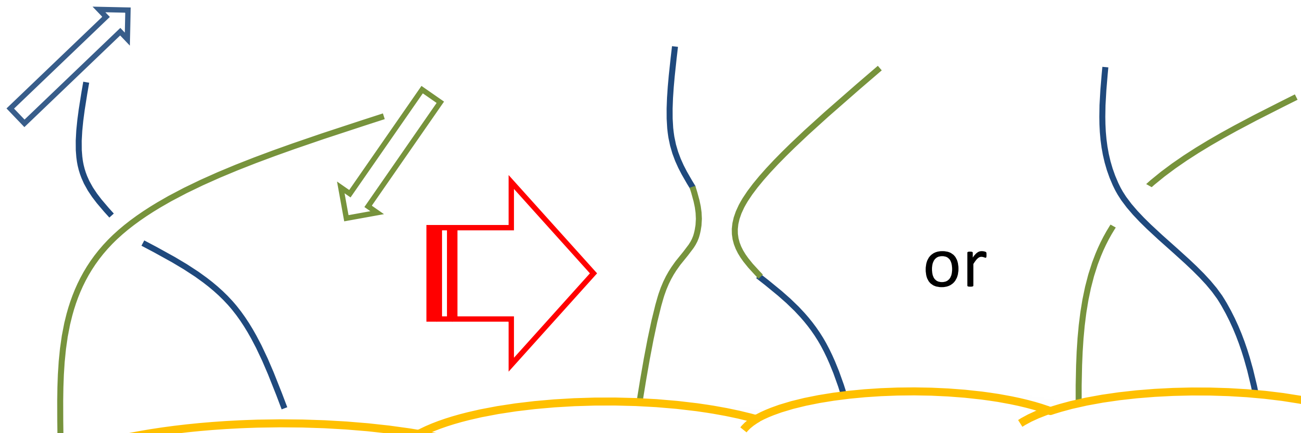
F-F	$10^{-3}<P_{FF}<1$
D-D	$10^{-1}<P_{DD}<1$
(p,q)-(p’,q’)	More complicated



intercomuting probability P
= a probe of the fundamental interactions

[numerical ... arty+(1988),
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[analytic: Eto+ (2007), Hashimoto+Tong (2005), Hanany+Hashimoto(2005)]

- cosmic superstrings

: P << 1

[Polchinski(1988),
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F-F	$10^{-3}<P_{FF}<1$
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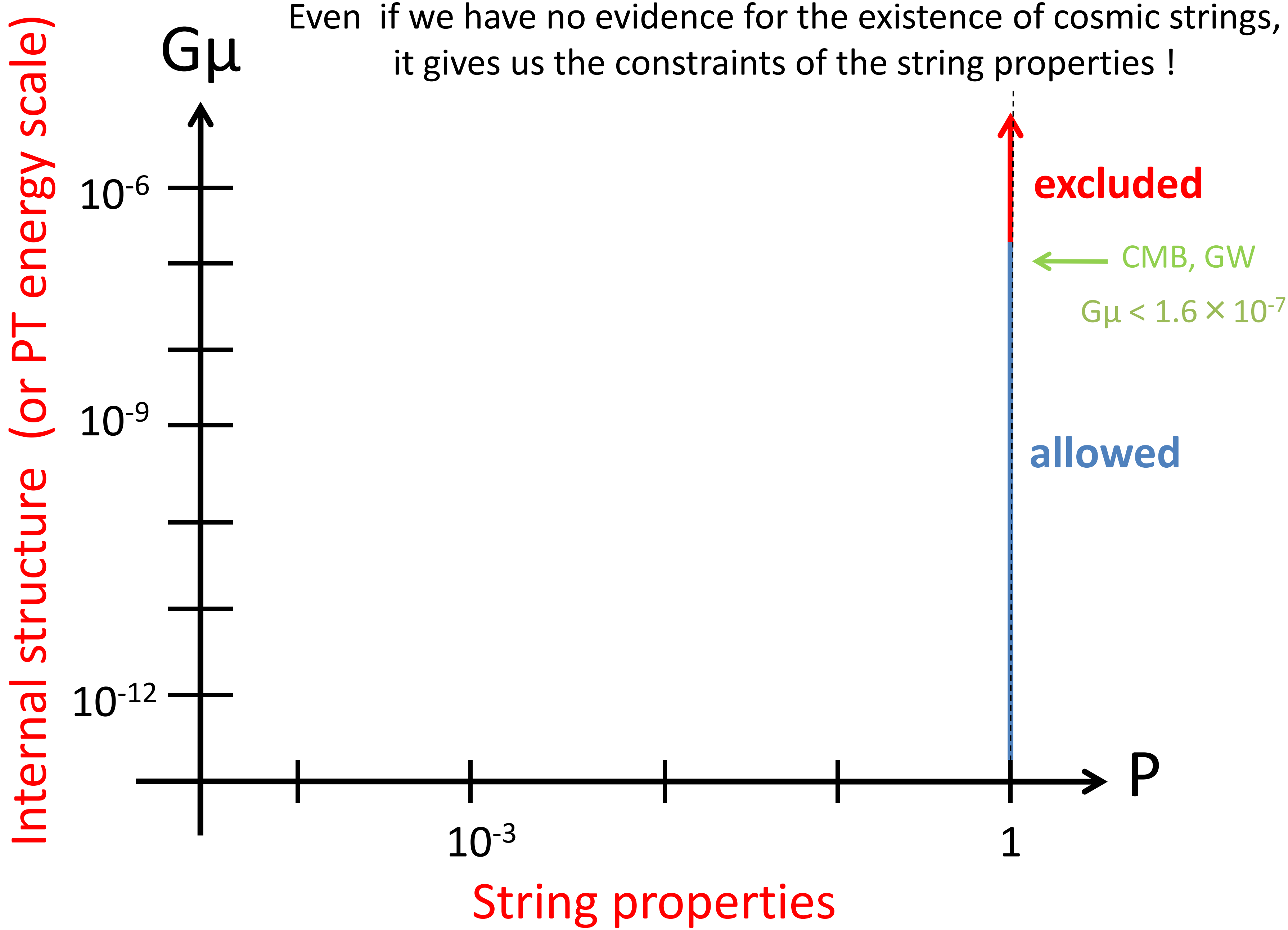
Even if we have no evidence for the existence of cosmic strings,
it gives us the constraints of the string properties !



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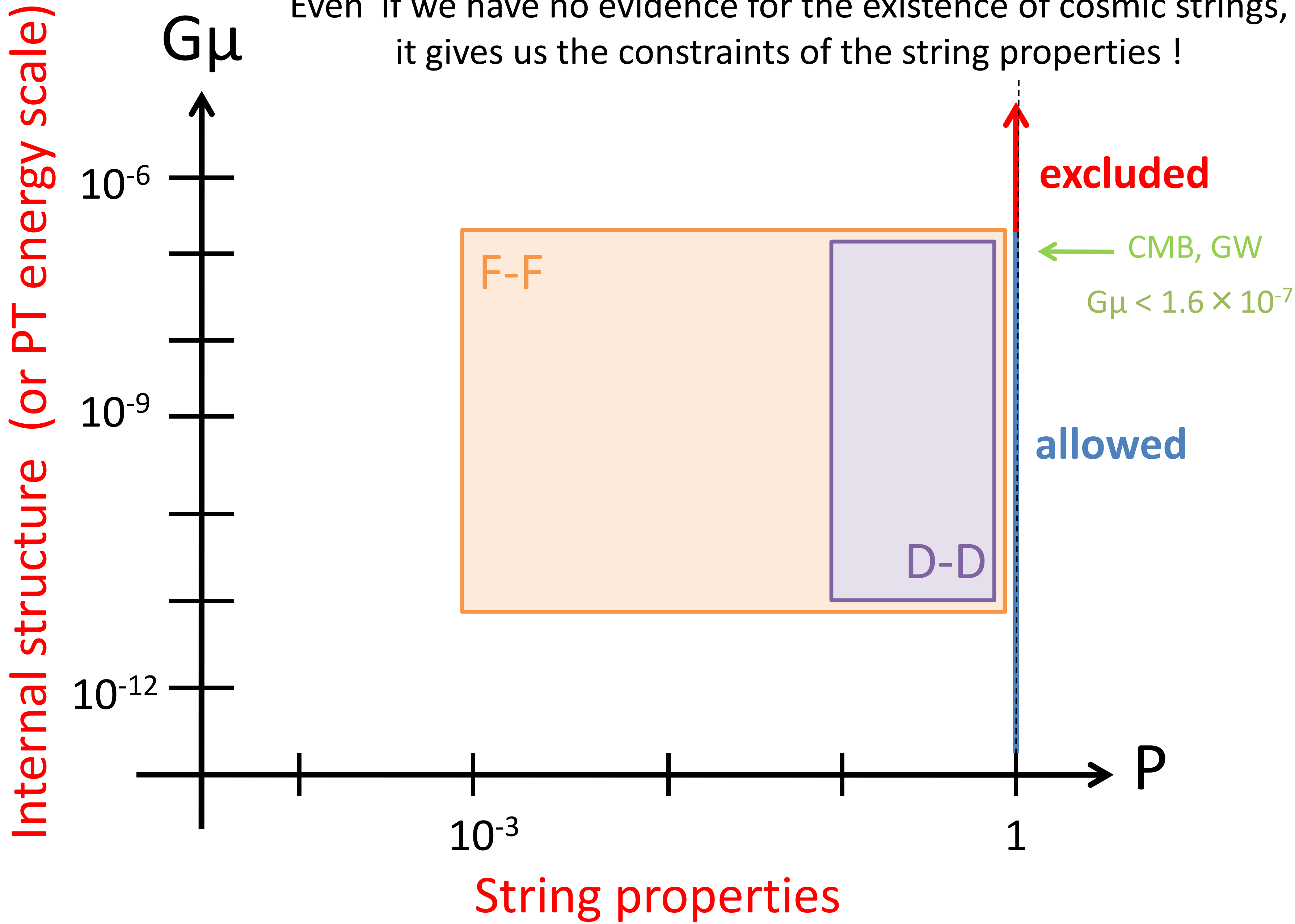

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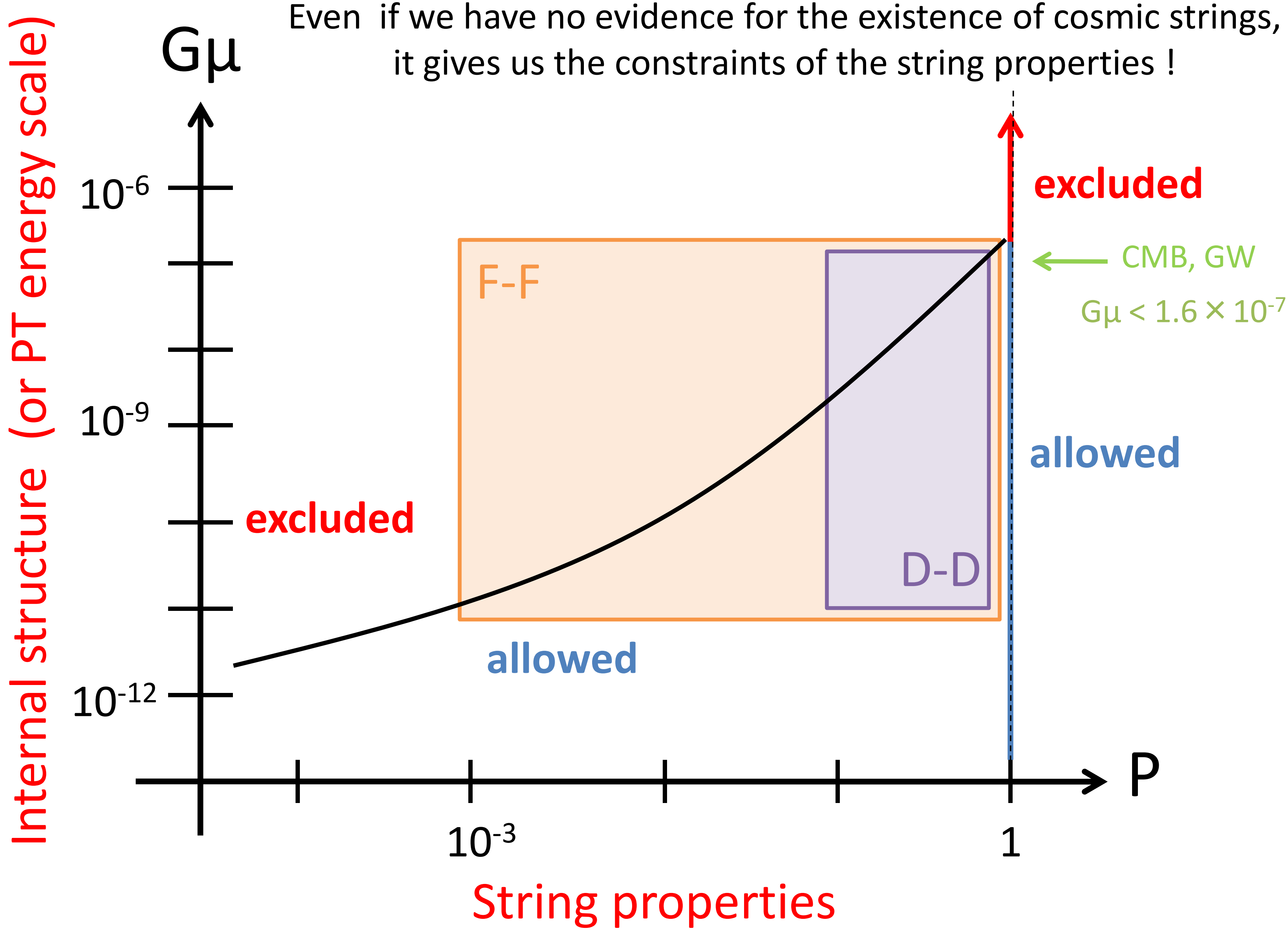

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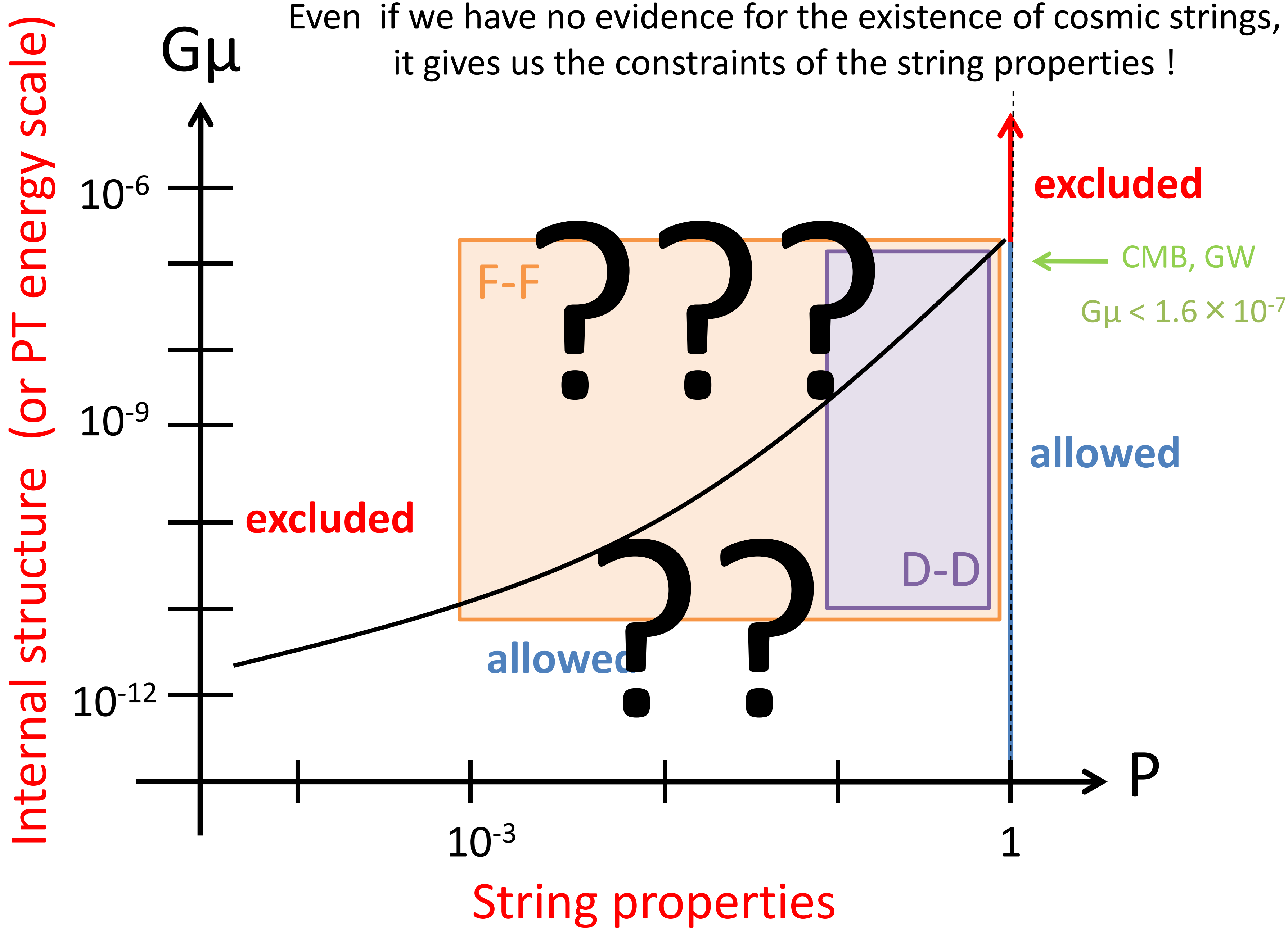


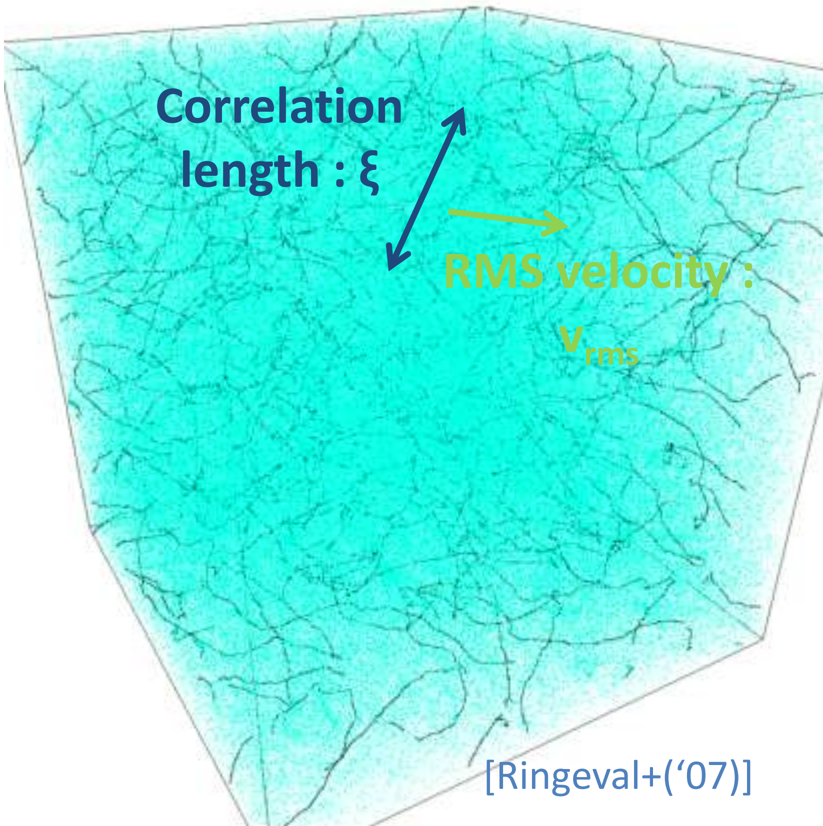
Evolution of string network

- Takahashi, Naruko, Sendouda, **DY**, Yoo, Sasaki, JCAP 0910, 003 (2009), arXiv:0811.4698
- work in progress with Hiramatsu and Nakao

1.5 : Analytic model ; Velocity-dependent one-scale model

➢ A string network is assumed to consist of string segment with the correlation length ξ , and the root-mean-square velocity v_{rms} :

$$\rho_{\rm str} = \frac{1}{\xi^3} \times \mu\xi = \frac{\mu}{\xi^2} \qquad \xi = \frac{1}{H\gamma}$$



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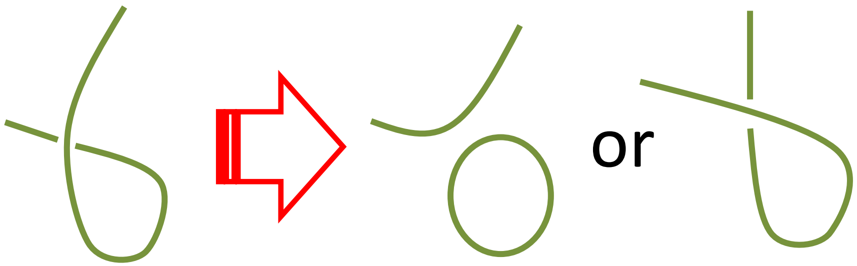
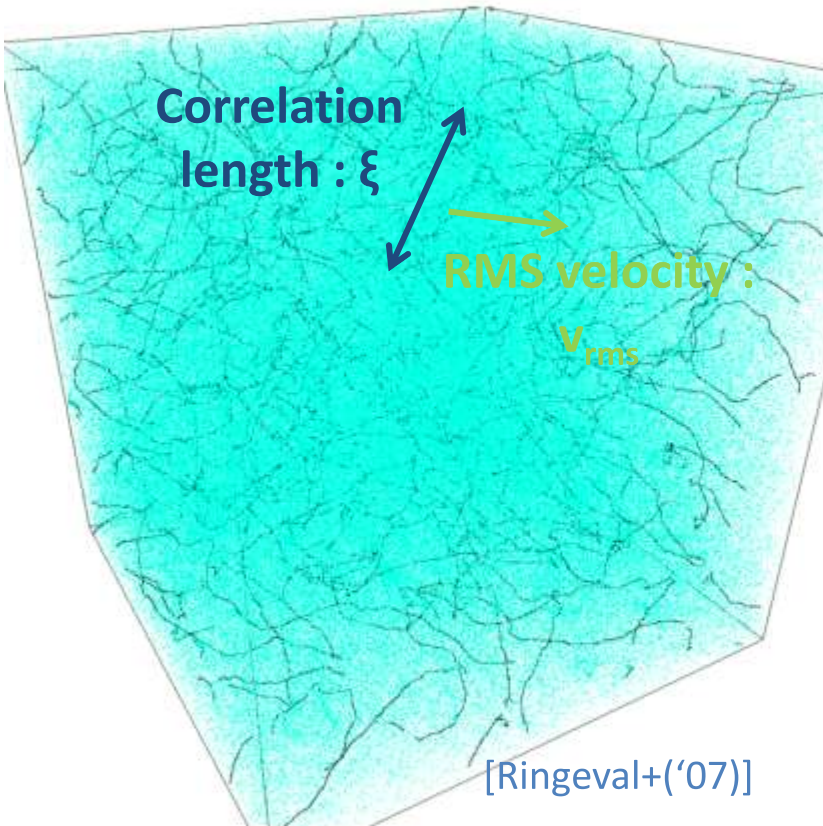
- Energy loss due to loop formation





[Ringeval+('07)]

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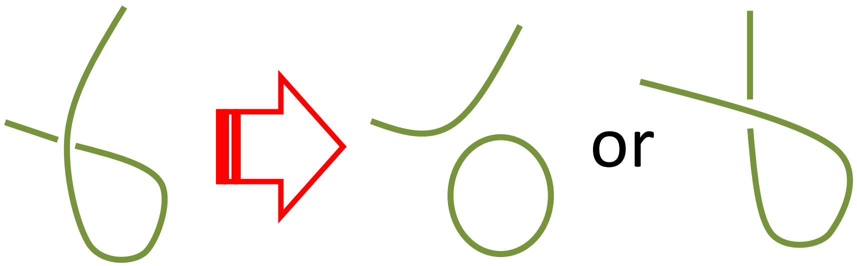

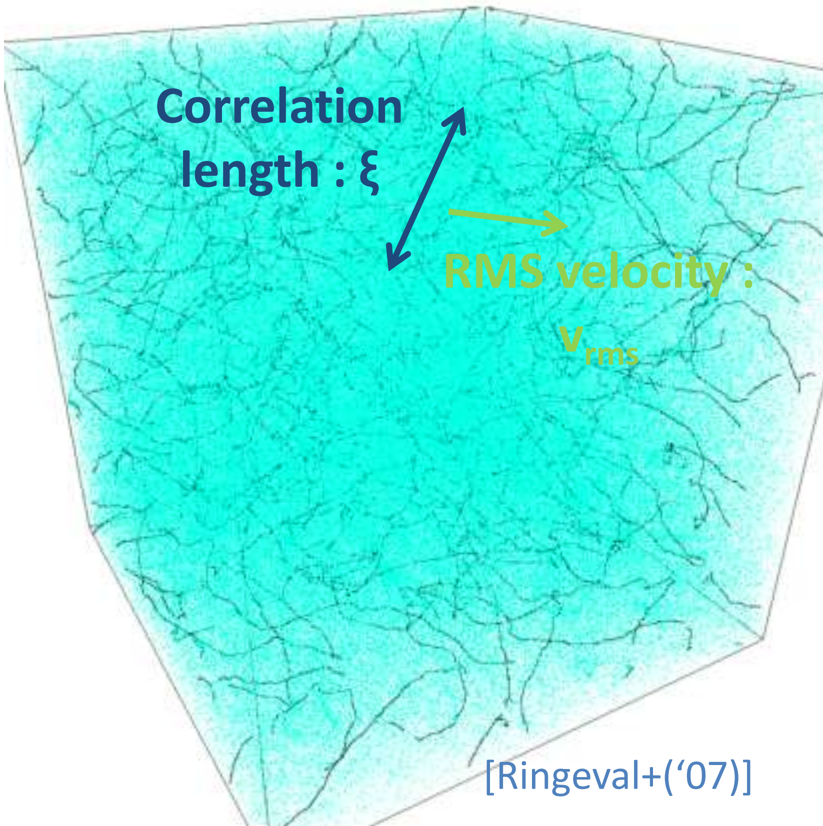

$$\frac{\Delta\rho_{\rm str}}{\Delta t} \approx -\frac{\mu\tilde{c}\xi \times (P v_{\rm rms}/\xi)}{\xi^3}$$

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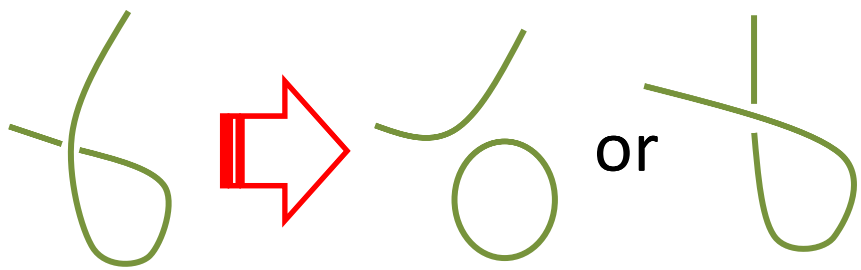

create a loop with length cξ

collision rate

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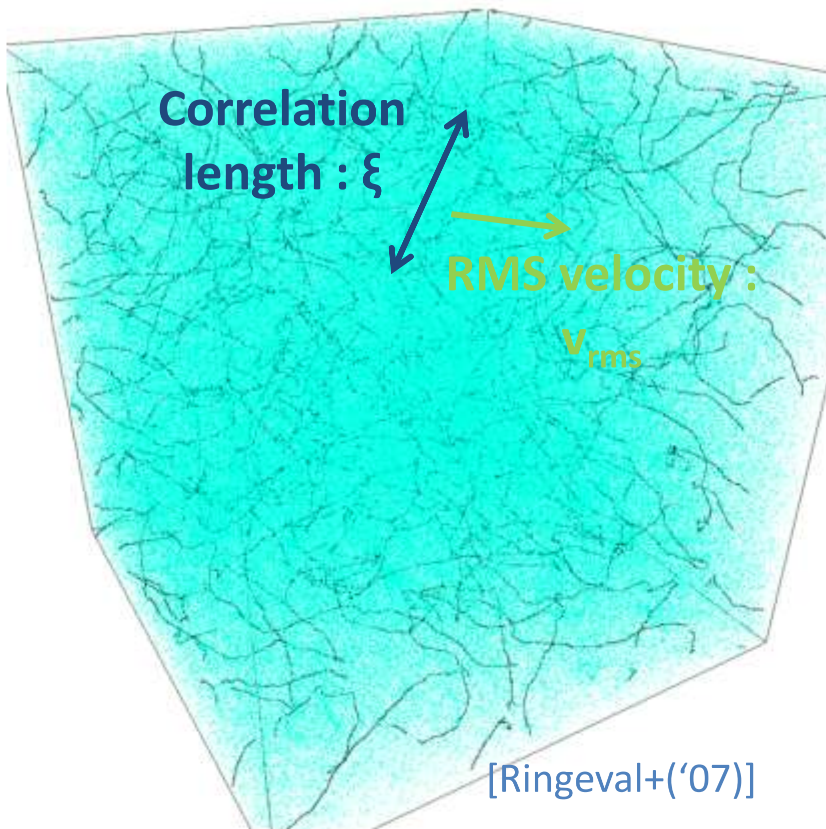

[Ringeval+('07)]

✓ From Nambu-Goto action, we have

$$\xi = \frac{1}{H\gamma}$$

$$\begin{cases} \dfrac{t}{\gamma}\dfrac{d\gamma}{dt} = \dfrac{1}{3}\left[\left(1 - v_{\rm rms}^2\right) - \underbrace{\tilde{c}Pv_{\rm rms}\gamma}_{\text{Loop formation}}\right] & \text{: Energy conservation} \\ \dfrac{dv_{\rm rms}}{dt} = \left(1 - v_{\rm rms}^2\right)H\left[\underbrace{k(v_{\rm rms})\gamma}_{\text{Curvature acceleration}} - 2v_{\rm rms}\right] & \text{: EOM} \end{cases}$$

[Takahashi, **DY** +(2009), **DY** +(2010a,b)]]

[see also Martins, Shellard (1996, 2002), Avgoustidis,Shellard (2006)]

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Assuming the SCALING (scale $\propto$ 1/H) is already realized by the last scattering surface, **γ** and **v_{rms}** are asymptotically constant in time:

[Takahashi, **DY** +(2009), **DY** +(2010a,b)]]

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$$\begin{cases} \cancelto{0}{\dfrac{t}{\gamma}\dfrac{d\gamma}{dt}} = \dfrac{1}{3}\left[\left(1 - v_{\rm rms}^2\right) - \underbrace{\tilde{c} P v_{\rm rms}\gamma}_{\text{Loop formation}}\right] & \text{: Energy conservation} \\ \cancelto{0}{\dfrac{dv_{\rm rms}}{dt}} = \left(1 - v_{\rm rms}^2\right) H \left[\underbrace{k(v_{\rm rms})\gamma}_{\text{Curvature acceleration}} - 2v_{\rm rms}\right] & \text{: EOM} \end{cases}$$

Assuming the SCALING (scale ∝ 1/H) is already realized by the last scattering surface, **γ** and **v_{rms}** are asymptotically constant in time:

$$\gamma \approx \sqrt{\frac{\pi\sqrt{2}}{3\tilde{c}P}} \quad \Longrightarrow \quad \rho_{\rm str} = \frac{\mu}{\xi^2} = \mu H^2 \gamma^2 \propto \frac{1}{P}$$

: Scaling solution incorporating P

[Takahashi, **DY** +(2009), **DY** +(2010a,b)]]

[see also Martins, Shellard (1996, 2002), Avgoustidis,Shellard (2006)]

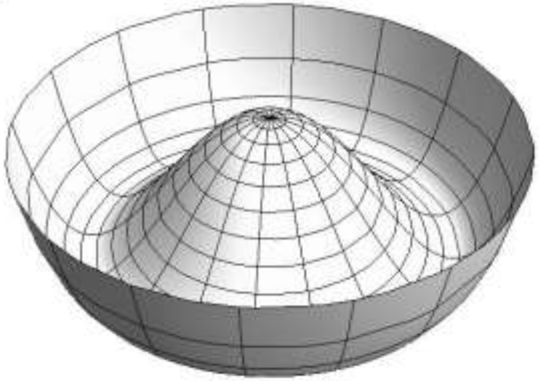
1.4 : Numerical approach; Abelian-Higgs model

To investigate the detail of the string network, we focus on the simplest model of cosmic strings, Abelian-Higgs model:

$$\mathcal{L}_{\rm AH} = (\partial_\mu + ieA_\mu)\,\Phi^*\,(\partial_\mu - ieA_\mu)\,\Phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\Phi)$$

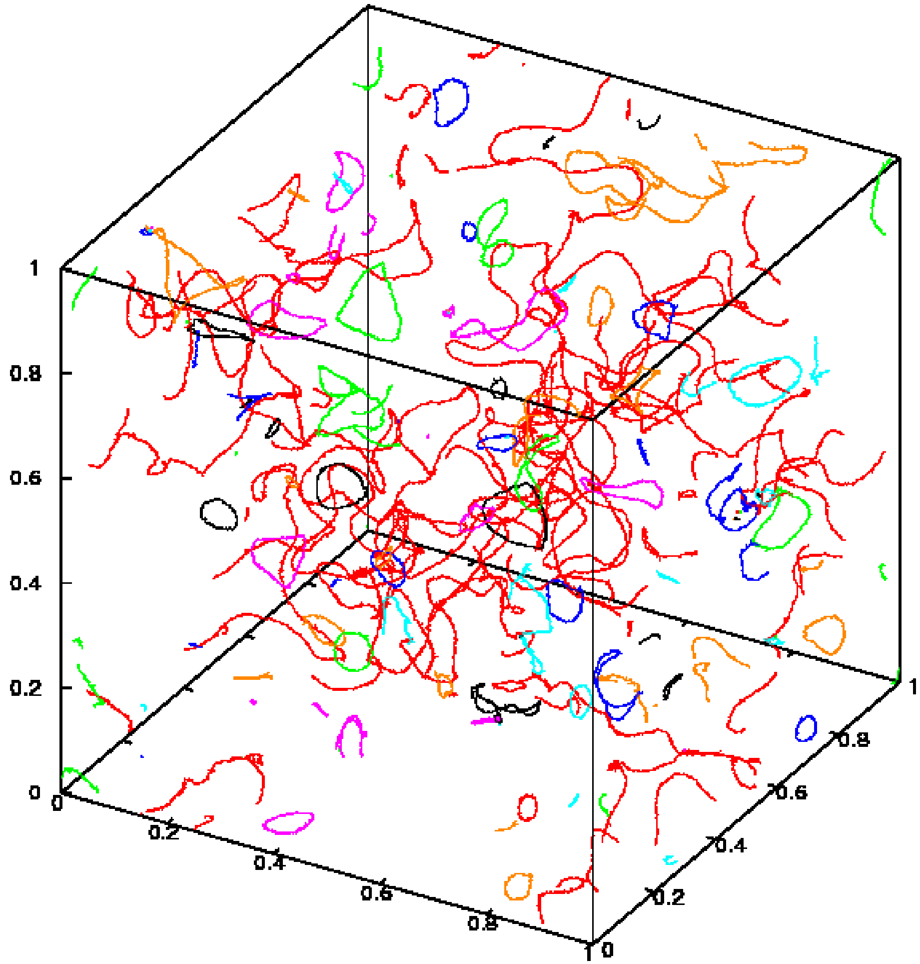
where $$V(\Phi) = \frac{\lambda}{4}\left(|\Phi|^2 - \eta_{\rm V}^2\right)^2$$

NOTICE: We just started this research. You cannot find the results, but you can SEE our simulations!



- **Conditions:**
  - ✓ Temperature : T=2Tc → 0.1Tc
  - ✓ Box size : $36/H_i \to 1.8/H_f$
  - ✓ 512 × 512 × 512, 256 × 256 × 256

An important parameter : $\beta = \dfrac{\lambda}{2e^2}$

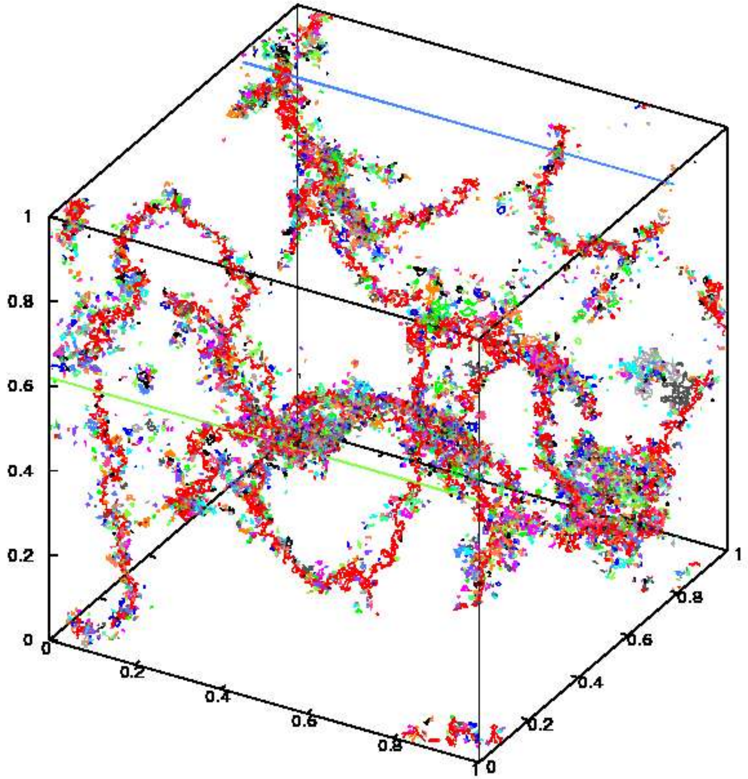


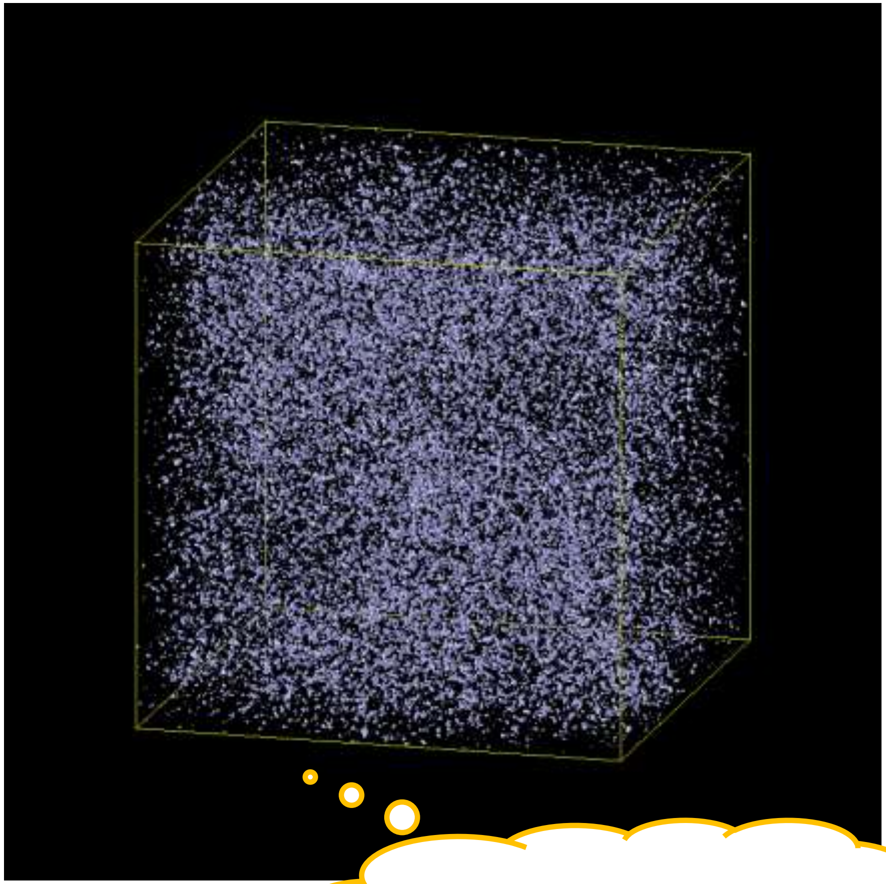
$$\beta = 1\,,\ \ \lambda = 0.1$$

Note:
Positions for string cores are found using phase information.

By Hiramatsu

An important parameter : $\beta = \frac{\lambda}{2e^2}$





Oscillation ???

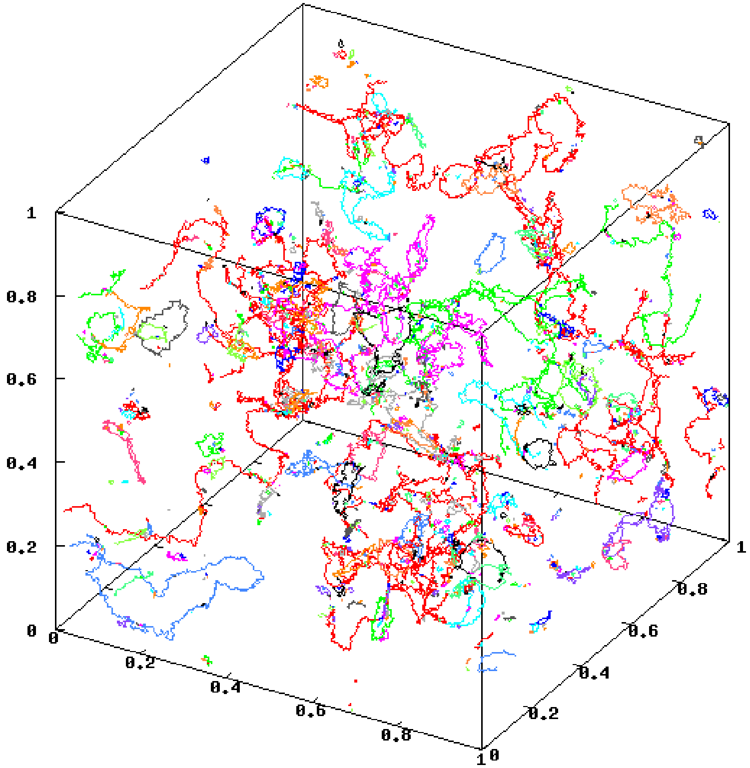
$$\beta = 0.25\,,\ \ \lambda = 0.01$$

Type-I strings

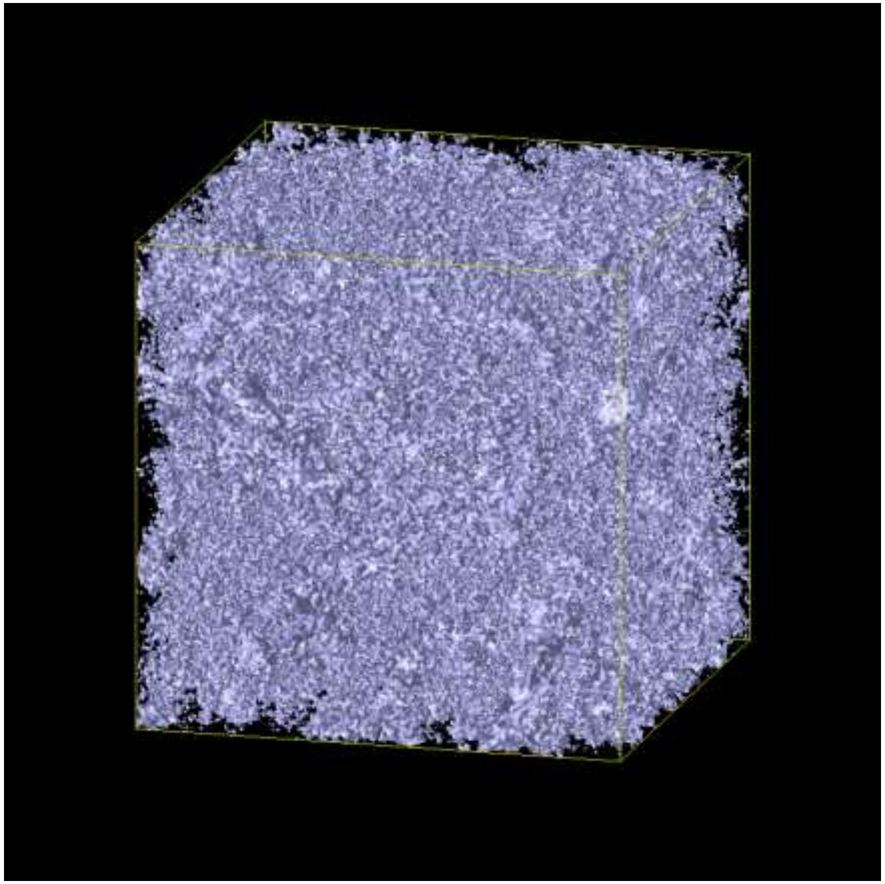
By Hiramatsu

An important parameter : $\beta = \dfrac{\lambda}{2e^2}$

String cores by phase information



Energy isosurface



$$\beta = 5 \times 10^{10}\,, \;\; \lambda = 0.1$$

Type-II strings

By Hiramatsu

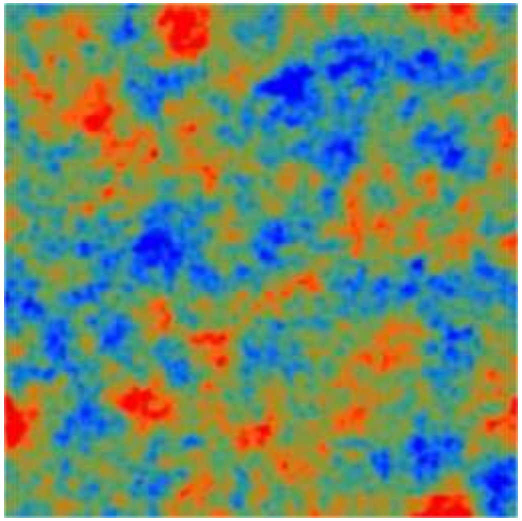
Part 2 : CMB from COSMIC STRINGS/COSMIC SUPERSTRINGS

Weak Lensing due to strings and CMB polarizations

- **DY**, Takahashi, Sendouda, Yoo, Sasaki, in prep.

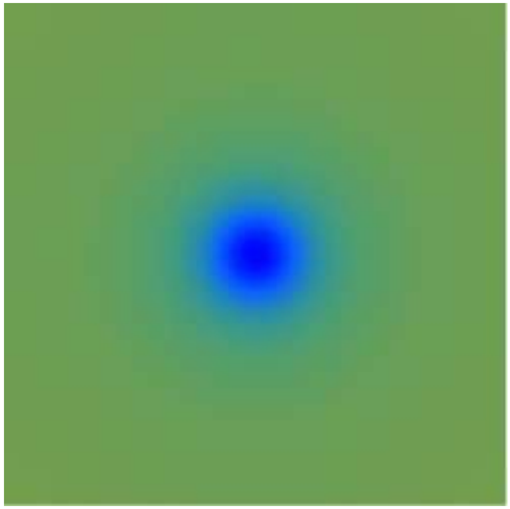
✓Foreground matter perturbations distort the CMB map !

Temperature fluctuations



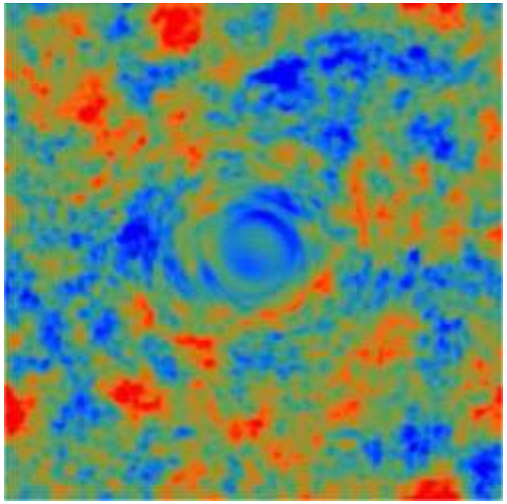
+

Foreground matter distribution



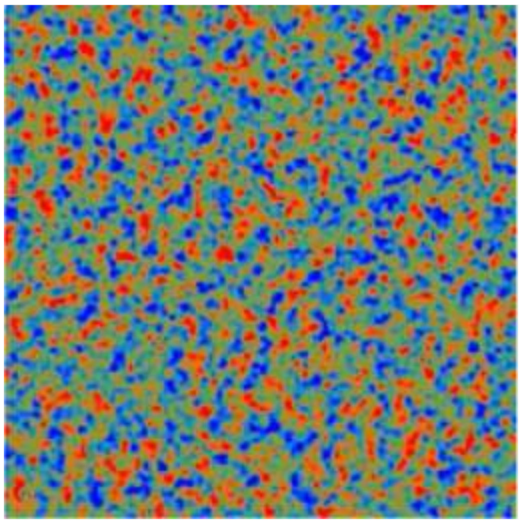
=

Lensed temperature fluctuations



[Hu & Okamoto (2002)]

Only E-mode



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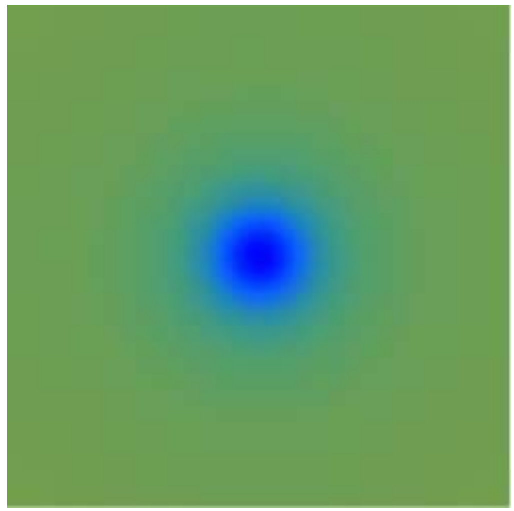
Foreground matter distribution



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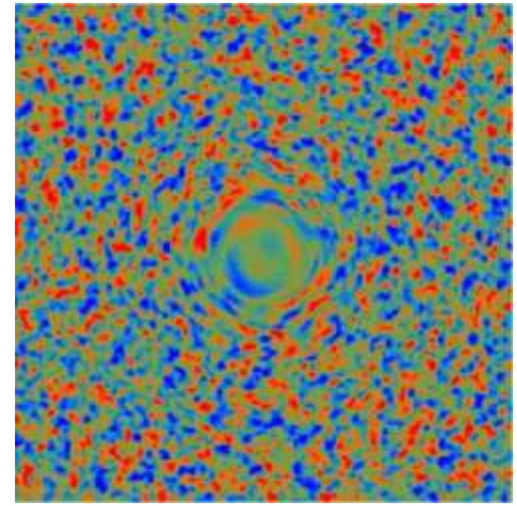
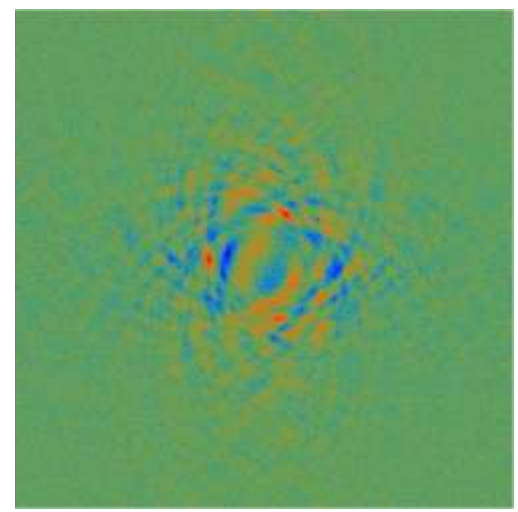
Lensed E-mode



Lensed B-mode !!!



Additional matter perturbation gives significant contribution of BB spectrum through the partial conversion of EE to BB !

2.1 : Gravitational Lensing

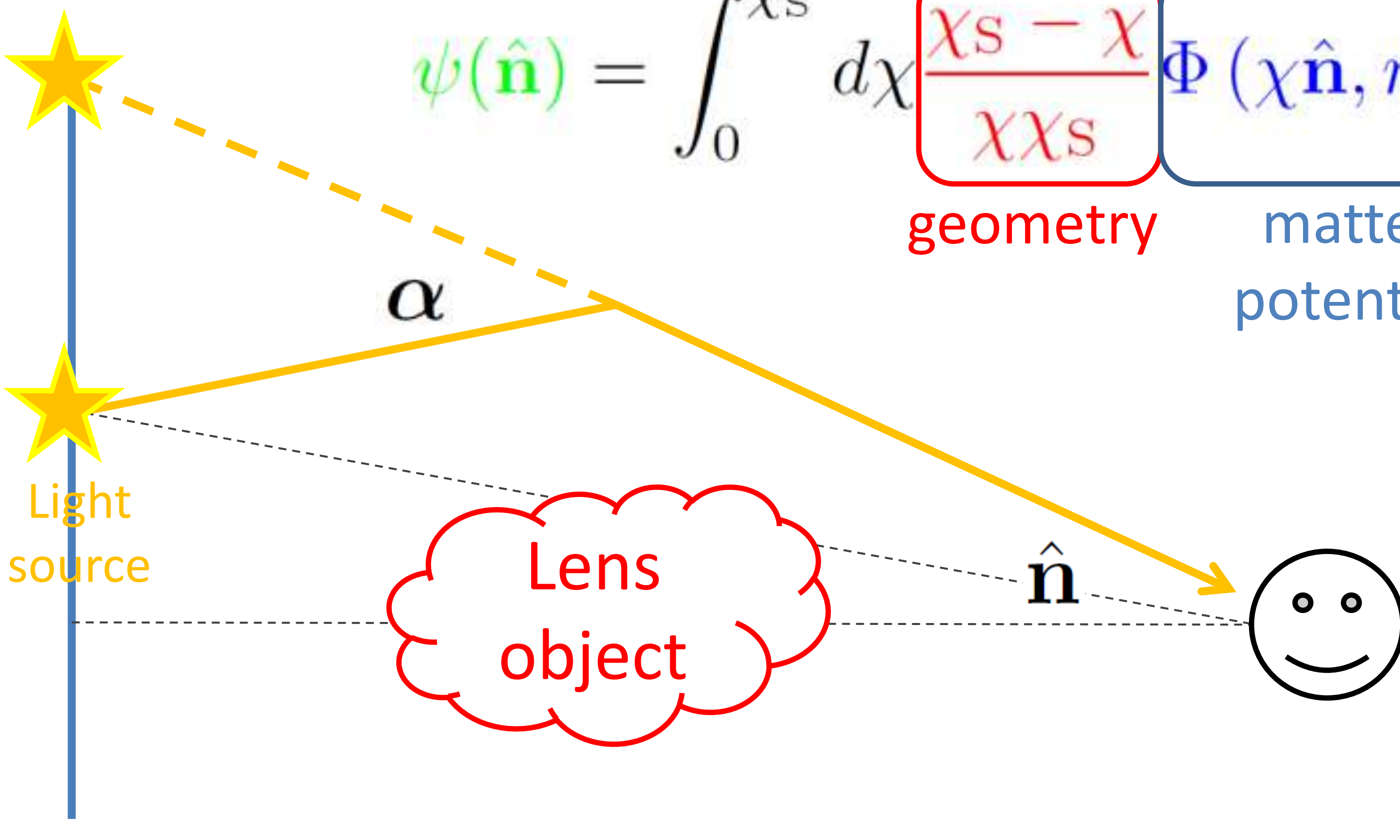
[Kaiser(1998), Bartelmann&Schneider(2001),Lewis&Charllinor(2006)]

$$\boldsymbol{\alpha} = \nabla_{\hat{\mathbf{n}}}\psi$$

$$\psi(\hat{\mathbf{n}}) = \int_0^{\chi_S} d\chi \boxed{\frac{\chi_S - \chi}{\chi\chi_S}} \boxed{\Phi\left(\chi\hat{\mathbf{n}}, \eta_\gamma(\chi)\right)}$$

geometry

matter potential

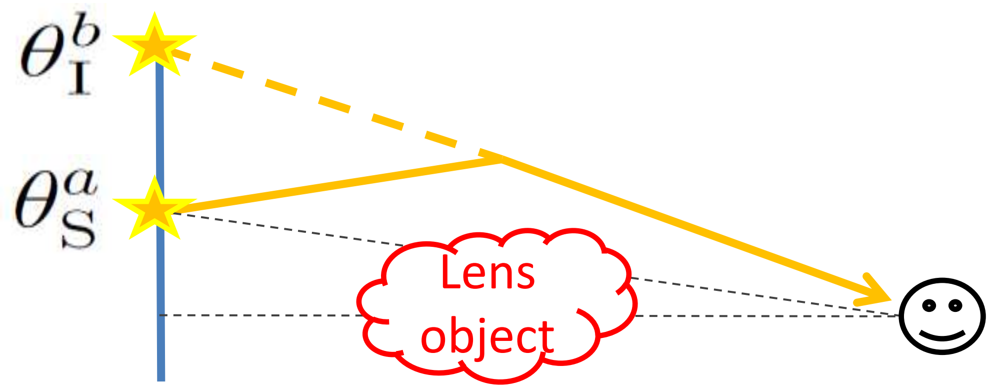


2.2 : Geodesic deviations

[see also Uzan&Bernardeau(2001), de Laix(1997)]

Solving the equation of geodesic deviation with an arbitrary metric perturbation, $h_{\mu\nu}$, in an expanding universe, we find the general expression:

$$\theta_{\mathrm{S}}^{a} = \mathcal{A}^{a}{}_{b}\theta_{\mathrm{I}}^{b}$$



$$\mathcal{A}_{ab} \approx \delta_{ab}\left[1 - \underbrace{\boxed{\frac{1}{\chi_{\mathrm{S}}}\int_0^{\chi_{\mathrm{S}}} d\chi \frac{\delta E(\chi\hat{\mathbf{n}}, \eta_\gamma(\chi))}{E_O}}}_{\text{①Gravitational time delay}}\right] + \int_0^{\chi_{\mathrm{S}}} d\chi \underbrace{\boxed{\frac{(\chi_{\mathrm{S}} - \chi)\chi}{\chi_{\mathrm{S}}}}}_{\text{geometry}} \left\{\bar{\nabla}_a\bar{\nabla}_b \underbrace{\boxed{\Phi(\chi\hat{\mathbf{n}}, \eta_\gamma(\chi))}}_{\text{②Lensing potential}} + \underbrace{\boxed{\Psi_{ab}(\chi\hat{\mathbf{n}}, \eta_\gamma(\chi))}}_{\text{③New term}}\right\}$$

① Gravitational time delay : $\frac{1}{\chi_{\rm S}}\int_0^{\chi_{\rm S}} d\chi \frac{\delta E(\chi\hat{\mathbf{n}},\eta_\gamma(\chi))}{E_O}$

The distance traveled by photon is perturbed, then this modulates the spatial surface of recombination.

Since the change of power spectrum is ~0.1%, we can neglect this contribution.

[Hu and Cooray (2000) for scalar perturbation]

① Gravitational time delay : $\frac{1}{\chi_S}\int_0^{\chi_S} d\chi \frac{\delta E(\chi\hat{\mathbf{n}},\eta_\gamma(\chi))}{E_O}$

The distance traveled by photon is perturbed, then this modulates the spatial surface of recombination.
Since the change of power spectrum is ~0.1%, we can neglect this contribution.
[Hu and Cooray (2000) for scalar perturbation]

③ New term: $\int_0^{\chi_S} d\chi \frac{(\chi_S-\chi)\chi}{\chi_S}\Psi_{ab}(\chi\hat{\mathbf{n}},\eta_\gamma(\chi))$

where $\Psi_{ab}=\frac{d}{d\chi}\left[\frac{1}{2}\frac{d}{d\chi}h_{ab}-p^\mu\nabla_{(a}h_{b)\mu}\right]$

③-1: This term introduces an unusual contribution. For vector and tensor perturbations, this term may become important.
③-2: Assuming “thin-lens approximation”, the contribution from this term reduces to boundary term !

⇒ Hereafter, we assume thin-lens approximation for simplicity.

② Lensing potential

Neglecting the gravitational time delay ① and the asymmetric term ③, we have the ordinary amplification matrix:

$$A_{ab}-\delta_{ab}\approx\nabla_{\hat{n}^a}\nabla_{\hat{n}^b}\psi(\hat{\mathbf{n}})$$

where $$\boxed{\psi(\hat{\mathbf{n}})=\int_0^{\chi_\mathrm{S}}d\chi\frac{\chi_\mathrm{S}-\chi}{\chi\chi_\mathrm{S}}\Phi\left(\chi\hat{\mathbf{n}},\eta_\gamma(\chi)\right)}$$

with $$\Phi=\frac{1}{2}h_{\mu\nu}p^\mu p^\nu$$

2.3 : Lensing potential due to cosmic (super-)strings

Assumption : Each scattering due to a string takes place locally, namely the Hubble expansion can be neglected:

$$\Box h_{\mu\nu} = 16\pi G \left(T_{\mu\nu} - \frac{1}{2} T \eta_{\mu\nu} \right)$$

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$$\Box h_{\mu\nu} = 16\pi G \left(T_{\mu\nu} - \frac{1}{2} T \eta_{\mu\nu} \right)$$

By using above linearized Einstein Eq., we can decompose the lensing potential into multipole moment analytically:

$$\psi(\hat{\mathbf{n}}) = \sum_{\ell m} \psi_{\ell m} Y_\ell^m(\hat{\mathbf{n}})$$

where

$$\psi_{\ell m} = \frac{8\pi G}{\ell(\ell+1)} \int d\sigma \frac{\chi_{\mathrm{S}} - \chi_{\mathrm{L}}(\sigma)}{\chi_{\mathrm{S}} \chi_{\mathrm{L}}(\sigma)} \mu_{\mathrm{proj}}(\sigma) Y_\ell^{m*}(\hat{\mathbf{n}}_{\mathrm{L}}(\sigma))$$

$$\mu_{\mathrm{proj}}(\sigma) = \mu \frac{(1+\dot{\chi}_{\mathrm{L}})^2 - {\chi'_{\mathrm{L}}}^2}{1+\dot{\chi}_{\mathrm{L}}}$$: projected string tension

2.4 : Segment formalism and lensing power spectrum

Since the observed sky map due to segments appears as a superposition of those due to each segment, then we can decompose

$$\psi_{\ell m}^{\mathrm{total}} = \sum_{i \in \mathrm{all\,segments}} \psi_{\ell m}^{(i)}$$

2.4 : Segment formalism and lensing power spectrum

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$$\psi_{\ell m}^{\text{total}} = \sum_{i \in \text{all segments}} \psi_{\ell m}^{(i)}$$

$$\Downarrow$$

$$C_\ell^{\psi\psi} = \frac{1}{2\ell+1} \sum_m \left[\left\langle \sum_{i \in \text{seg}} \left| \psi_{\ell m}^{(i)} \right|^2 \right\rangle + \left\langle \sum_{i \neq j \in \text{seg}} \psi_{\ell m}^{(i)} \psi_{\ell m}^{(j)} \right\rangle \right]$$

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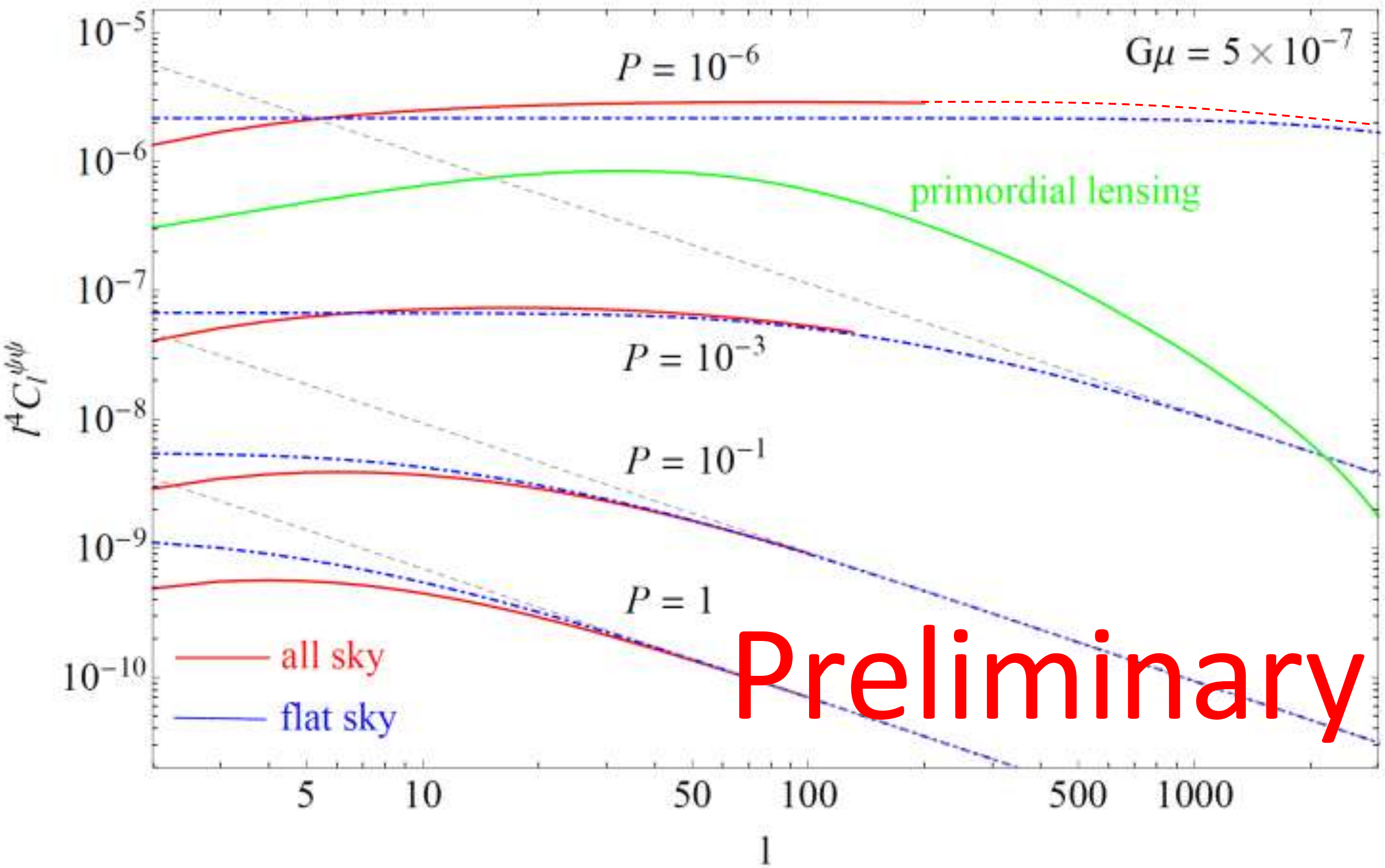
$$\to \int dz \underbrace{\boxed{\frac{dV}{dz} n_{\mathrm{seg}}}}_{\text{Weight factor depending on the matter distribution}} \underbrace{\boxed{\frac{1}{2\ell+1}\sum_m \left|\psi_{\ell m}\right|^2}}_{\text{Power spectrum due to a scattering}}$$

2.2-3 : Lensing Potential



✓Strings leads to broader lensing spectrum than those due to the primordial scalar perturbations.

✓ The contributions from large scale dominates the spectrum.

✓ As P degreases, the amplitude increases and the spectrum becomes broader.

2.3 : Weak lensing of CMB

$$\frac{\Delta T}{T}(\hat{\mathbf{n}}) \equiv \Theta(\hat{\mathbf{n}})$$

Weak lensing of the CMB remaps the primary anisotropy according to the deflection angle :

$$\tilde{\Theta}(\hat{\mathbf{n}}) \equiv \Theta(\hat{\mathbf{n}} + \nabla_{\hat{\mathbf{n}}}\psi(\hat{\mathbf{n}}))$$

$$\approx \Theta(\hat{\mathbf{n}}) + \nabla^{\hat{n}^a}\psi(\hat{\mathbf{n}})\nabla_{\hat{n}^a}\Theta(\hat{\mathbf{n}}) + \frac{1}{2}\nabla^{\hat{n}^a}\psi(\hat{\mathbf{n}})\nabla^{\hat{n}^b}\psi(\hat{\mathbf{n}})\nabla_{\hat{n}^a}\nabla_{\hat{n}^b}\Theta(\hat{\mathbf{n}}) + \cdots$$

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2D Fourier decomposition :

$$\tilde{\Theta}_{\boldsymbol{\ell}} = \Theta_{\boldsymbol{\ell}} - \int \frac{d^2\boldsymbol{\ell}'}{(2\pi)^2}\left[\boldsymbol{\ell}' \cdot (\boldsymbol{\ell} - \boldsymbol{\ell}')\right]\Theta_{\boldsymbol{\ell}'}\psi_{\boldsymbol{\ell}-\boldsymbol{\ell}'}$$

$$-\frac{1}{2}\int \frac{d^2\boldsymbol{\ell}' d\boldsymbol{\ell}''}{(2\pi)^4}\left[(\boldsymbol{\ell}' \cdot \boldsymbol{\ell}'')\left\{\boldsymbol{\ell}' \cdot (\boldsymbol{\ell}' + \boldsymbol{\ell}'' - \boldsymbol{\ell})\right\}\right]\Theta_{\boldsymbol{\ell}'}\psi_{\boldsymbol{\ell}''}\psi^*_{\boldsymbol{\ell}''+\boldsymbol{\ell}'-\boldsymbol{\ell}}$$

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2D Fourier decomposition :

$$\tilde{\Theta}_{\boldsymbol{\ell}} = \Theta_{\boldsymbol{\ell}} - \int \frac{d^2\boldsymbol{\ell}'}{(2\pi)^2}\left[\boldsymbol{\ell}'\cdot(\boldsymbol{\ell}-\boldsymbol{\ell}')\right]\Theta_{\boldsymbol{\ell}'}\psi_{\boldsymbol{\ell}-\boldsymbol{\ell}'}$$

$$-\frac{1}{2}\int \frac{d^2\boldsymbol{\ell}'d\boldsymbol{\ell}''}{(2\pi)^4}\left[(\boldsymbol{\ell}'\cdot\boldsymbol{\ell}'')\left\{\boldsymbol{\ell}'\cdot(\boldsymbol{\ell}'+\boldsymbol{\ell}''-\boldsymbol{\ell})\right\}\right]\Theta_{\boldsymbol{\ell}'}\psi_{\boldsymbol{\ell}''}\psi^*_{\boldsymbol{\ell}''+\boldsymbol{\ell}'-\boldsymbol{\ell}}$$

The TT angular power spectrum at lowest order of Clψψ can be written as

$$\boxed{\tilde{C}_\ell^{\Theta\Theta} \approx \left(1-\ell^2 R^{\psi\psi}\right)C_\ell^{\Theta\Theta} + \underbrace{\int \frac{d^2\boldsymbol{\ell}'}{(2\pi)^2}\left[\boldsymbol{\ell}'\cdot(\boldsymbol{\ell}-\boldsymbol{\ell}')\right]^2 C_{\ell'}^{\psi\psi}C_{|\boldsymbol{\ell}-\boldsymbol{\ell}'|}^{\Theta\Theta}}_{\text{Convolution of Cl}\Theta\Theta\text{ and Cl}\psi\psi}}$$

with $R^{\psi\psi} = \dfrac{1}{4\pi}\displaystyle\int \frac{d\ell'}{\ell'}\ell'^4 C_{\ell'}^{\psi\psi}$

By following the same step as for the temperature fluctuations,

$$
\begin{aligned}
\tilde{C}_\ell^{BB} &= \left(1-\ell^2 R^{\psi\psi}\right) C_\ell^{BB} \\
&\quad \underbrace{+\int \frac{d^2\boldsymbol{\ell}'}{(2\pi)^2}\left[\boldsymbol{\ell}'\cdot(\boldsymbol{\ell}-\boldsymbol{\ell}')\right]^2 C_{|\boldsymbol{\ell}-\boldsymbol{\ell}'|}^{\psi\psi}\left\{C_{\ell'}^{EE}\sin^2\left(2\varphi_{\ell,\ell'}\right)+C_{\ell'}^{BB}\cos^2\left(2\varphi_{\ell,\ell'}\right)\right\}}_{\text{Convolution of ClEE, BB and Cl}\psi\psi}
\end{aligned}
$$

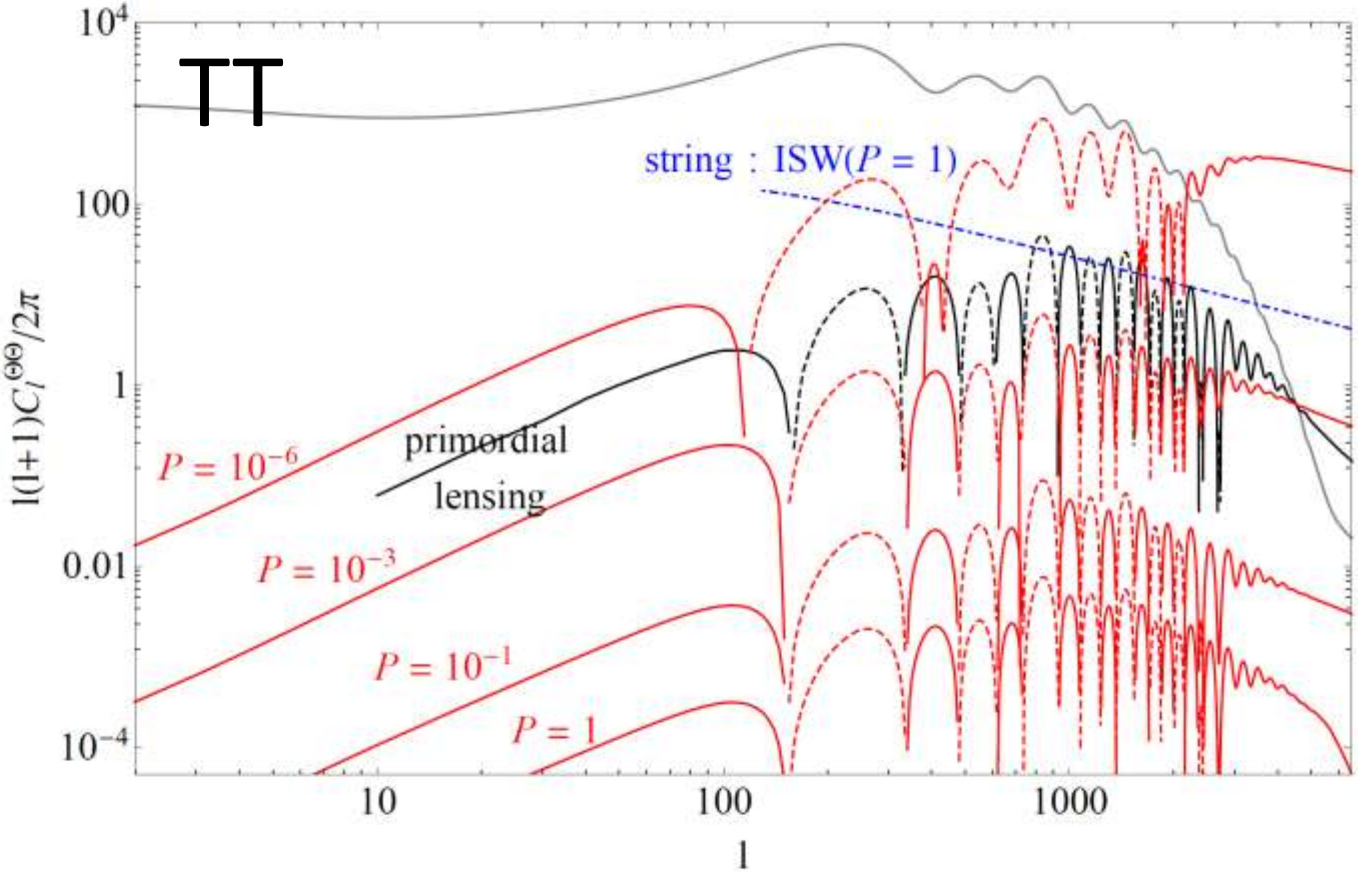
By following the same step as for the temperature fluctuations,

$$
\tilde{C}_\ell^{BB} = \cancelto{0}{\left(1-\ell^2 R^{\psi\psi}\right) C_\ell^{BB}}
$$

$$
\underbrace{+\int \frac{d^2\boldsymbol{\ell}'}{(2\pi)^2}\left[\boldsymbol{\ell}'\cdot(\boldsymbol{\ell}-\boldsymbol{\ell}')\right]^2 C_{|\boldsymbol{\ell}-\boldsymbol{\ell}'|}^{\psi\psi}\left\{C_{\ell'}^{EE}\sin^2\left(2\varphi_{\ell,\ell'}\right)+\cancelto{0}{C_{\ell'}^{BB}\cos^2\left(2\varphi_{\ell,\ell'}\right)}\right\}}_{\text{Convolution of ClEE, BB and Cl}\psi\psi}
$$

If no primordial BB spectrum, the partial conversion of EE to BB !

2.2-4 : Lensed spectrum in flat-sky Preliminary



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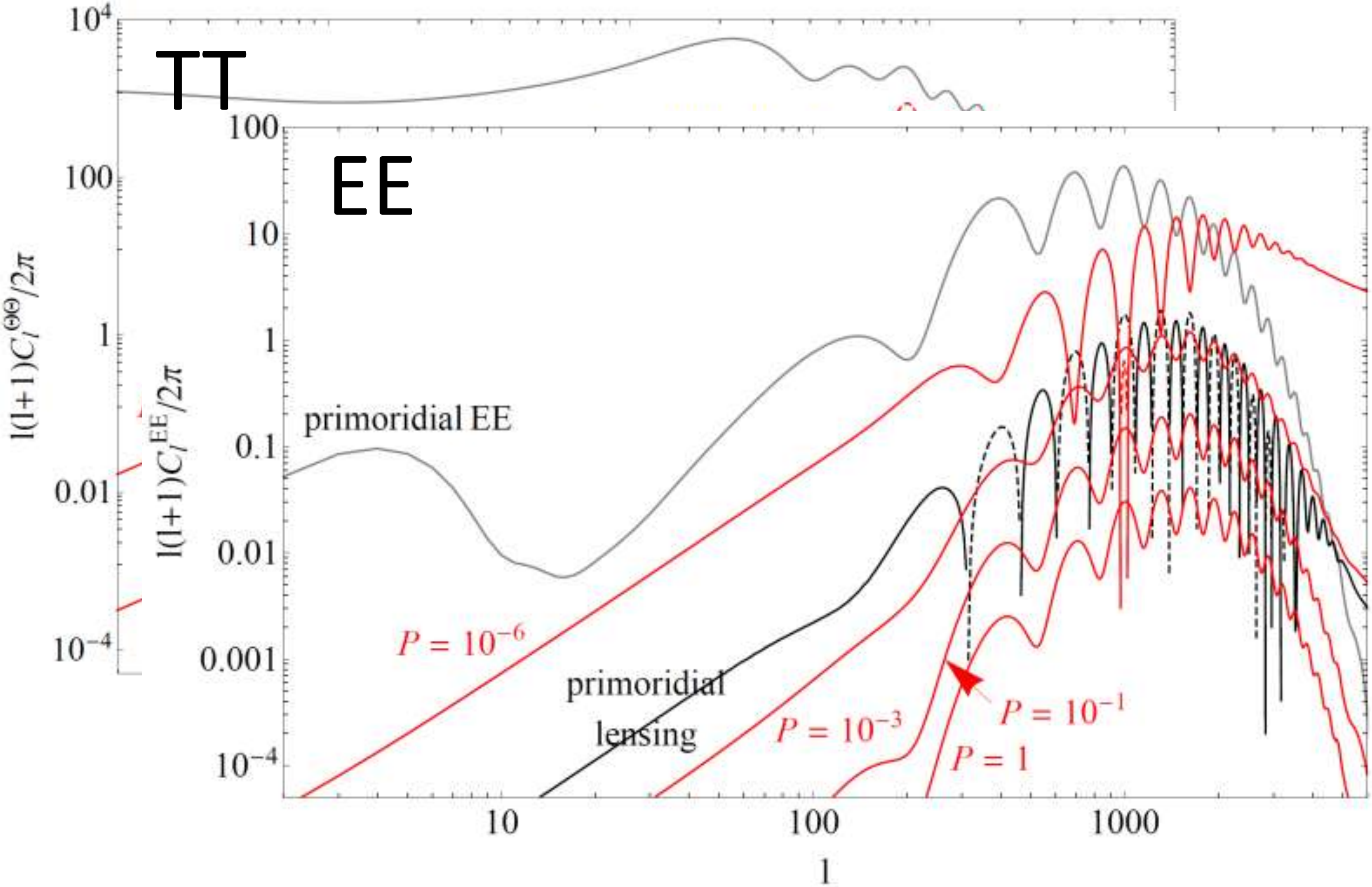

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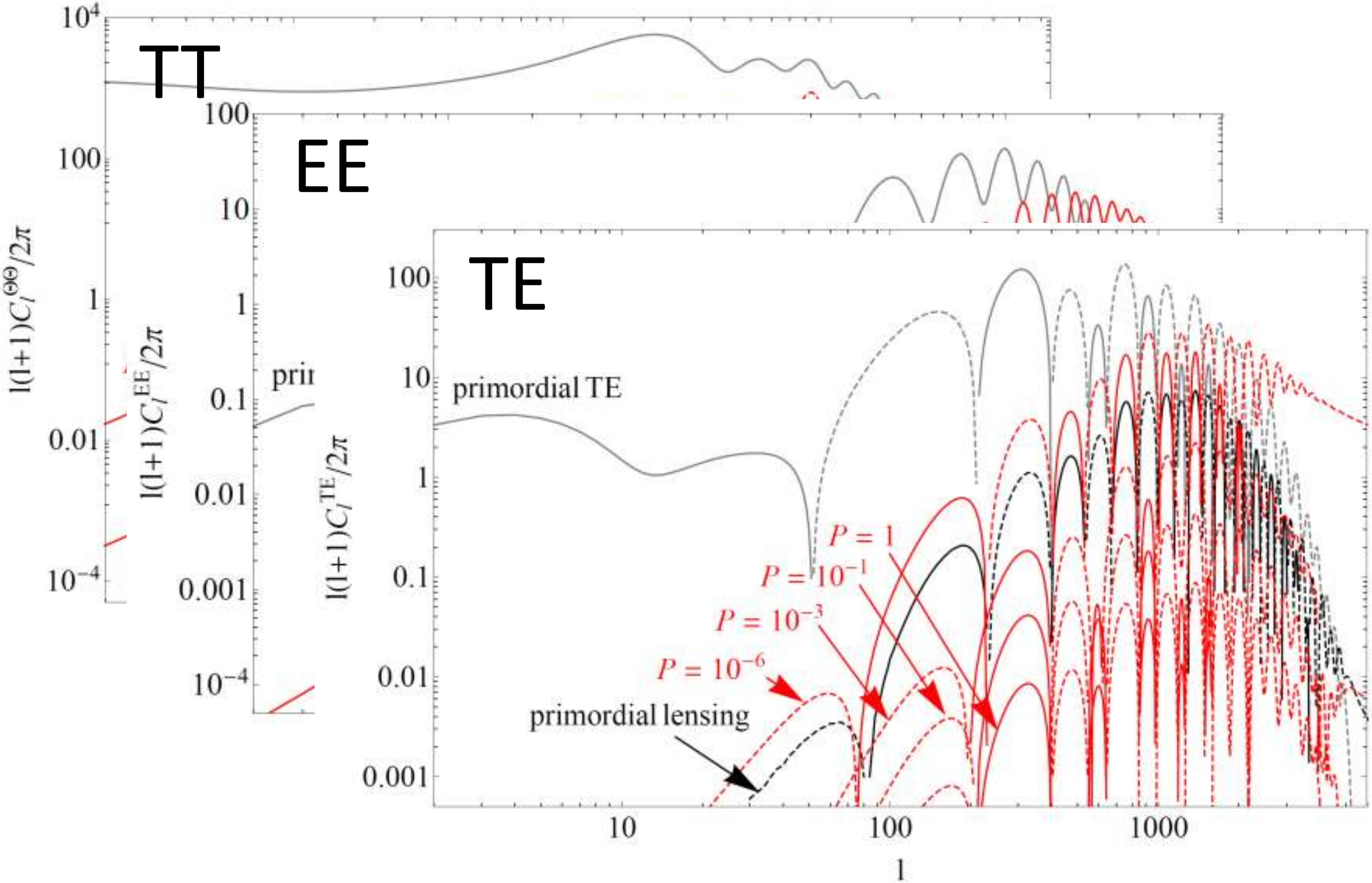
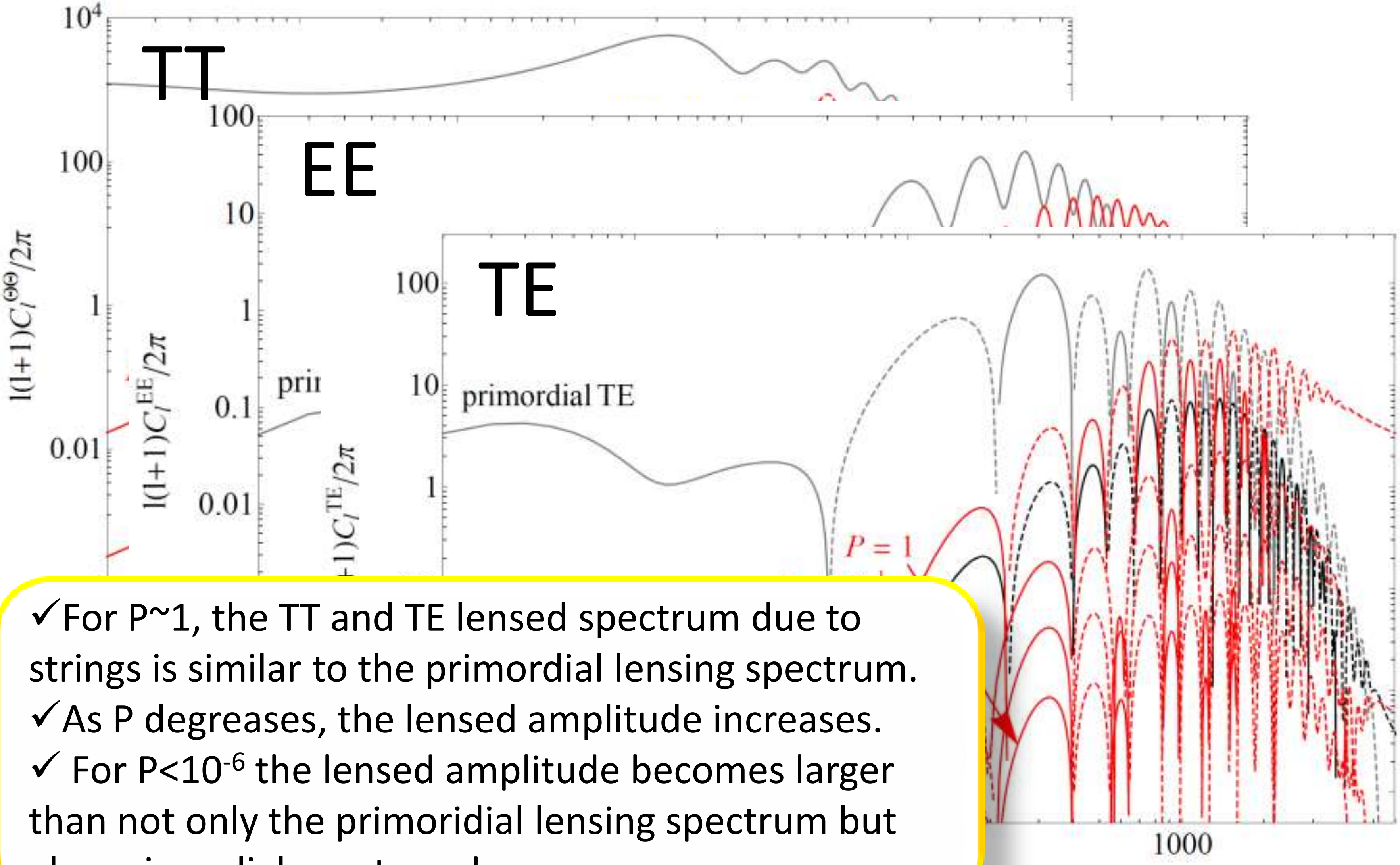


2.2-4 : Lensed spectrum in flat-sky Preliminary

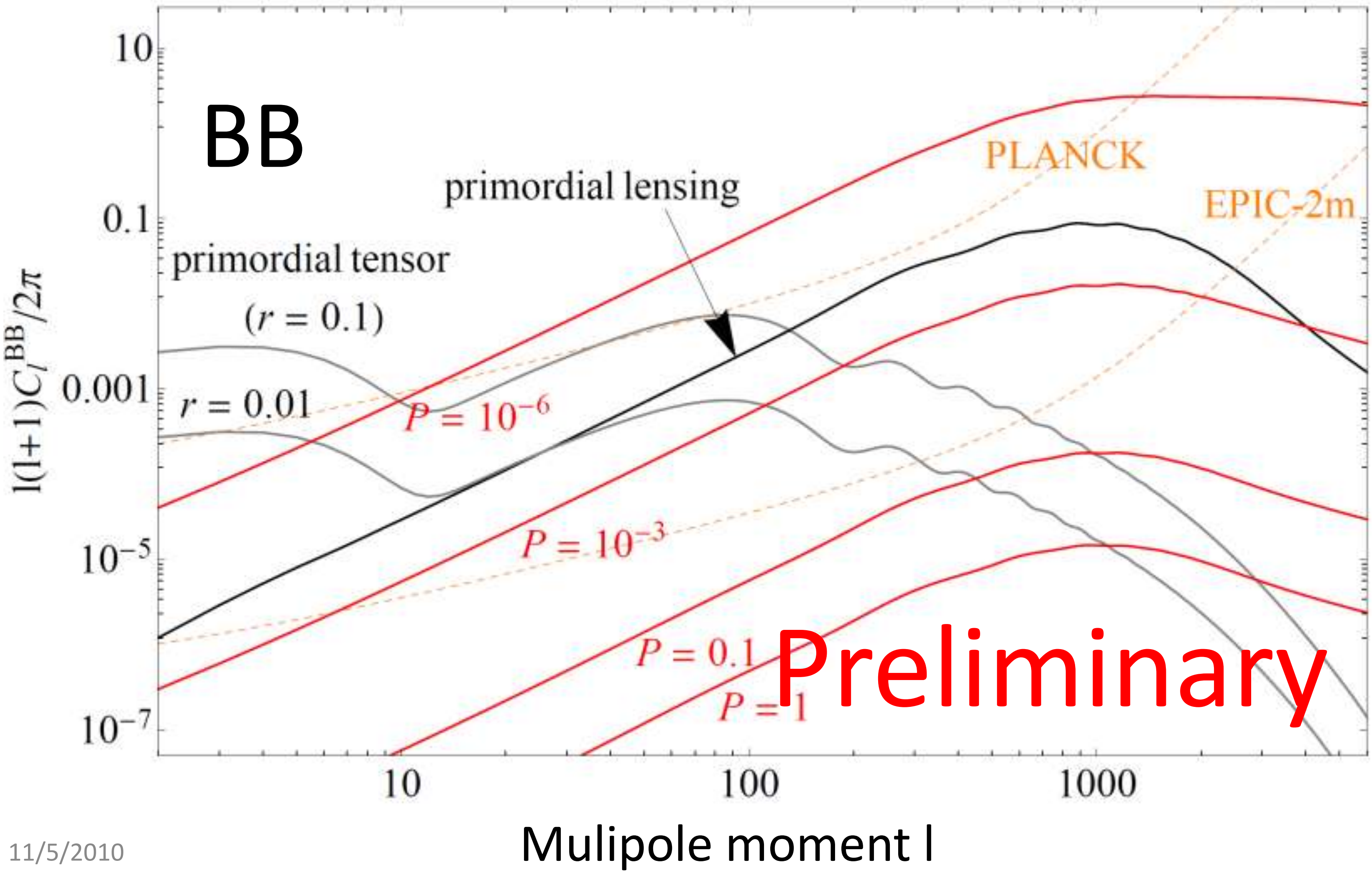


✓For P~1, the TT and TE lensed spectrum due to strings is similar to the primordial lensing spectrum.
✓As P degreases, the lensed amplitude increases.
✓ For P<10^{-6} the lensed amplitude becomes larger than not only the primoridial lensing spectrum but also primordial spectrum !

2.2-5 : Lensed BB spectrum in flat-sky

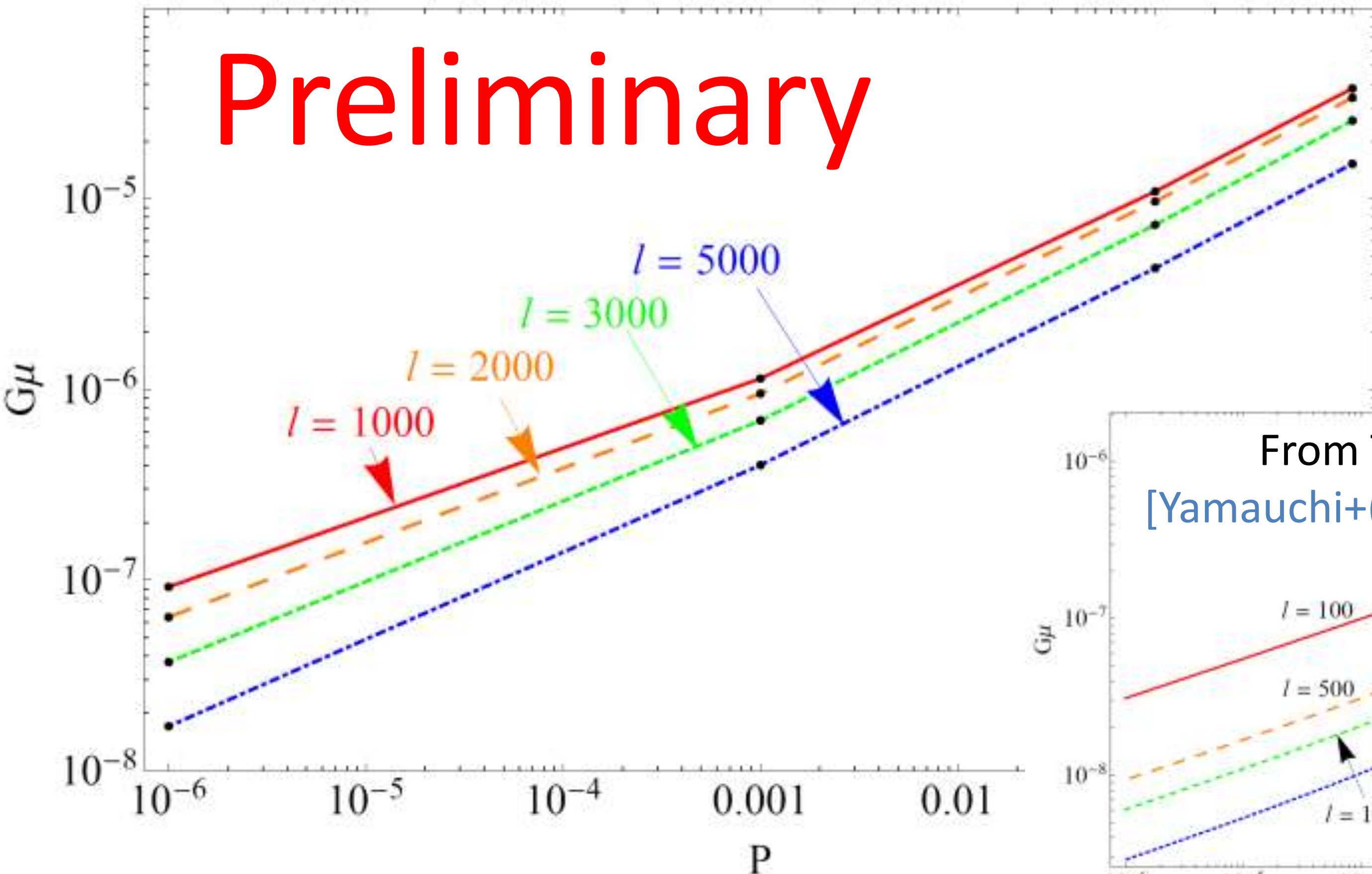
The signal from the weak lensing due to cosmic superstring with P<<1 can be detected by PLANCK !!!

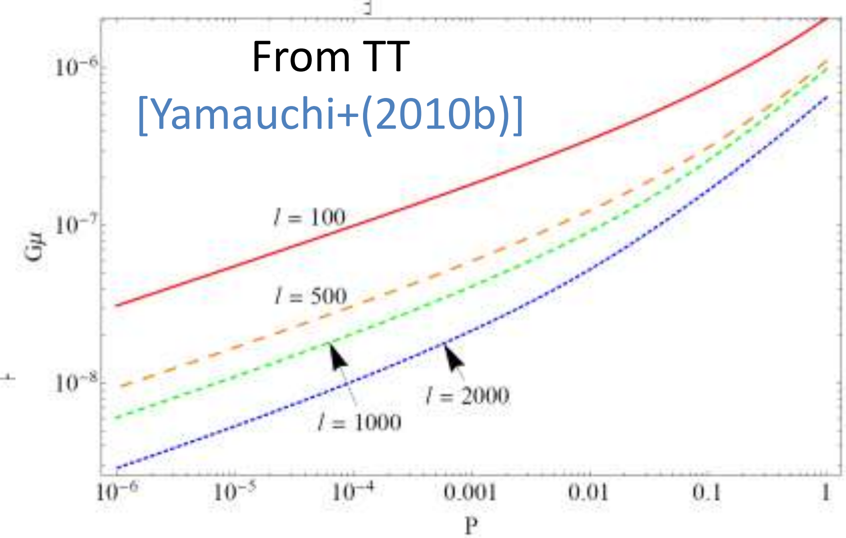
→ Lensed BB spectrum gives the independent constraint on Gμ and P !



2.2-6 : Constraints on string tension Gμ

- As P degreases, the amplitude due to strings increases, hence the tension of strings with smaller P is tightly constrained.
- Assuming that the amplitude of BB spectrum due to weak lensing for various l has to be smaller than the primordial lensing, we have the constraint on Gμ:





- We estimated the contributions of the weak lensing due to cosmic (super-)strings to cosmic microwave background temperature anisotropy and polarizations.

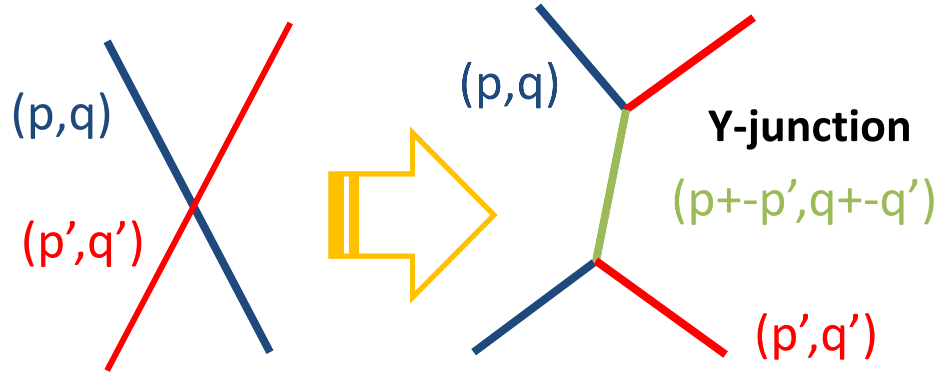
- Lensed BB spectrum gave the independent constraints on the string tension $G\mu$ and the intercommuting probability P.

4 : Summary

A cosmic string is a new smoking gun for string cosmology !

- tensions "μ" ⇔ internal geometry (warping)
- intercommuting probabilities "P" ⇔ string interactions

4 : Summary : Future



Stringy effect ::

- Y-junctions
- small scale structures (cusps, kinks, …)

Observations ::

- Gravitational waves from Y-junction, cusps, kinks, …
- Vector modes in weak lensing survey

THANK YOU !