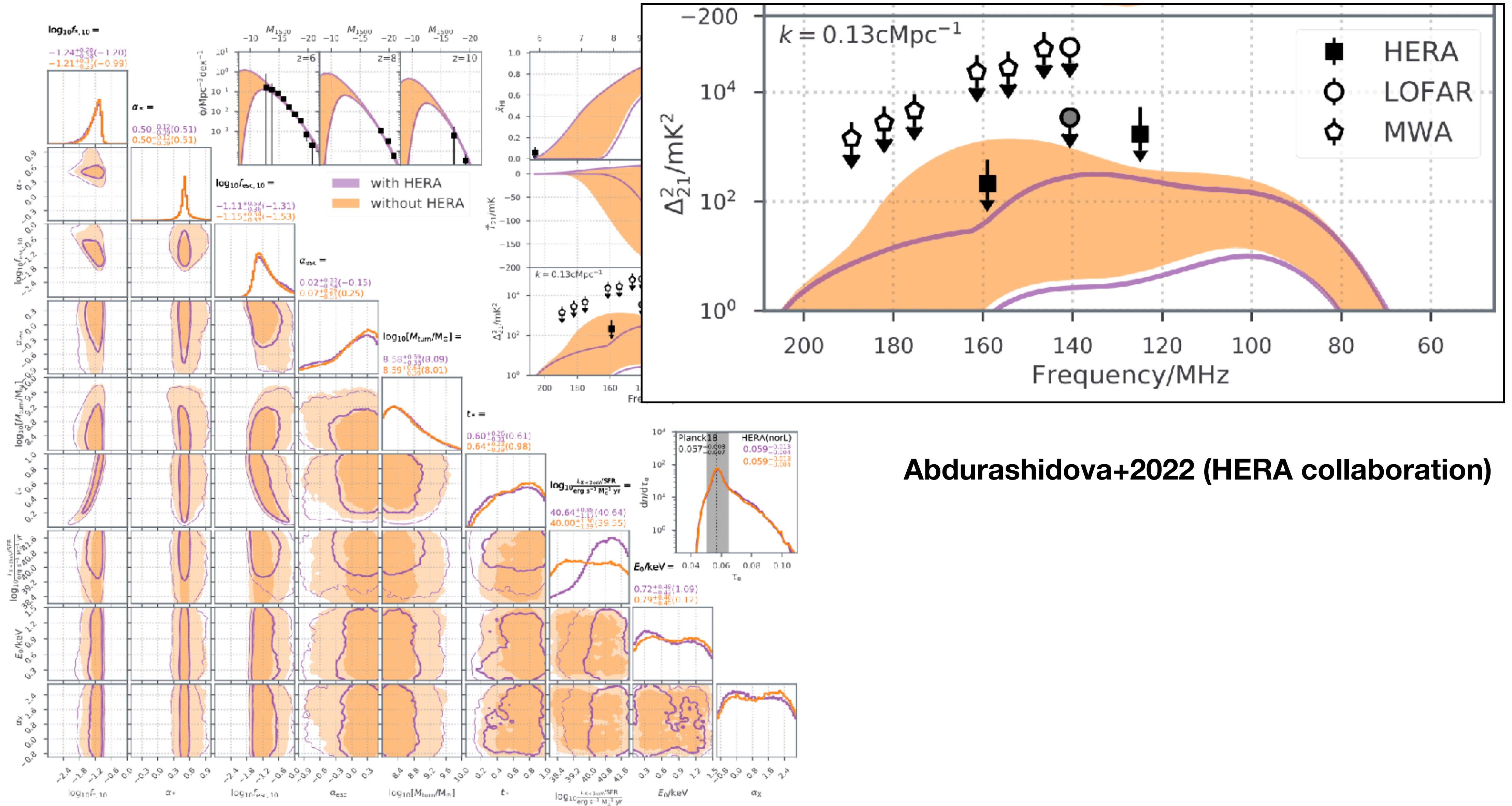


# Estimating EoR parameters with deep learning

**Kana Moriwaki (The University of Tokyo)**

**Hongo 21cm workshop  
3-4 Oct. 2024**

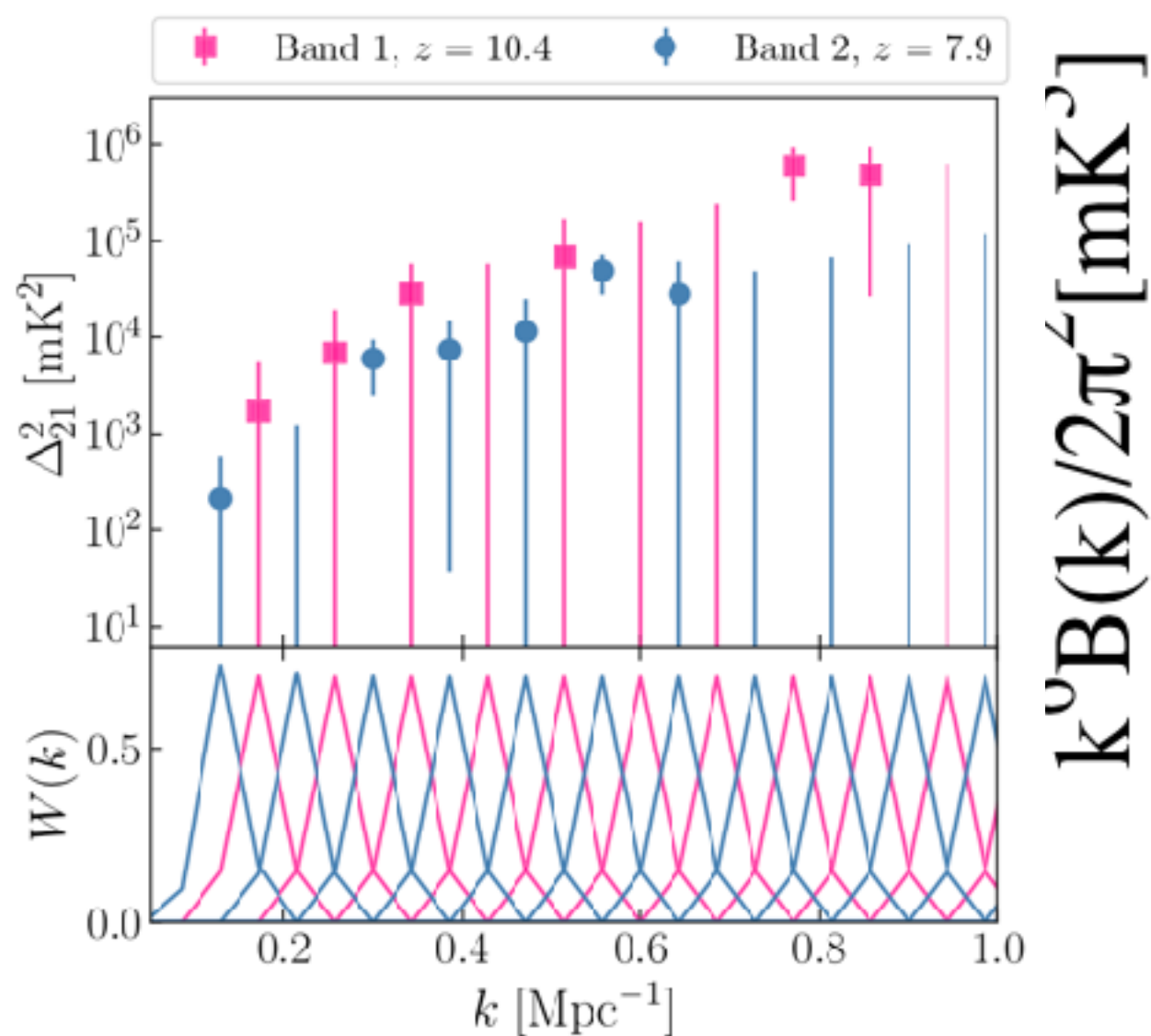
# Estimating EoR parameters



Abdurashidova+2022 (HERA collaboration)

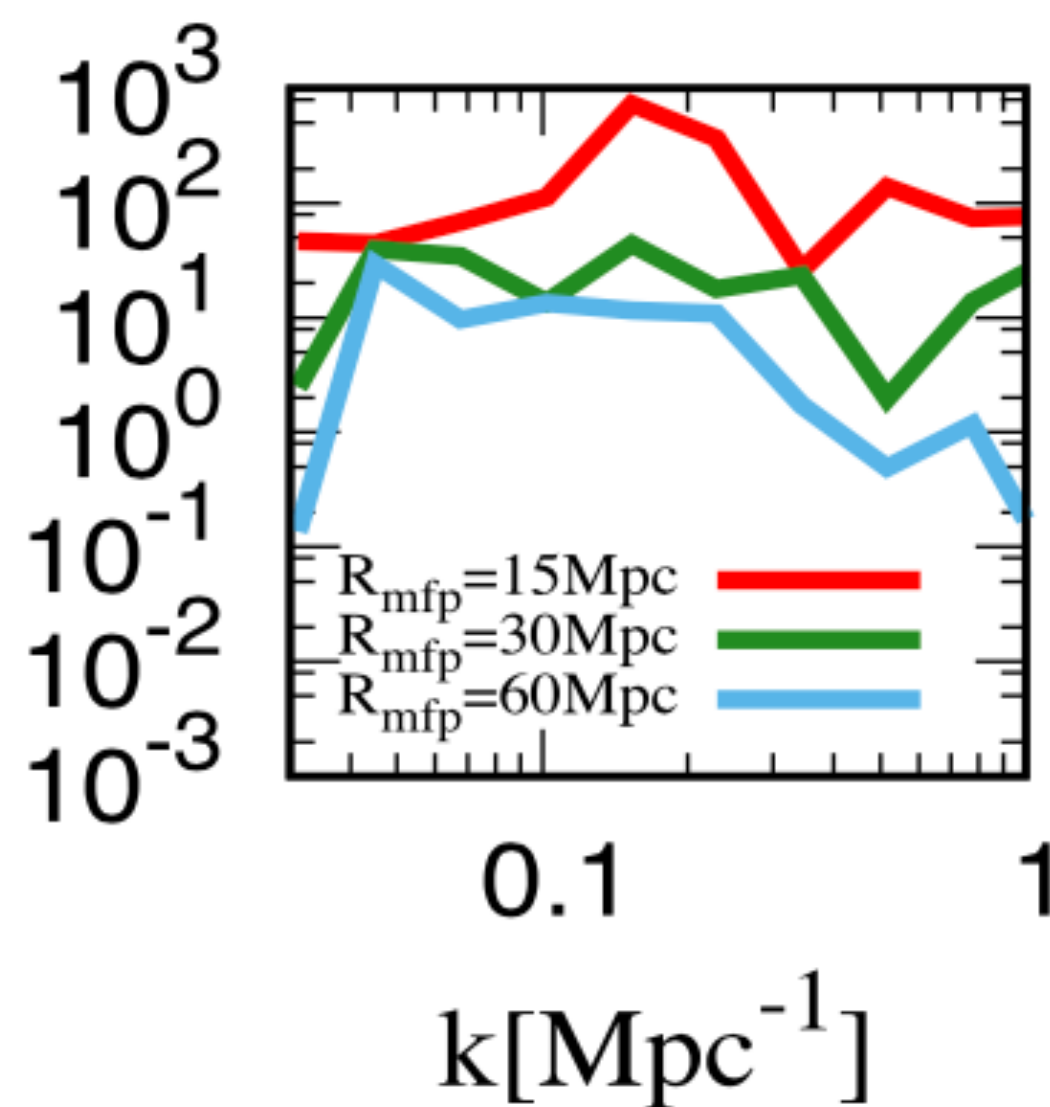
# What summary statistics to use?

## Power spectrum



HERA 2022

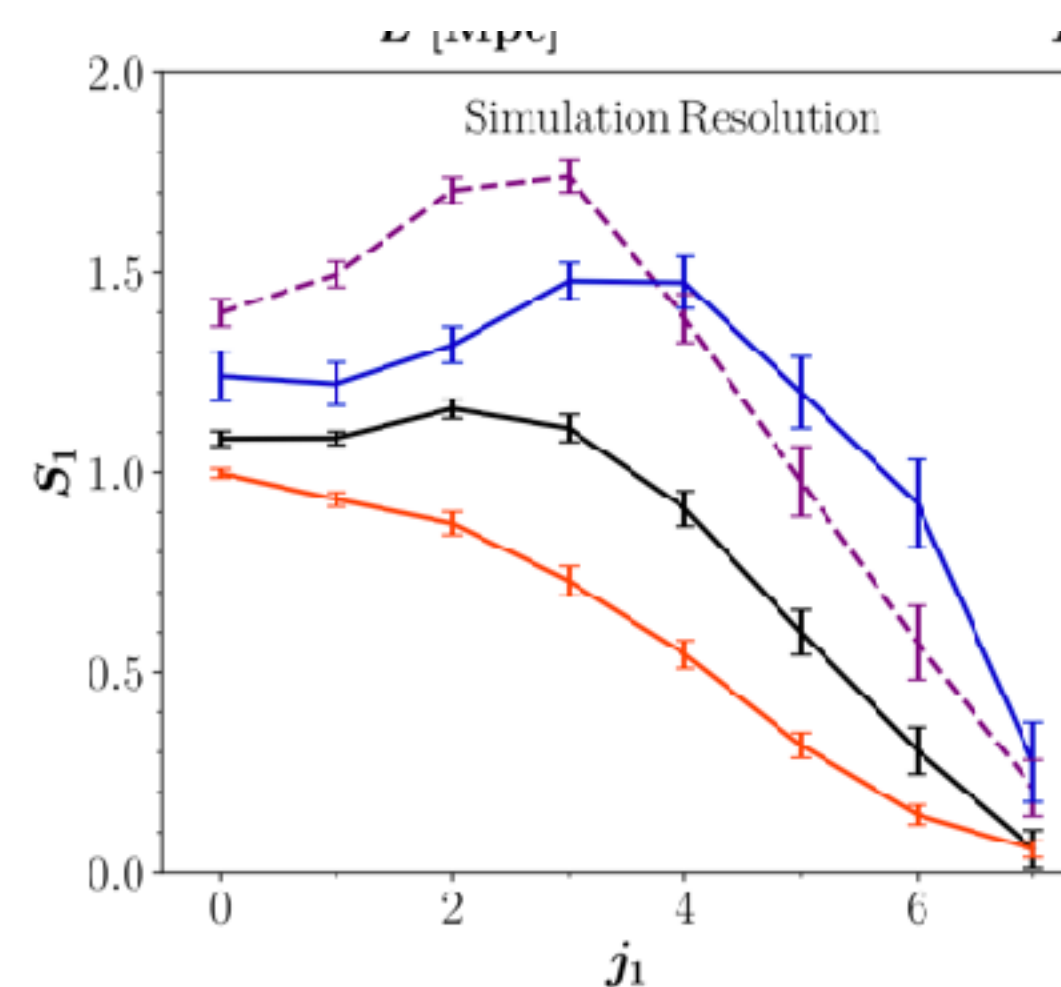
## Bispectrum



Shimabukuro+2017

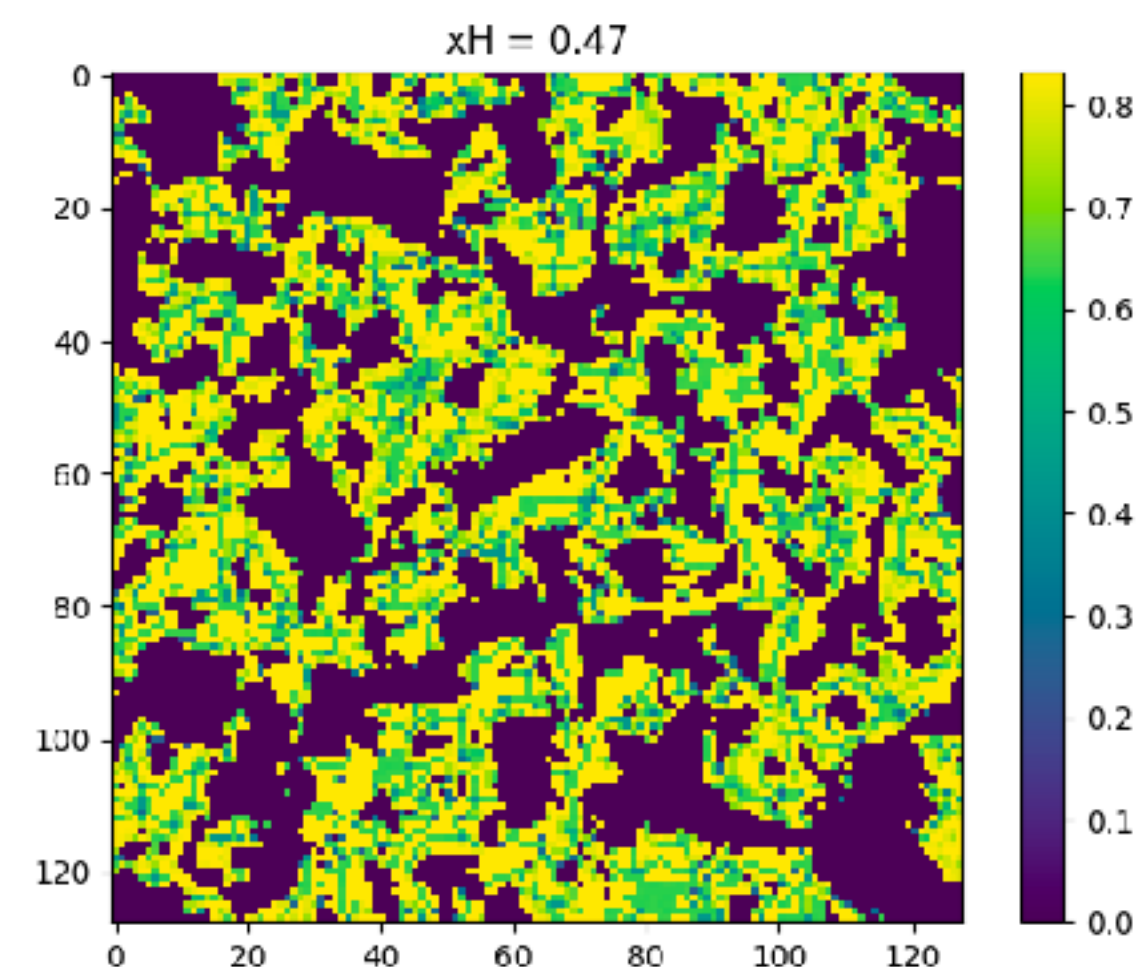
## WST

wavelet scattering transform



Greig+2023

## Image?



Can we directly deal with the image cubes?

Imaging is expected to be possible at the late stage of SKA1 or in SKA2

# Generative models



stability.ai

Stability AIについて

モデル

API

ニュース

English

using generative modeling has improved significantly in the last four years<sup>4</sup>

## Stable Diffusion XL

Create and inspire using the worlds fastest growing open source AI platform.

With Stable Diffusion XL, you can create descriptive images with shorter prompts and generate words within images. The model is a significant advancement in image generation capabilities, offering enhanced image composition and face generation that results in stunning visuals and realistic aesthetics.

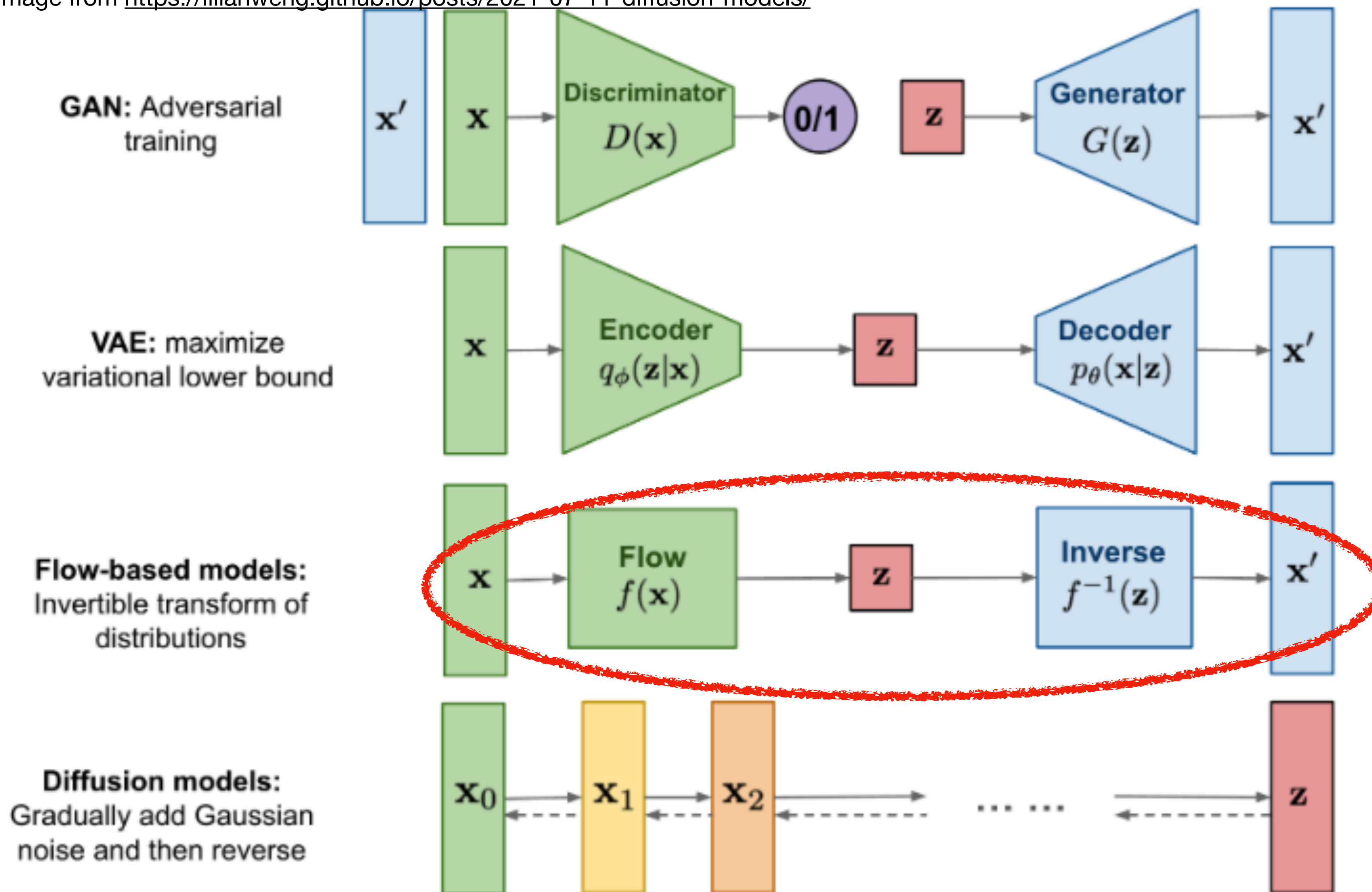
Stable Diffusion XL is currently in beta on DreamStudio and other leading imaging applications. Like all of Stability AI's foundation models, Stable Diffusion XL will be released as open source for optimal accessibility in the near future.

DreamStudio



# There are several different generative models

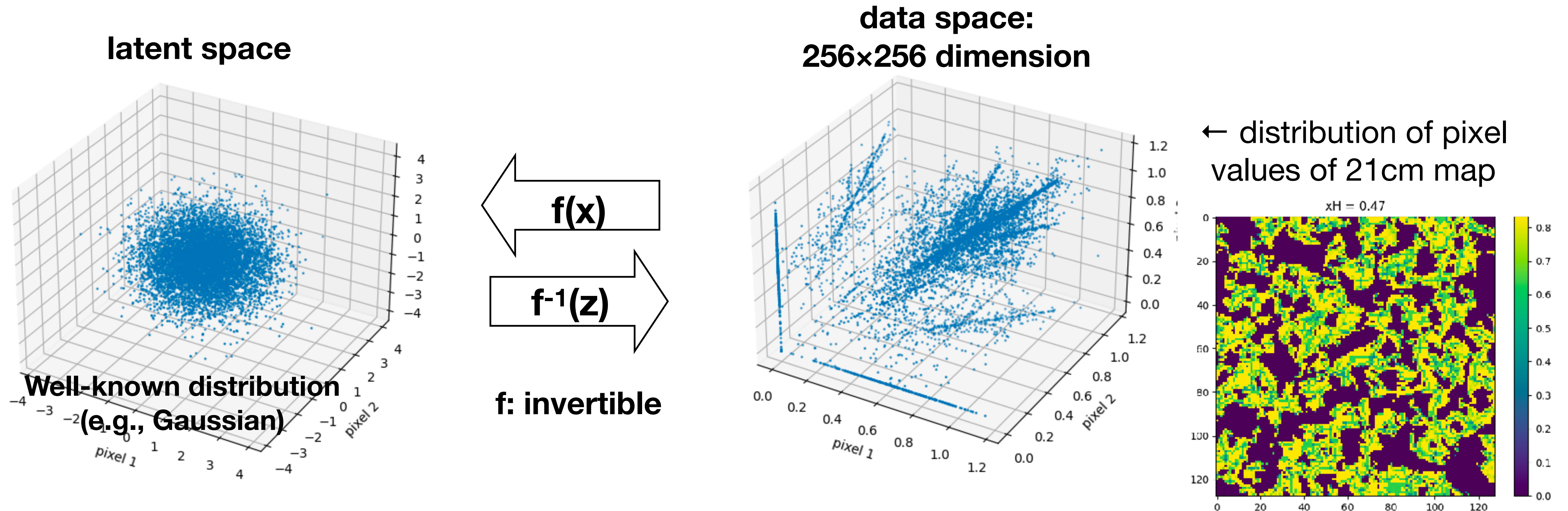
Image from <https://lilianweng.github.io/posts/2021-07-11-diffusion-models/>



**Might not be the SOTA in terms of generation quality... But NF tells us about the probability density**

# Data space $\Leftrightarrow$ Latent space

Flow-based models transfer data into latent space with invertible functions.



- When generating a new data, one can sample a random point in the latent space and apply “ $f^{-1}$ ”
- Invertible functions can be used to infer the probability density of a given data

# What transformation to use?

**Transformations are invertible and their determinants can be (easily) computed**

**Examples:**

$$f(x) = \begin{pmatrix} 3 & 2 \\ 0 & 4 \end{pmatrix} x \longrightarrow f^{-1}(x) = \begin{pmatrix} 1/3 & -1/6 \\ 0 & 1/4 \end{pmatrix} x, \quad \det \left( \frac{\partial f(x)}{\partial x} \right) = 3 \times 4$$

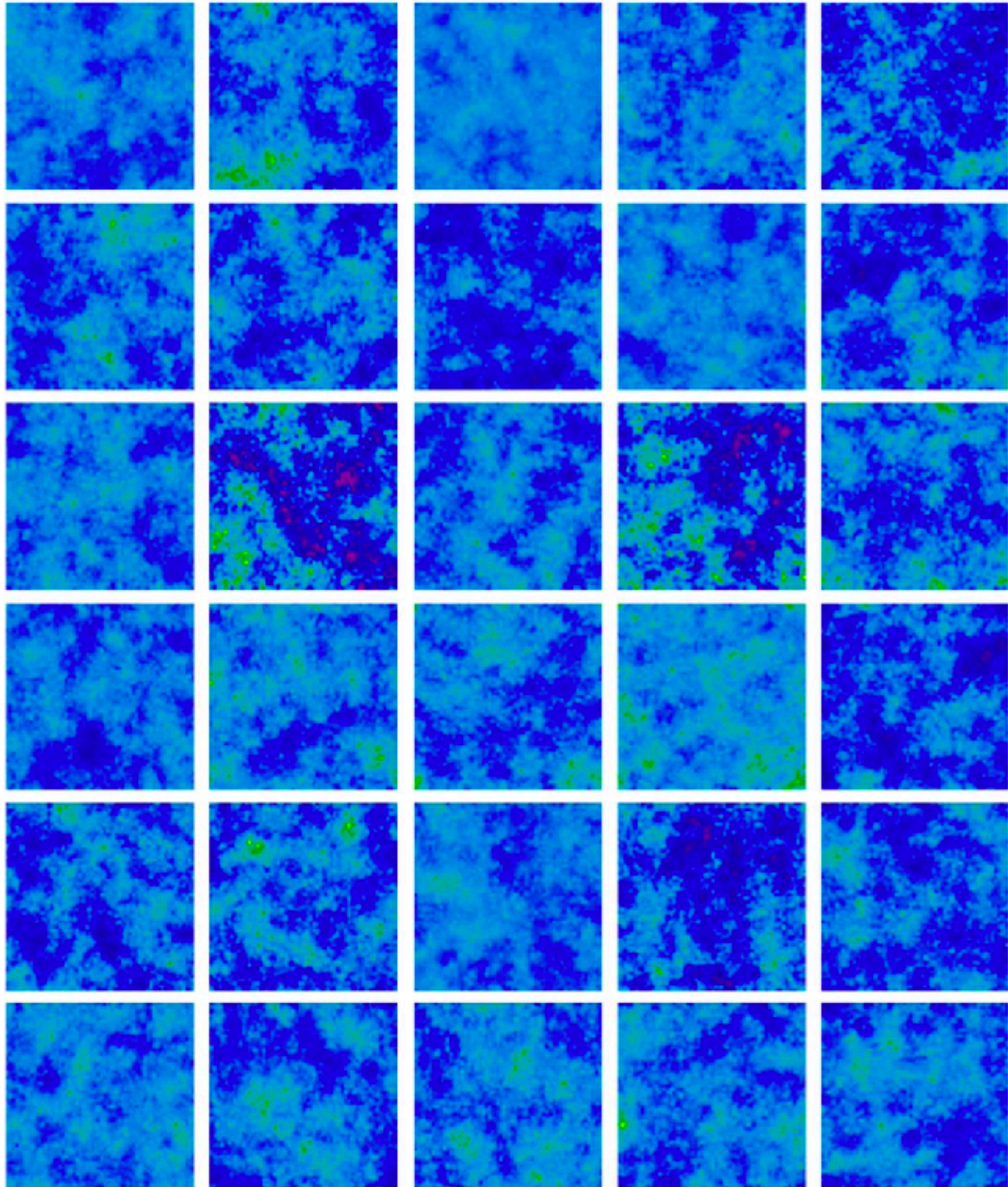
$$f : \text{permutation} \longrightarrow f^{-1} : \text{inverse permutation}, \quad \det \left( \frac{\partial f(x)}{\partial x} \right) = 1$$



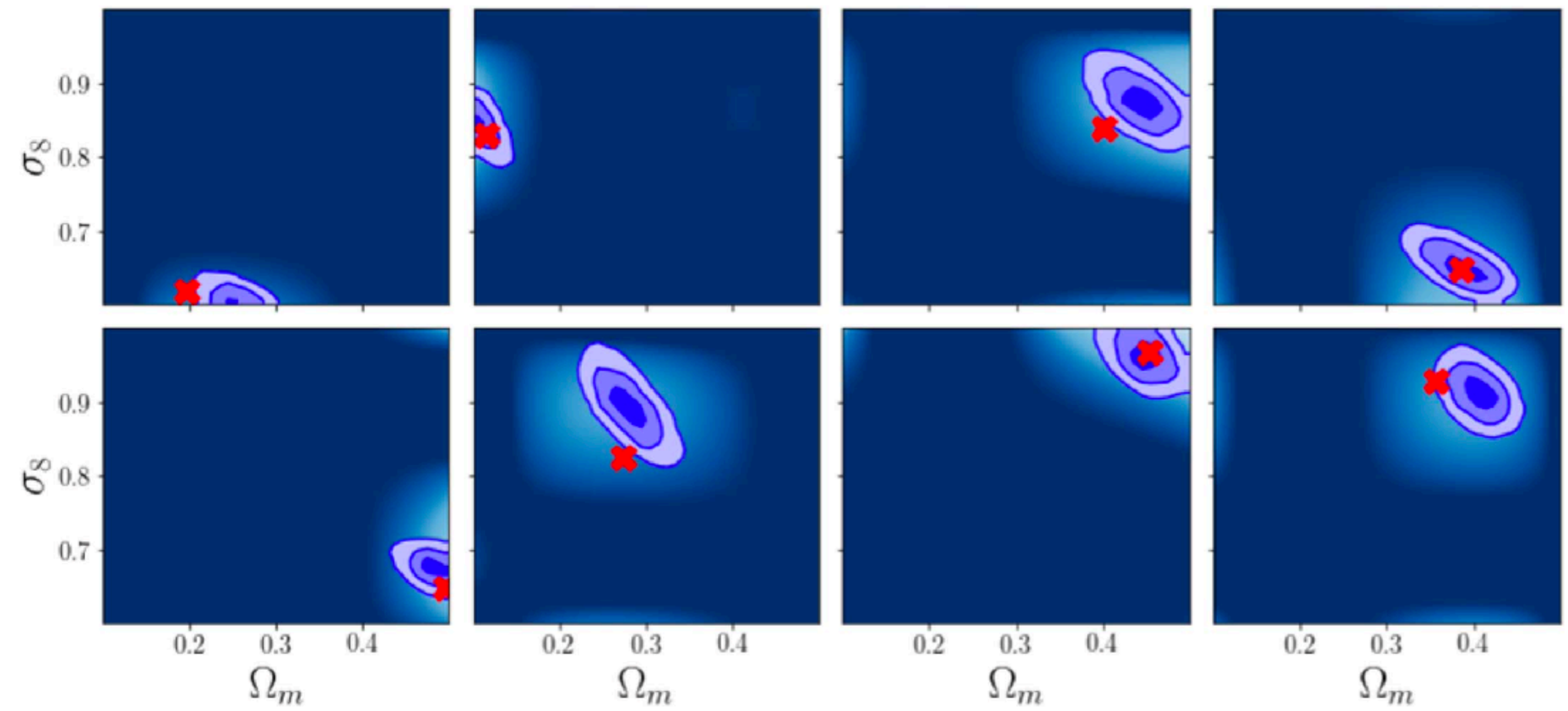
permute pixel 0 and 3

# Hassan+ 2022: application of NF to post-reionization 21cm maps

## Generated LSS images



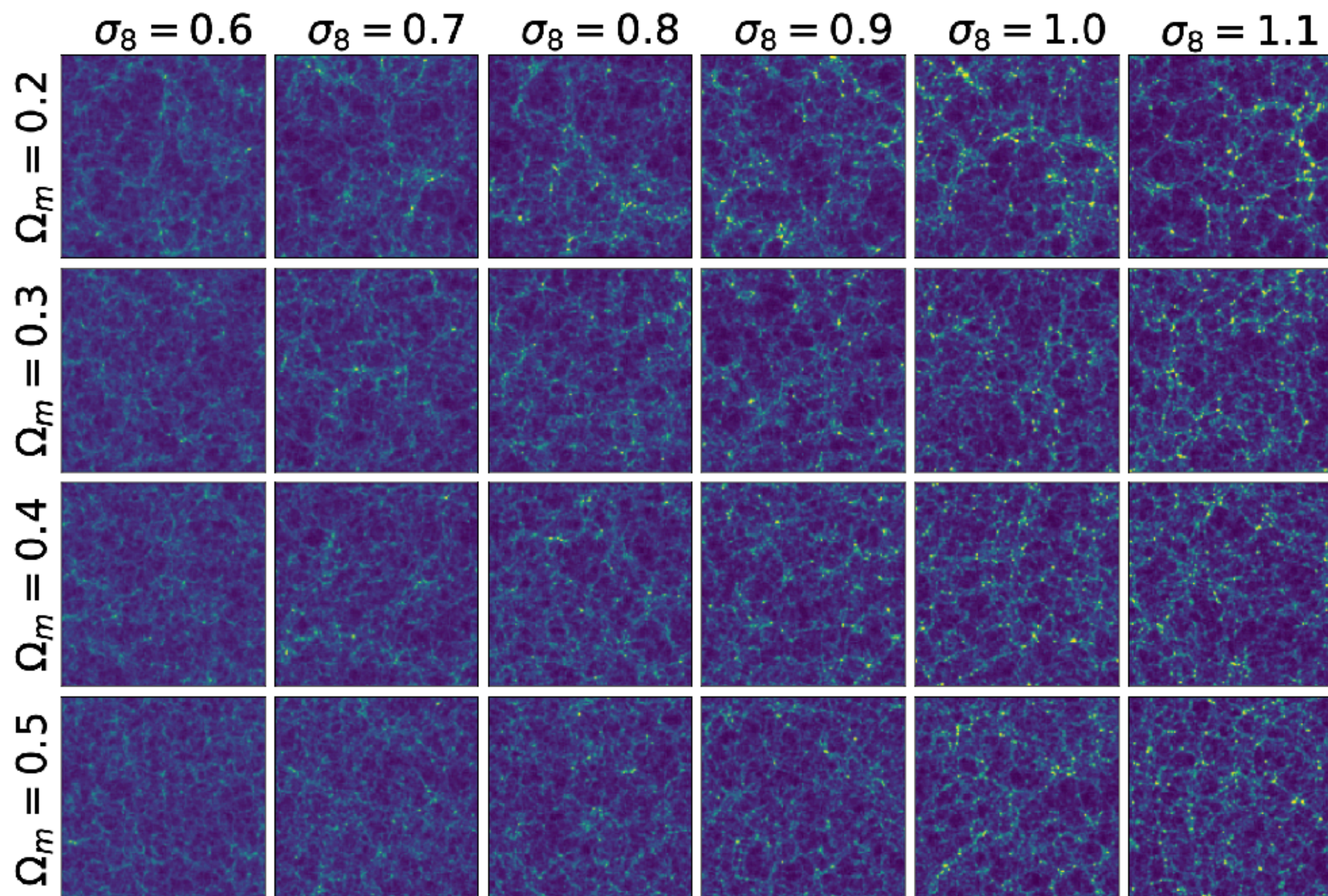
## Cosmological parameter inference



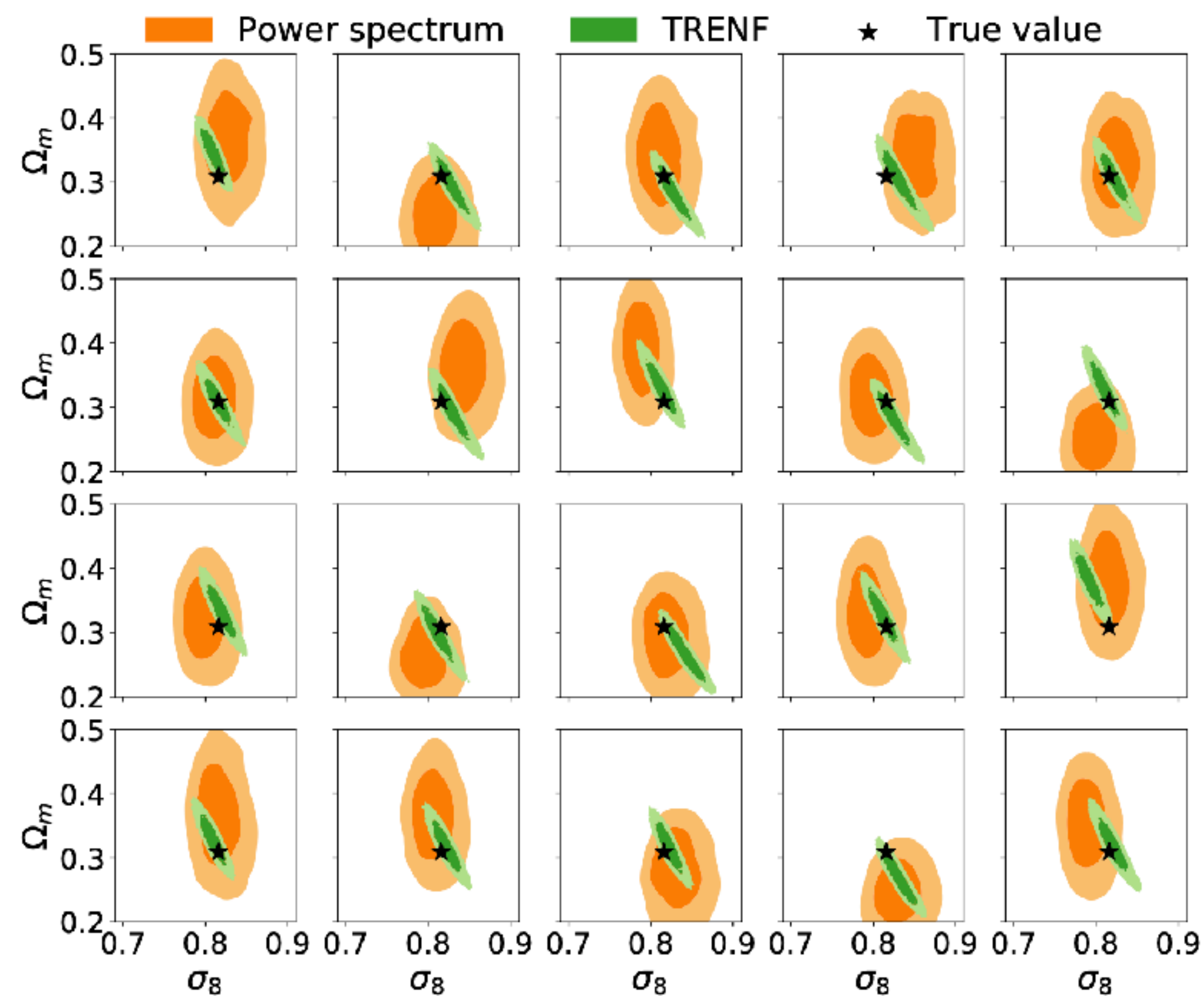


# Dai+2022: Translation and Rotation Equivariant Normalizing Flow

## Generated LSS images



## Cosmological parameter inference



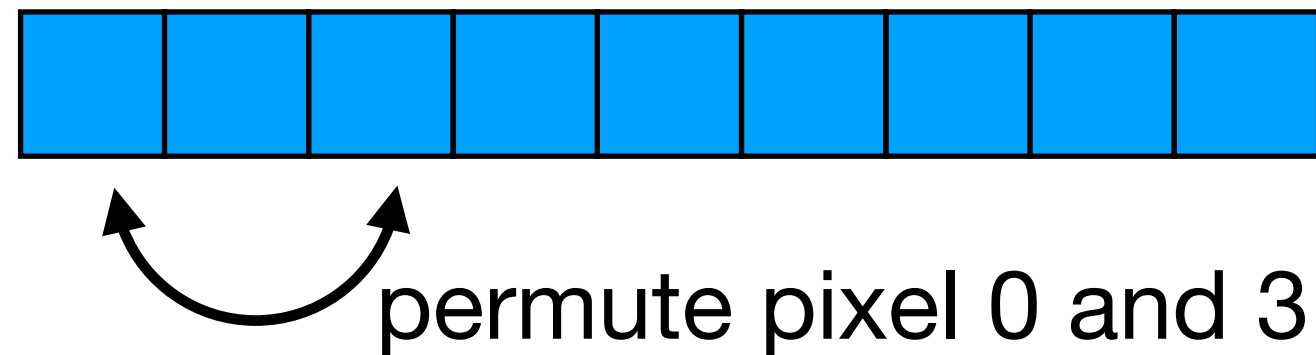
# What transformation to use?

Transformations are invertible and their determinants can be (easily) computed

Examples:

$$f(x) = \begin{pmatrix} 3 & 2 \\ 0 & 4 \end{pmatrix} x \longrightarrow f^{-1}(x) = \begin{pmatrix} 1/3 & -1/6 \\ 0 & 1/4 \end{pmatrix} x, \quad \det \left( \frac{\partial f(x)}{\partial x} \right) = 3 \times 4$$

$$f : \text{permutation} \longrightarrow f^{-1} : \text{inverse permutation}, \quad \det \left( \frac{\partial f(x)}{\partial x} \right) = 1$$

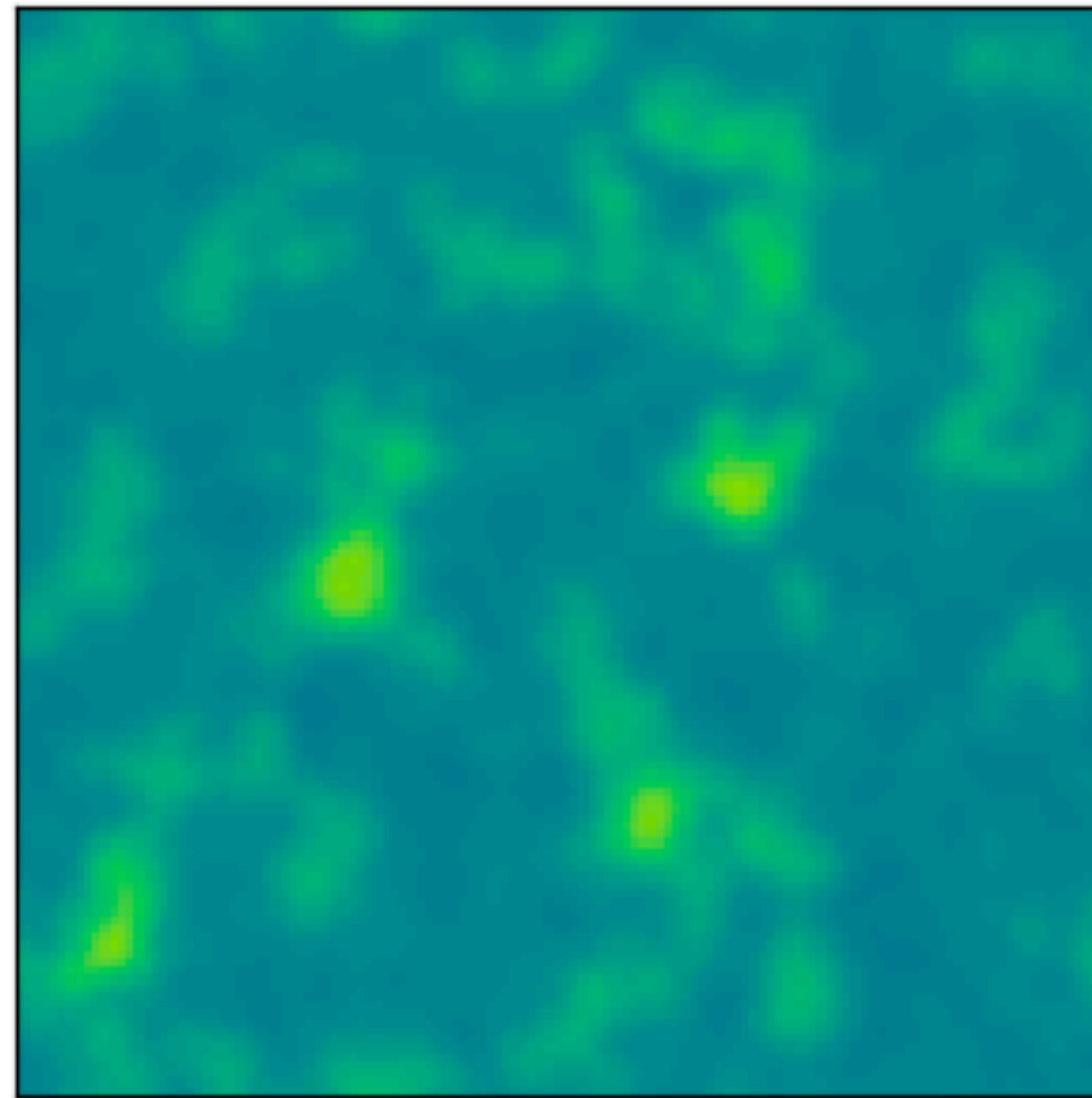


**Translation and Rotation Equivariant Normalizing Flow (TRENF; Dai+2022)**

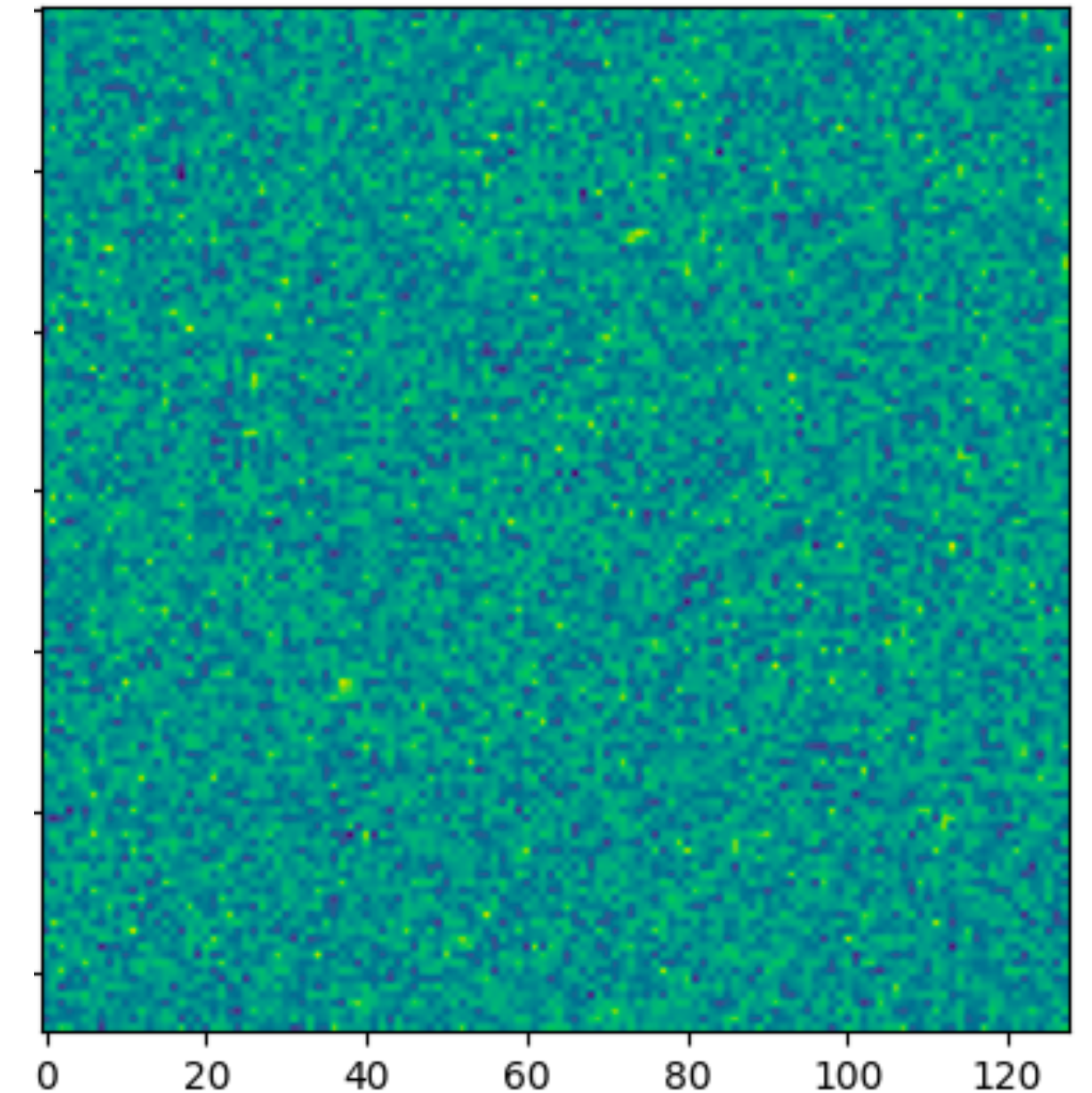
$$f = \Psi \left( \hat{F}^{-1} \tilde{T}(k) \hat{F} x \right) \quad \begin{array}{l} \Psi : \text{Monotonic nonlinear function} \\ \tilde{T}(k) : \text{non-zero function} \end{array}$$

# Let us use NF for EoR 21cm map!

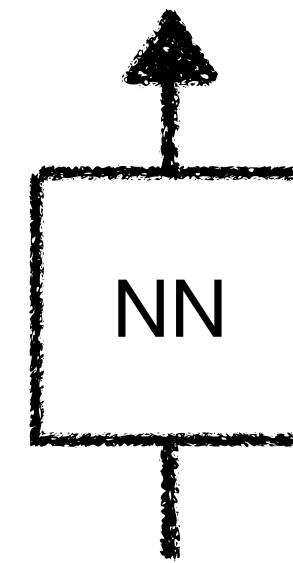
21cm map + noise + smoothing



Latent variable



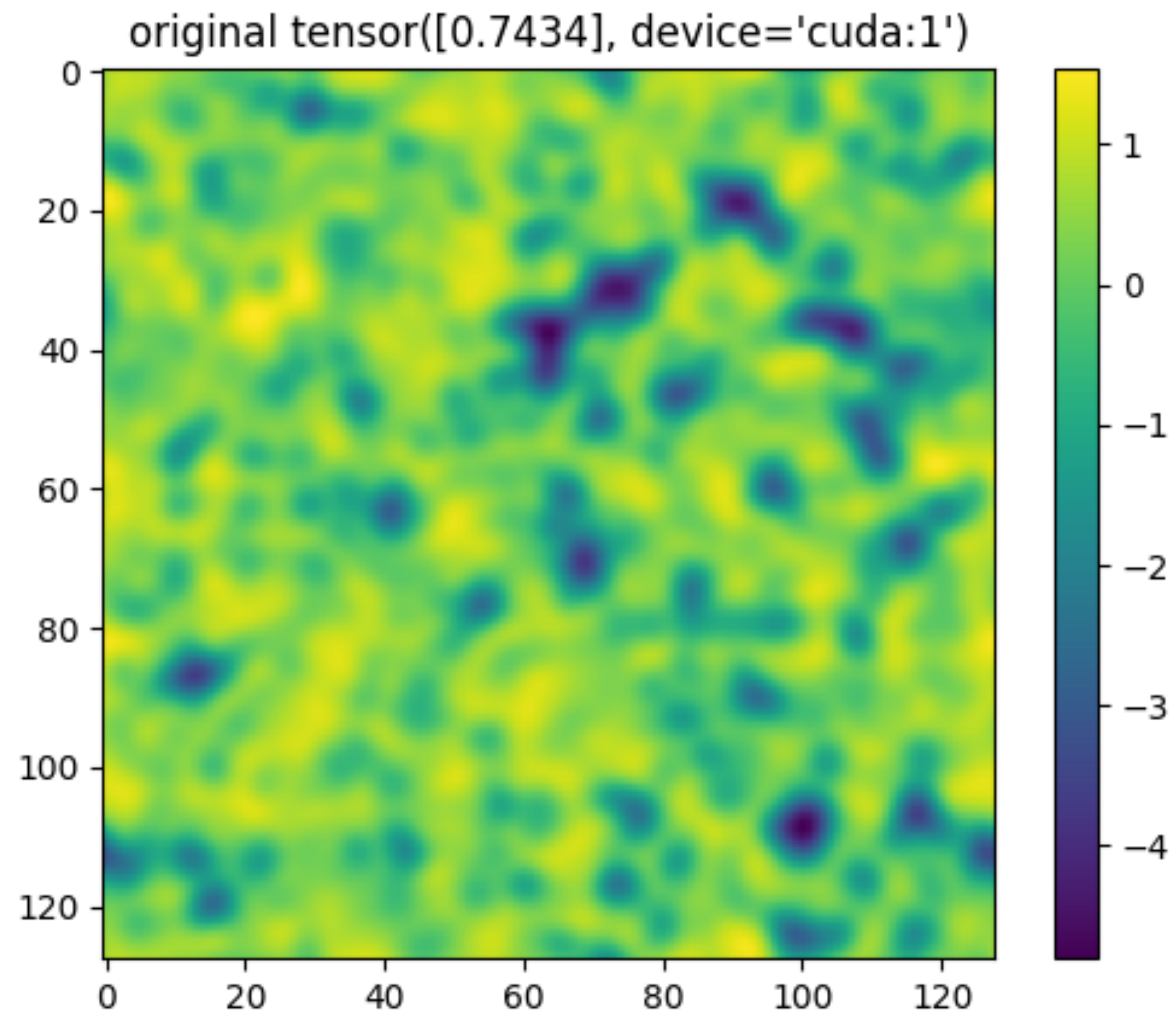
Transformation  
parameters  
 $T_1, T_2, \dots, T_N$



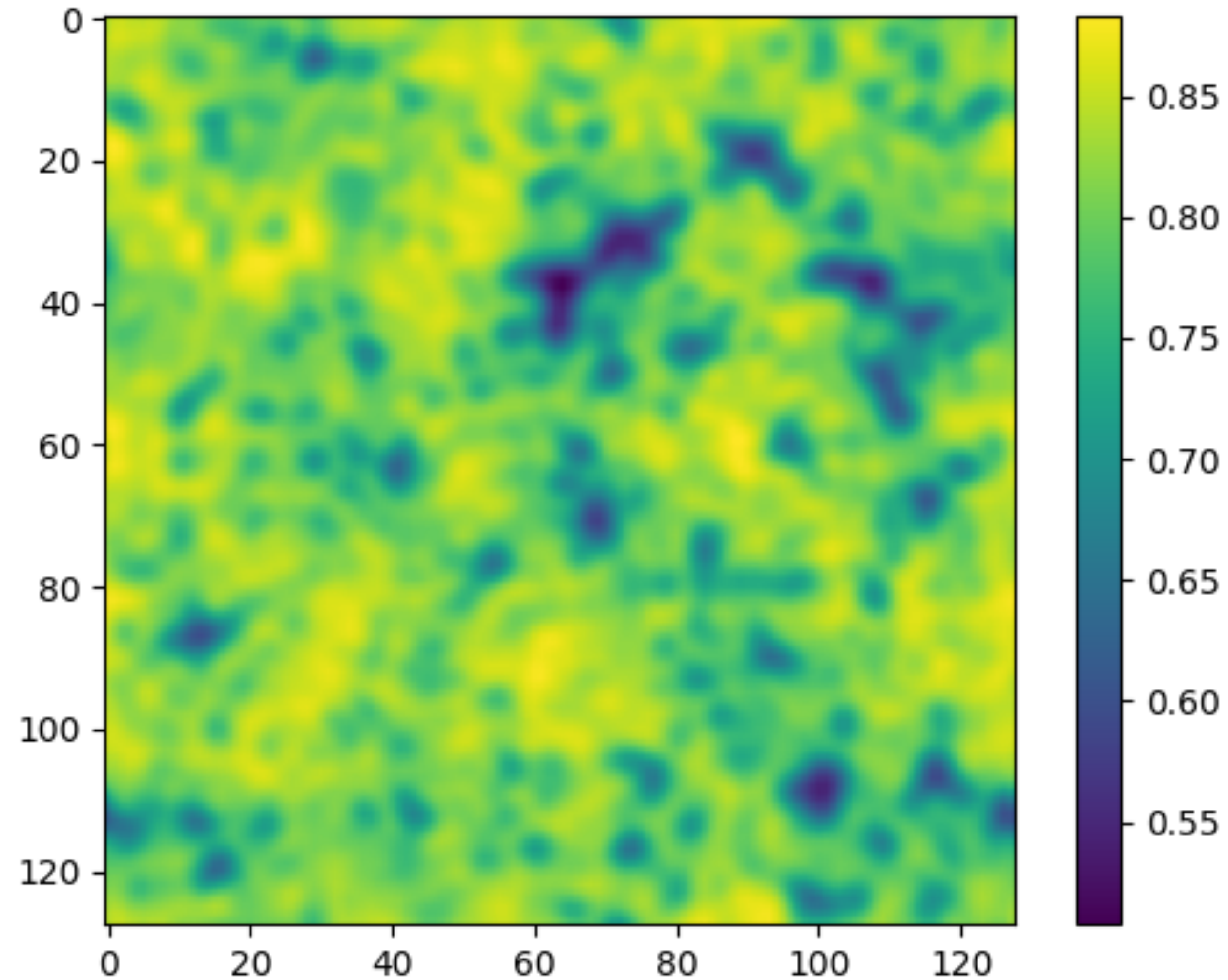
neutral fraction

Training data:  
21cmFAST with different  
parameters of  $f^*$ ,  $\alpha^*$ ,  $f_{\text{esc}}$ ,  
 $\alpha_{\text{esc}}$ ,  $t^*$ ,  $M_{\text{turn}}$ ,  $L_X$ ,  $E_0$ ,  $\alpha_X$

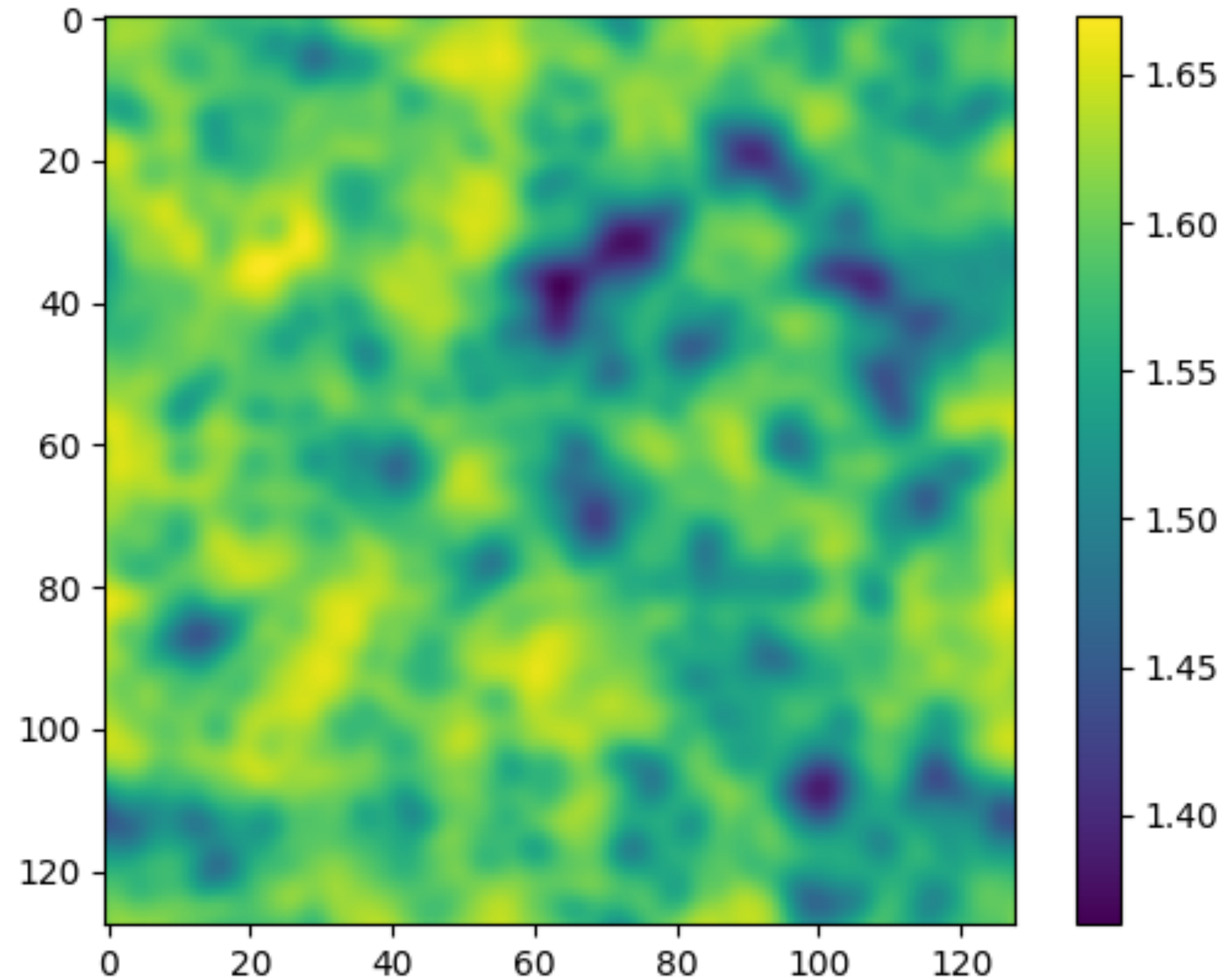
# Result: transform to noise



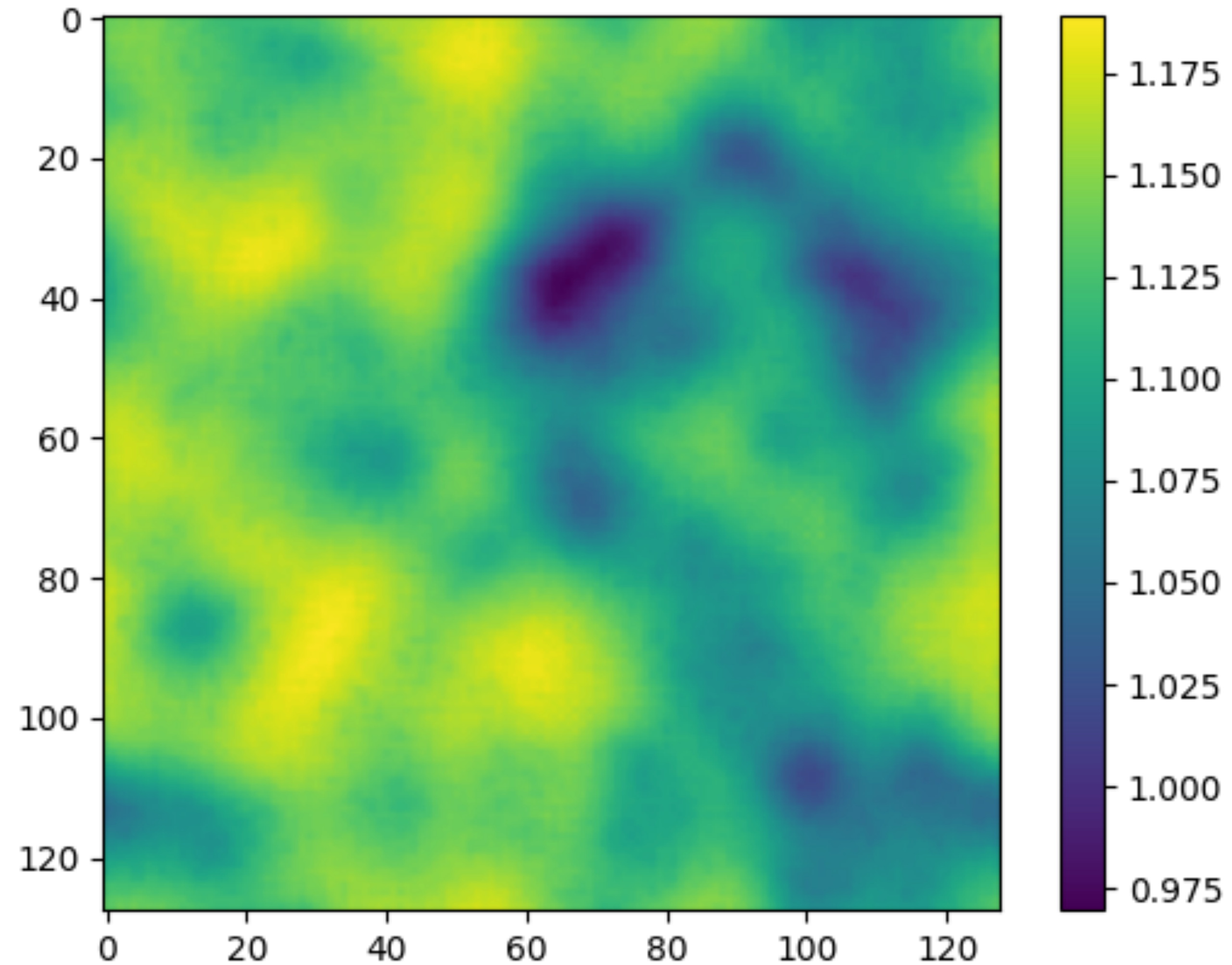
# Result: transform to noise



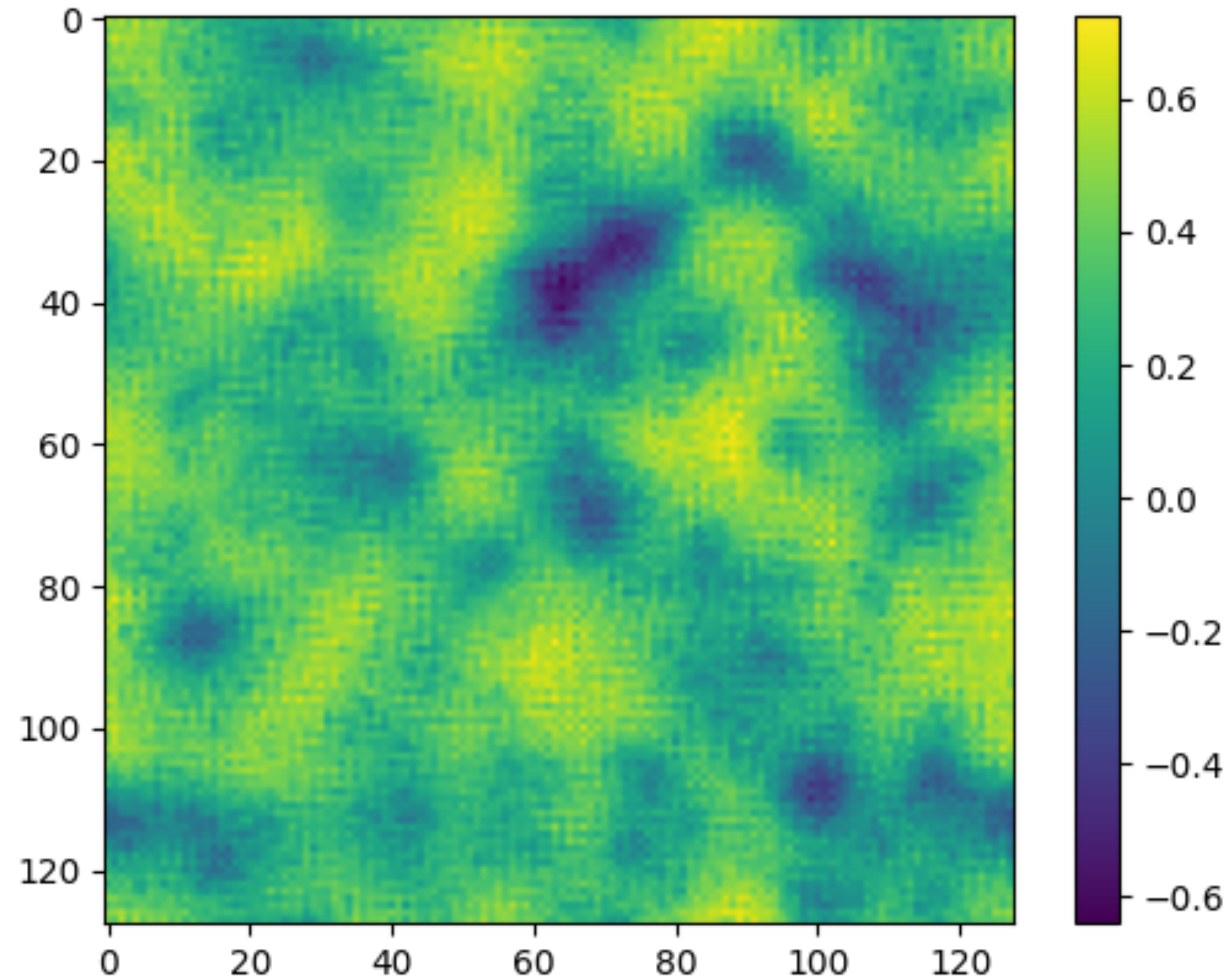
# Result: transform to noise



# Result: transform to noise

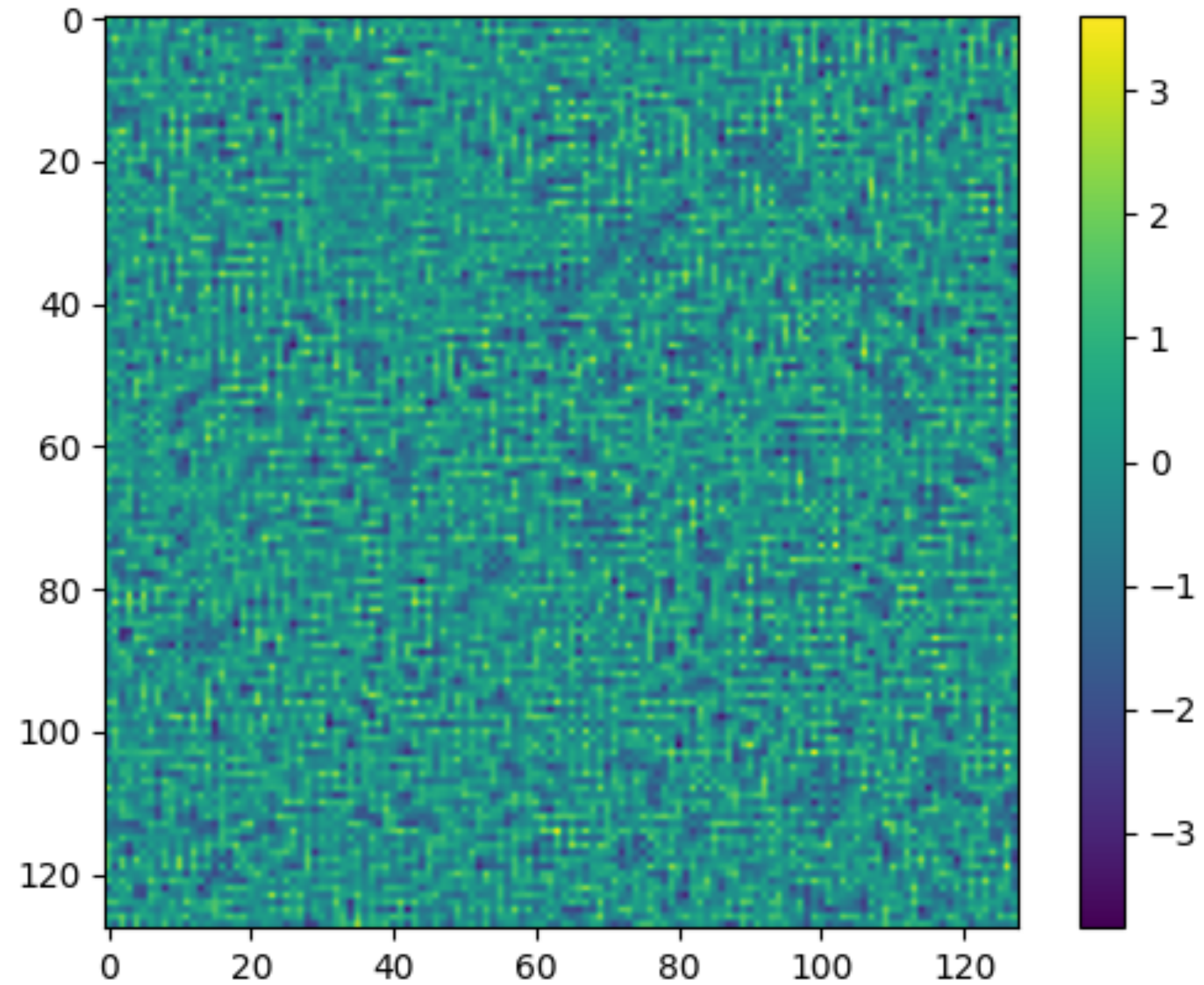


# Result: transform to noise



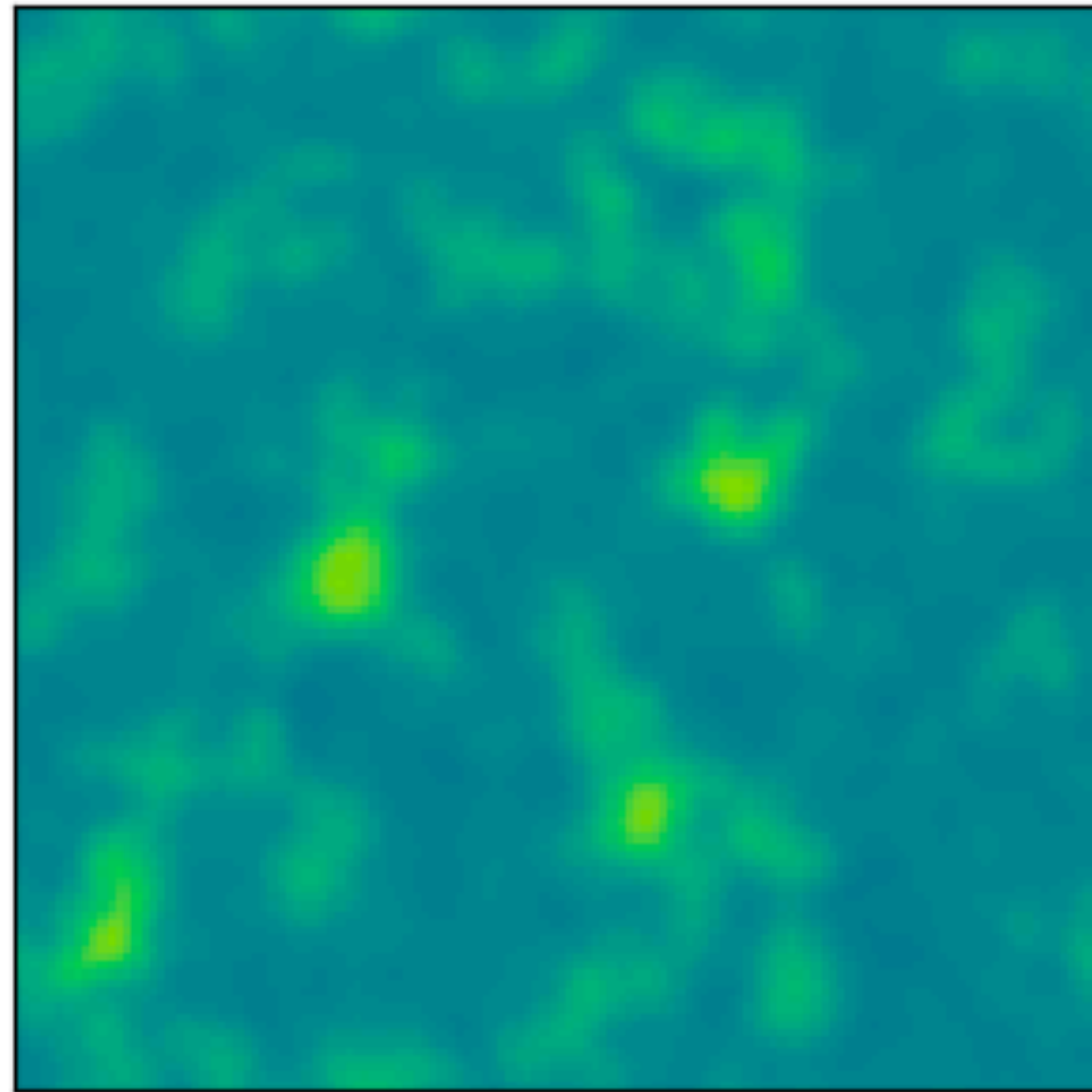


# Result: transform to noise

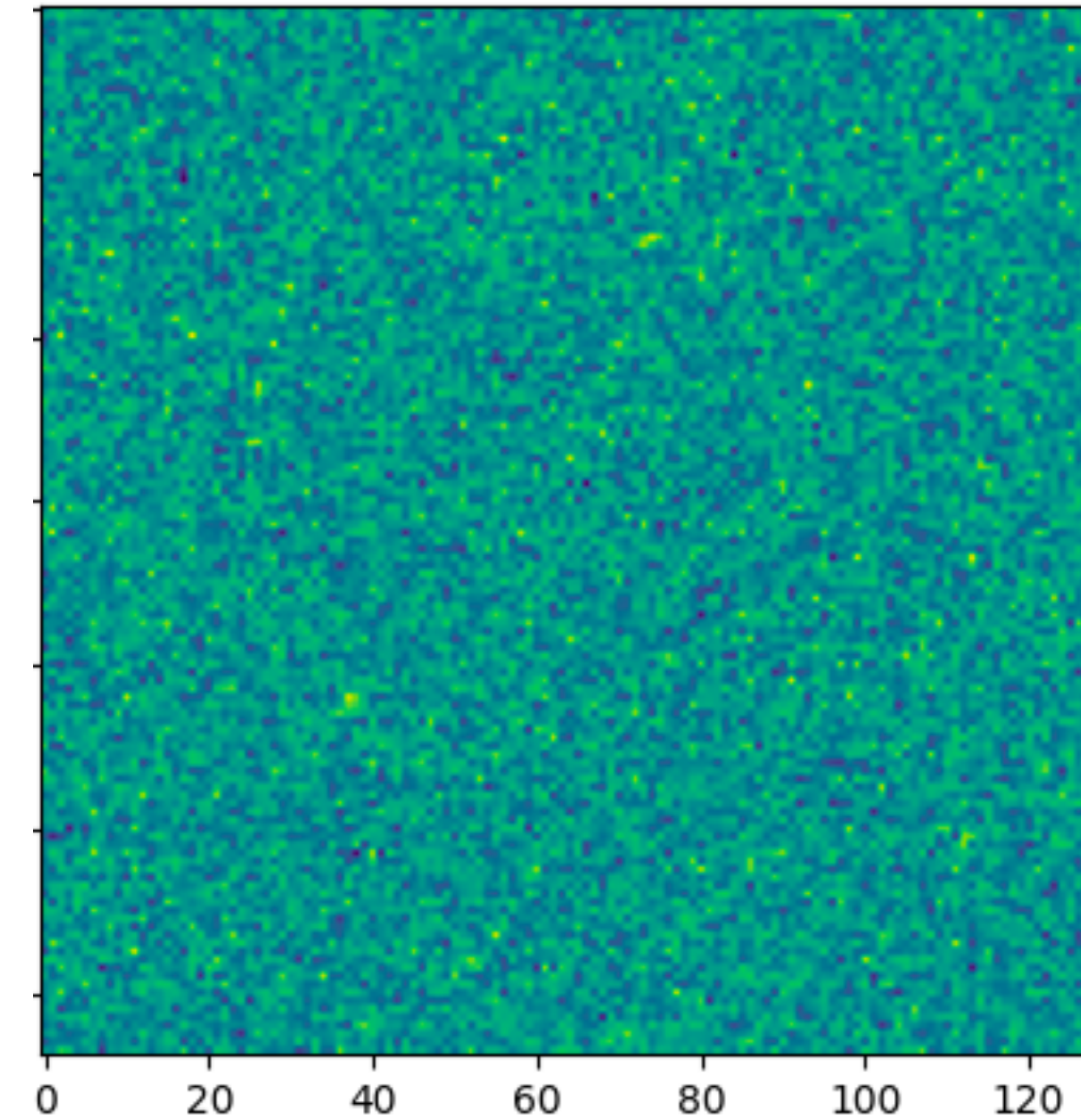


# Generation of new images

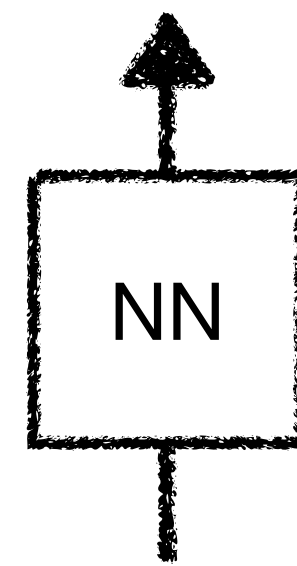
21cm map + noise + smoothing



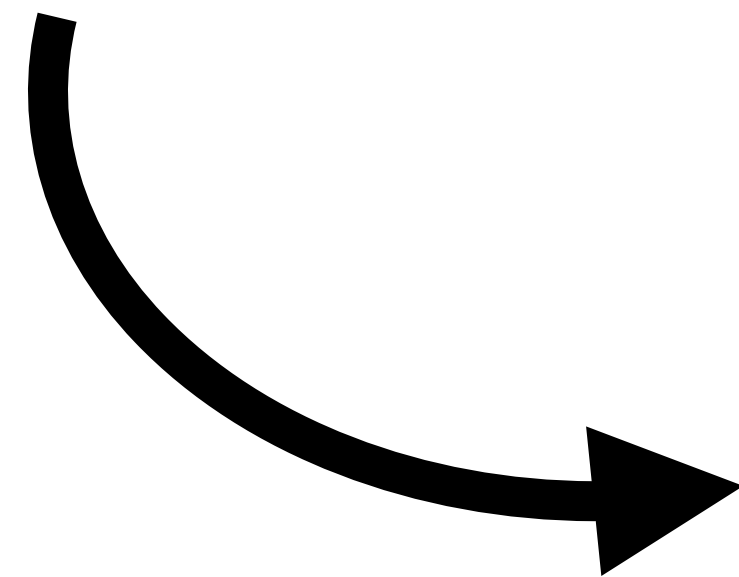
Latent variable



Transformation  
parameters  
 $T_1, T_2, \dots, T_N$

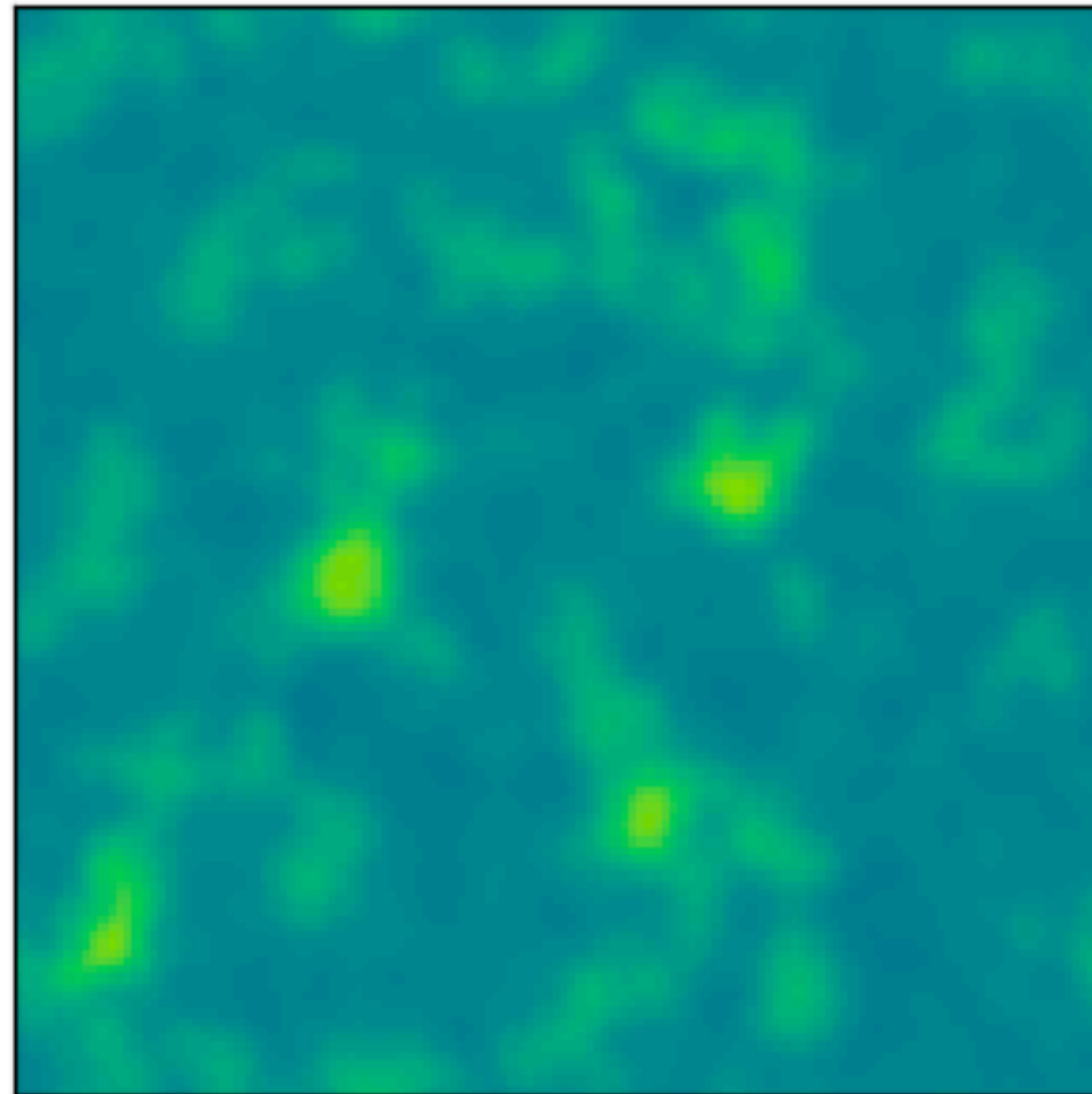


neutral fraction

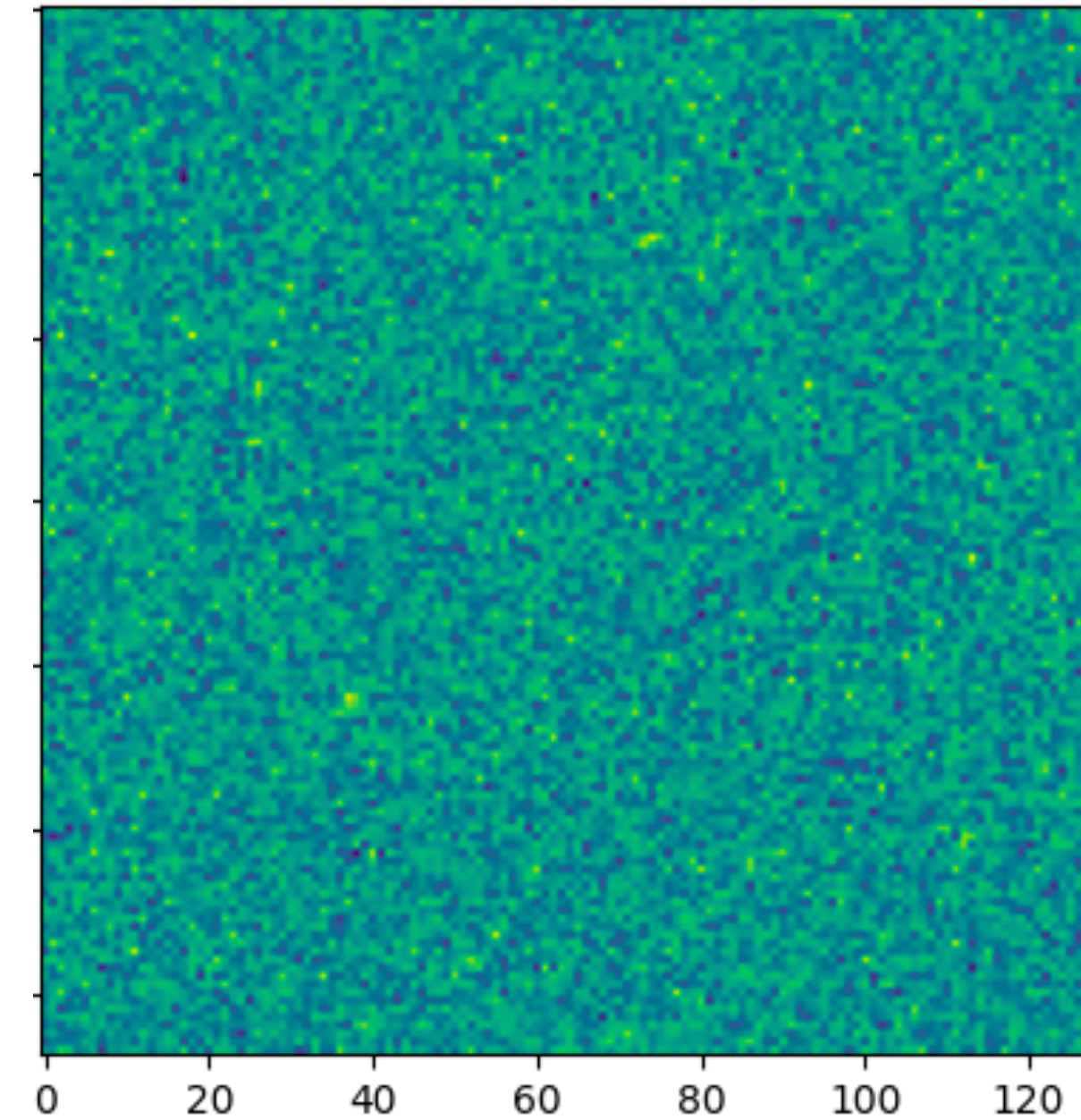


# Generation of new images

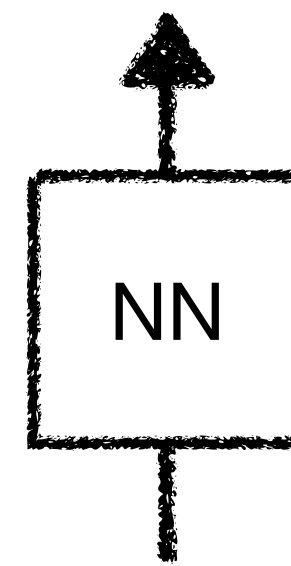
21cm map + noise + smoothing



Latent variable

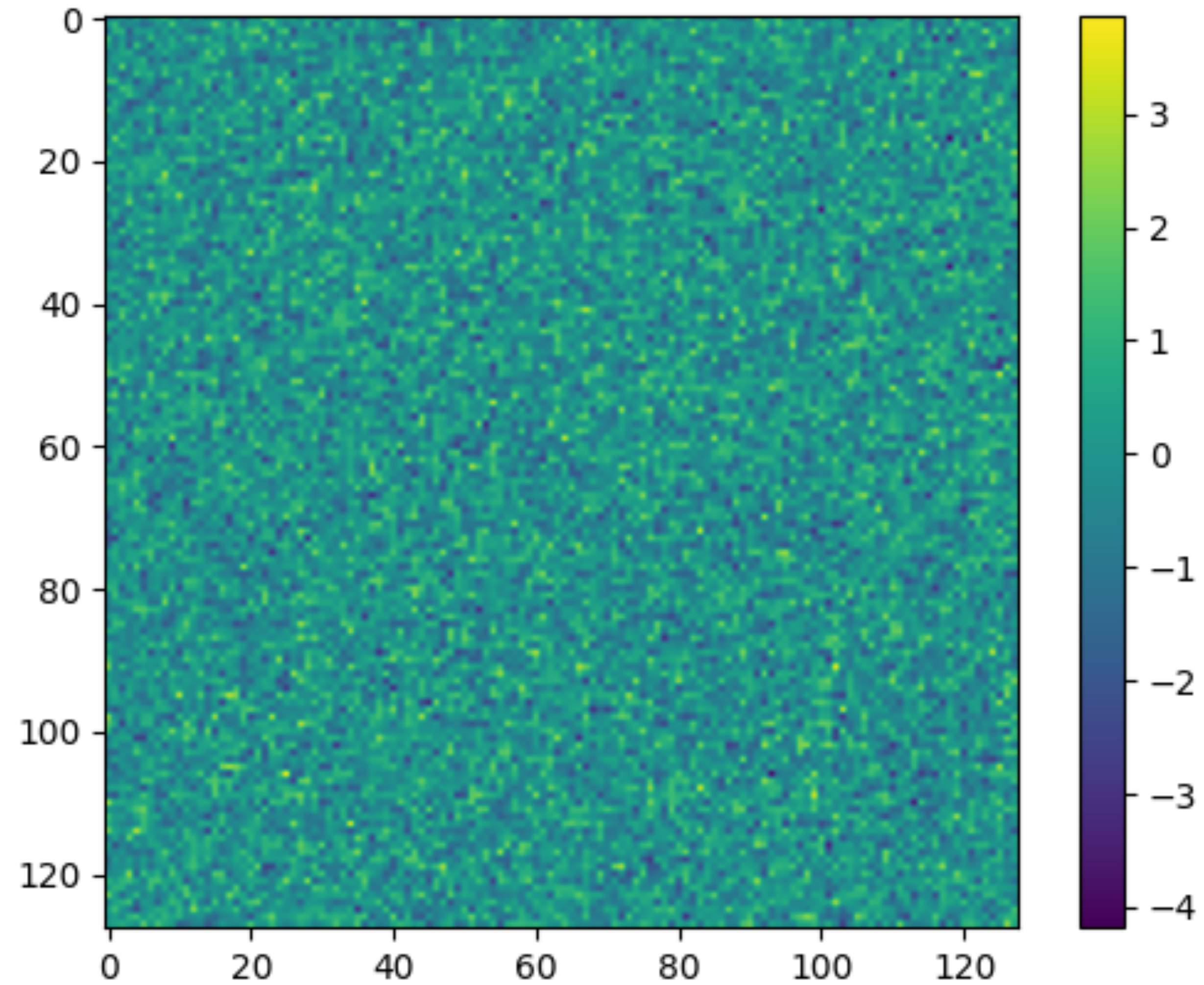


Transformation  
parameters  
 $T_1, T_2, \dots, T_N$

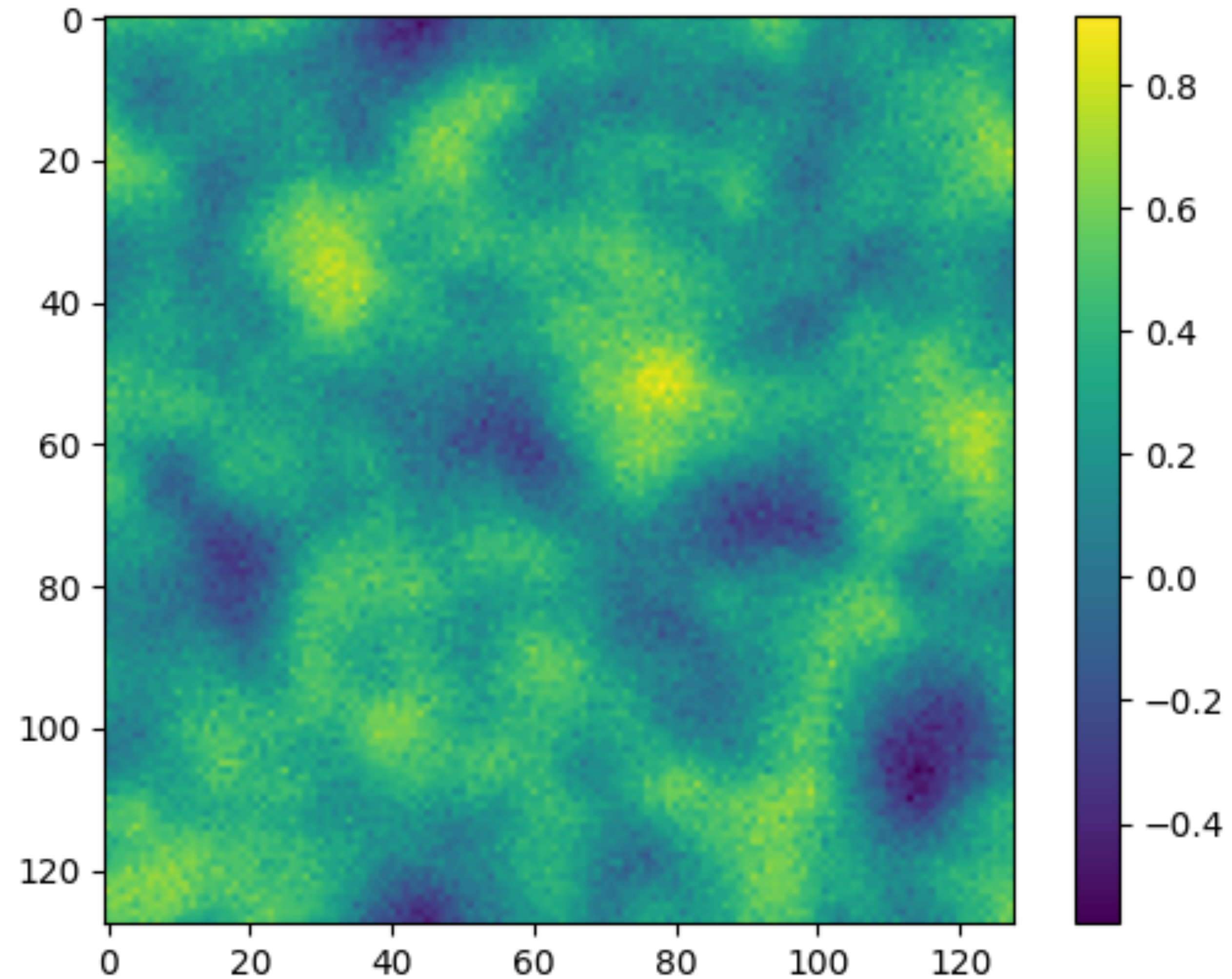


neutral fraction

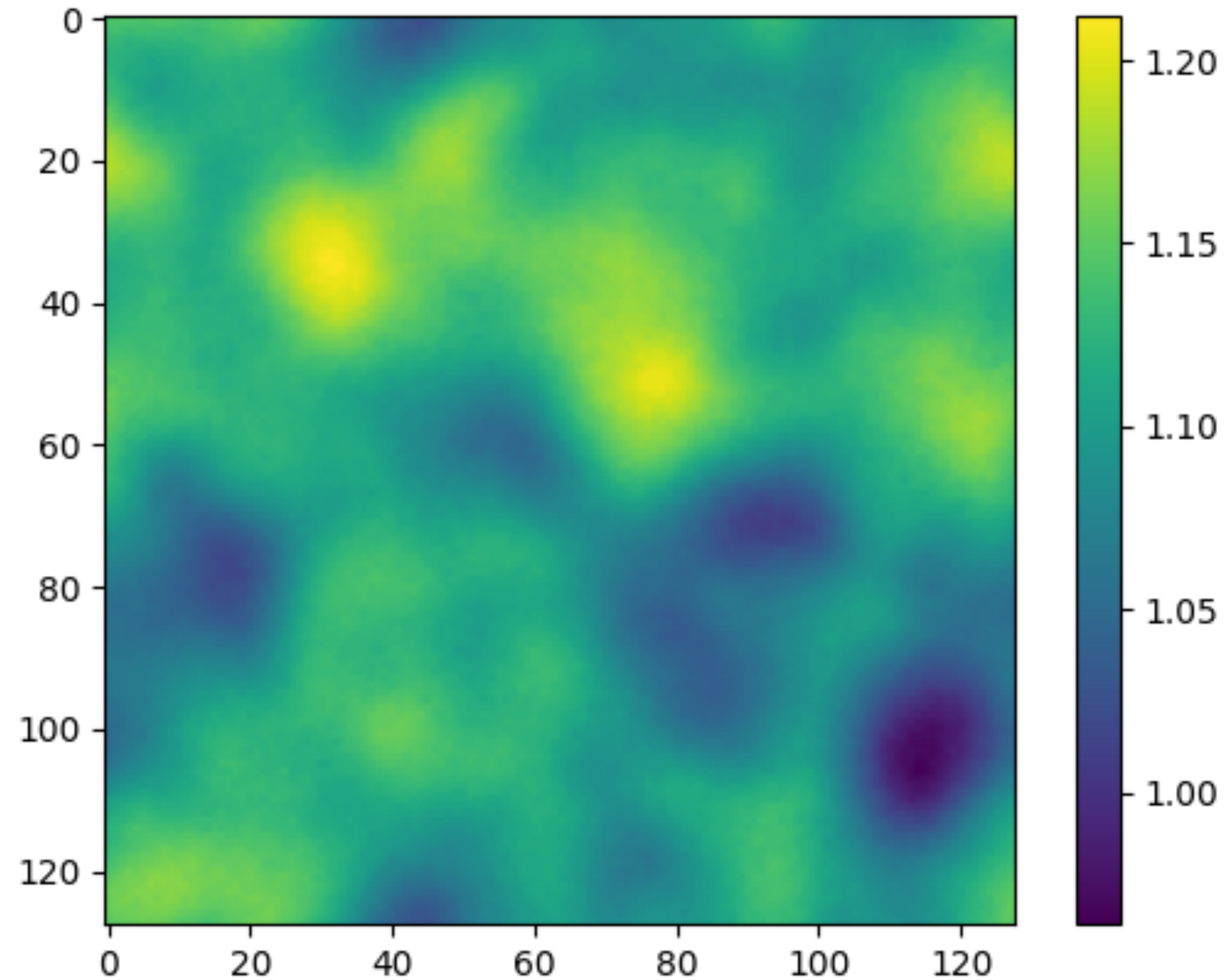
# Result: transform from random noise



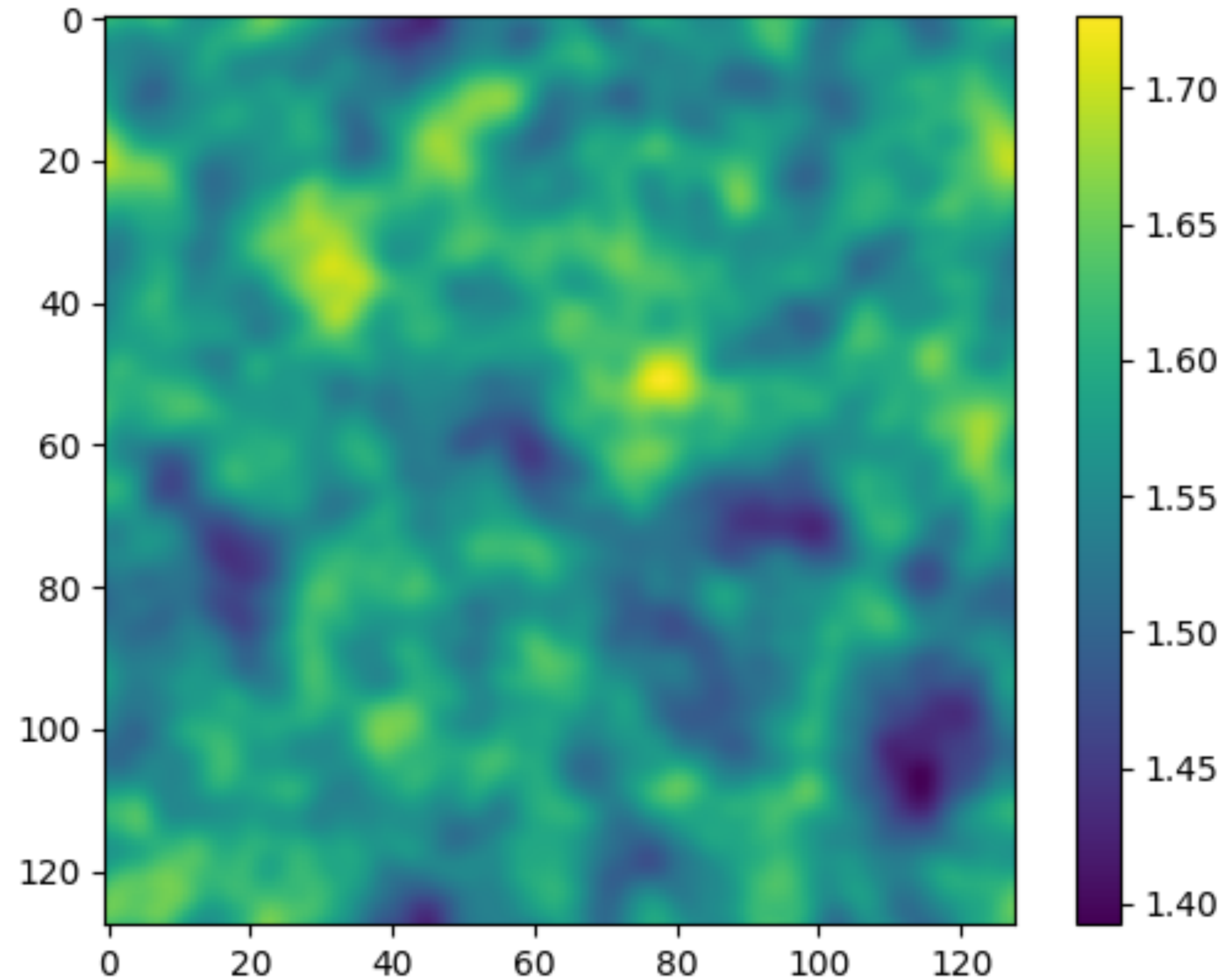
# Result: transform from random noise



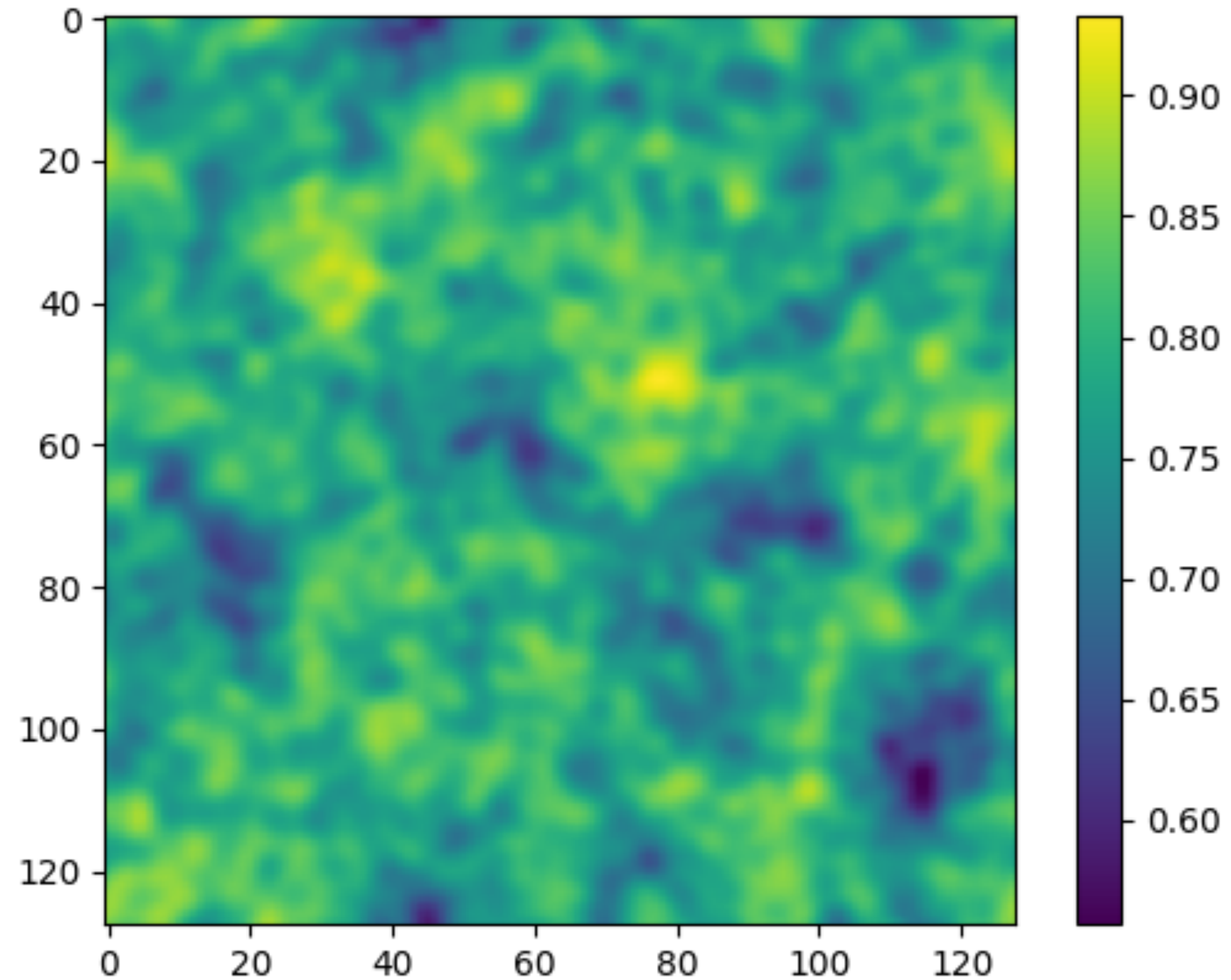
# Result: transform from random noise



# Result: transform from random noise



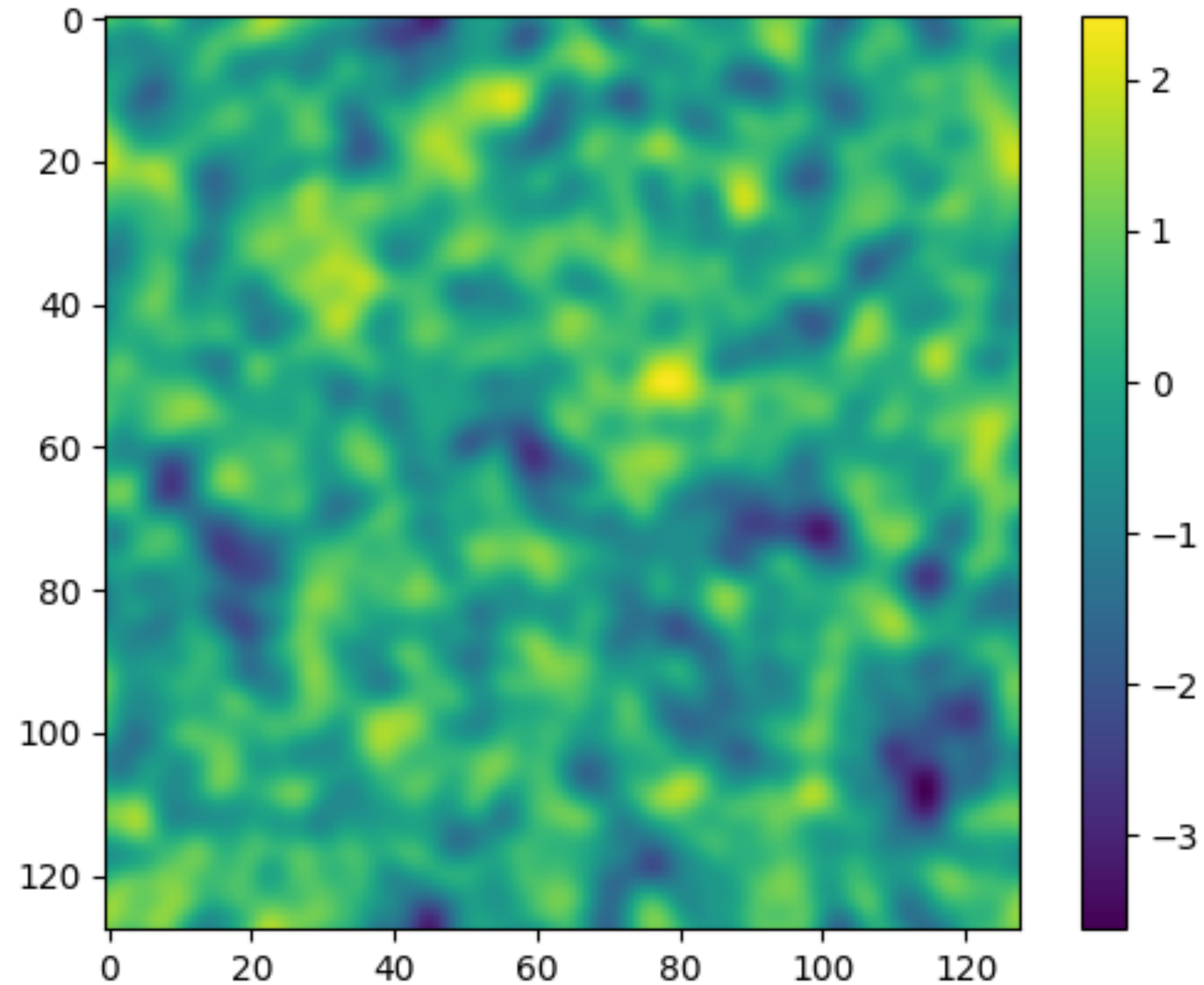
# Result: transform from random noise





# Result: transform from random noise

Generated image



# Result: generated images

$\langle x_H \rangle = 0.1$

$\langle x_H \rangle = 0.3$

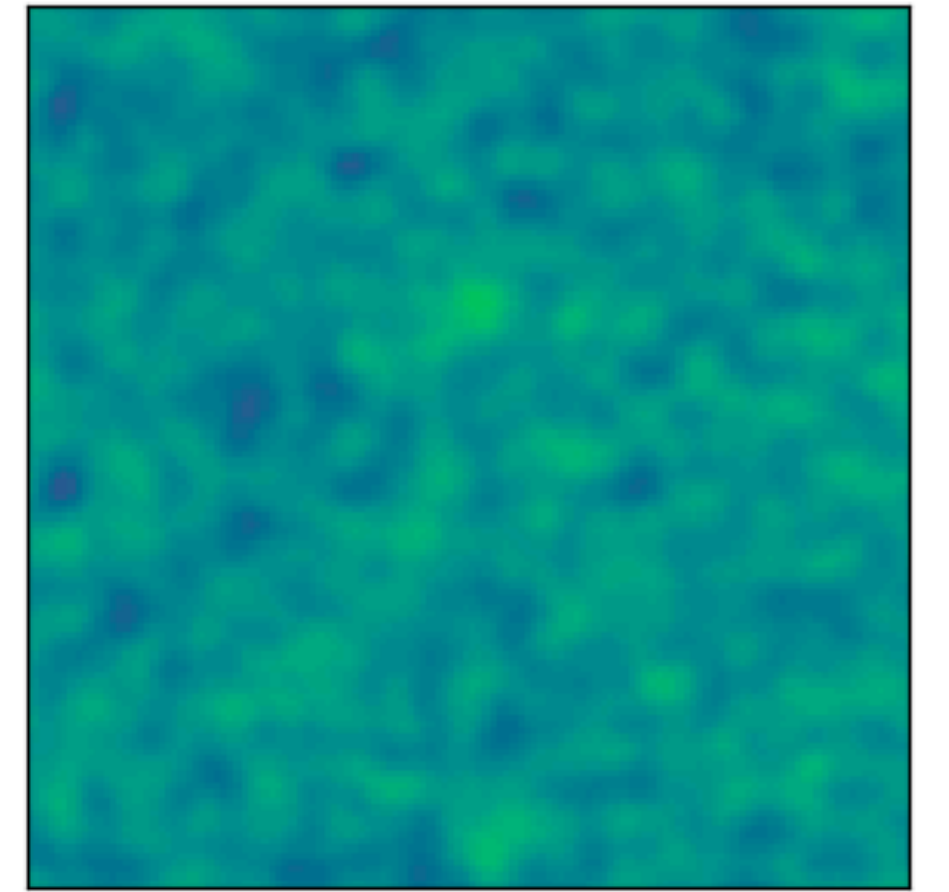
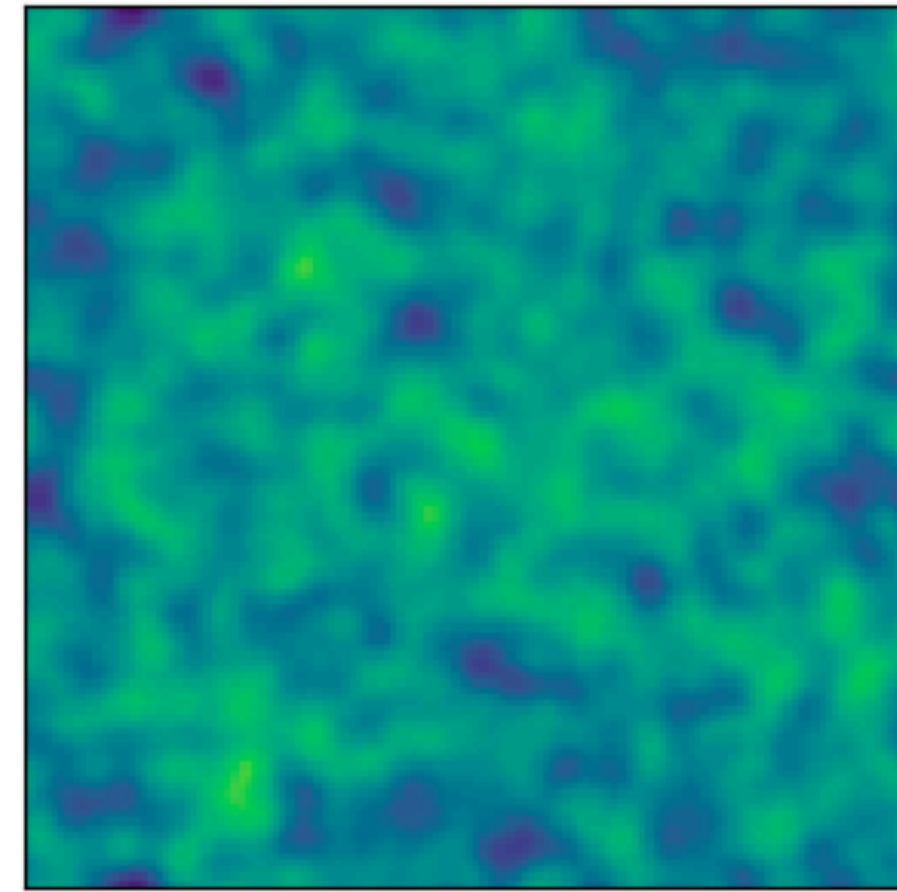
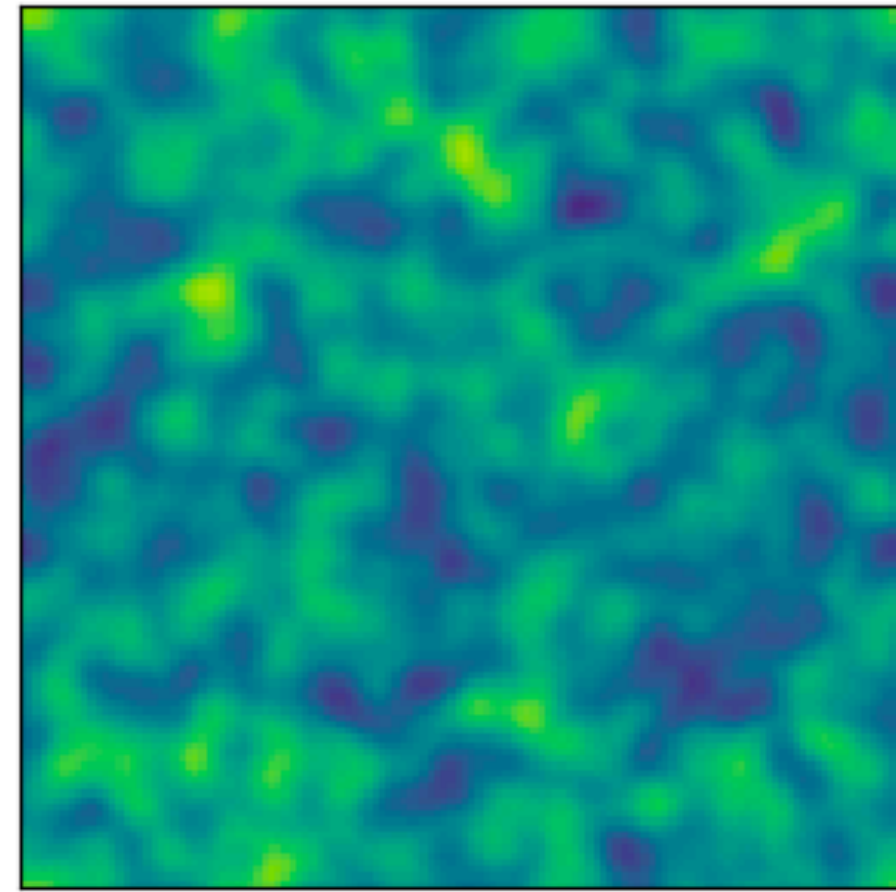
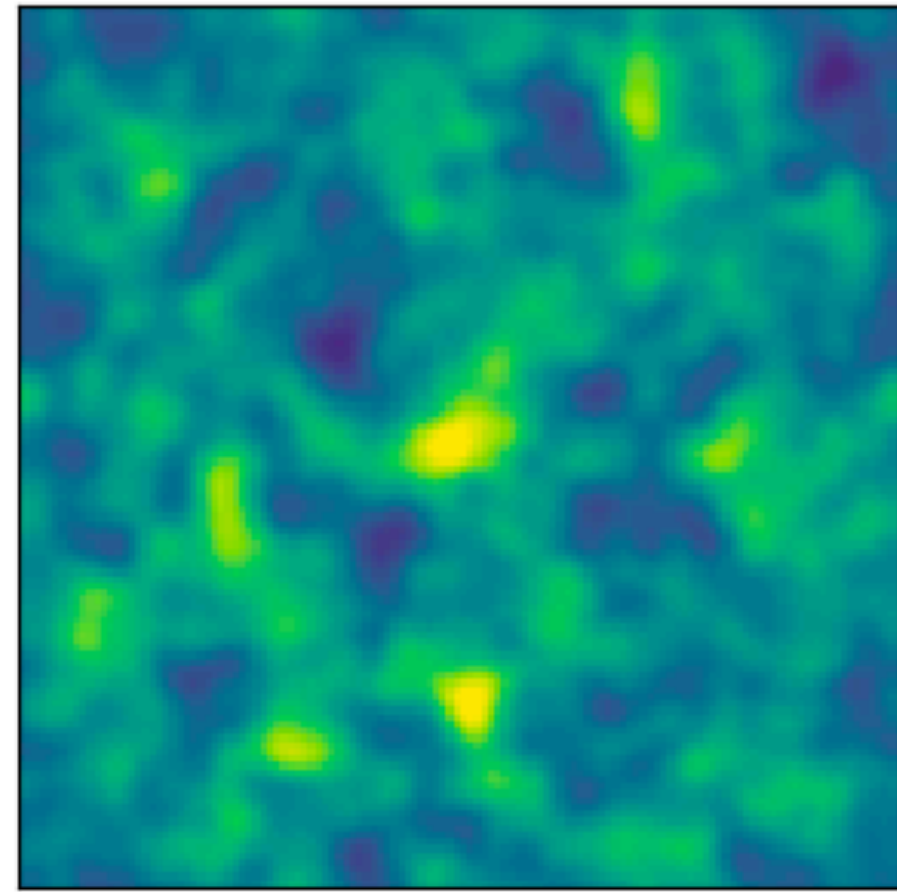
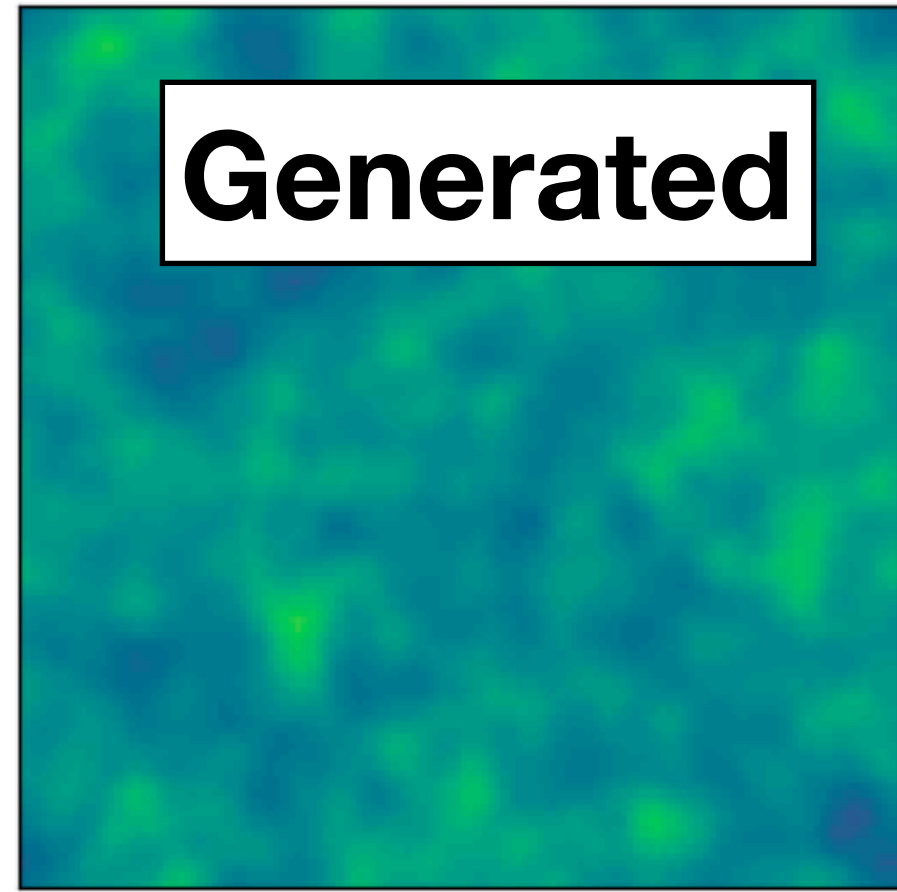
$\langle x_H \rangle = 0.5$

$\langle x_H \rangle = 0.7$

$\langle x_H \rangle = 0.9$

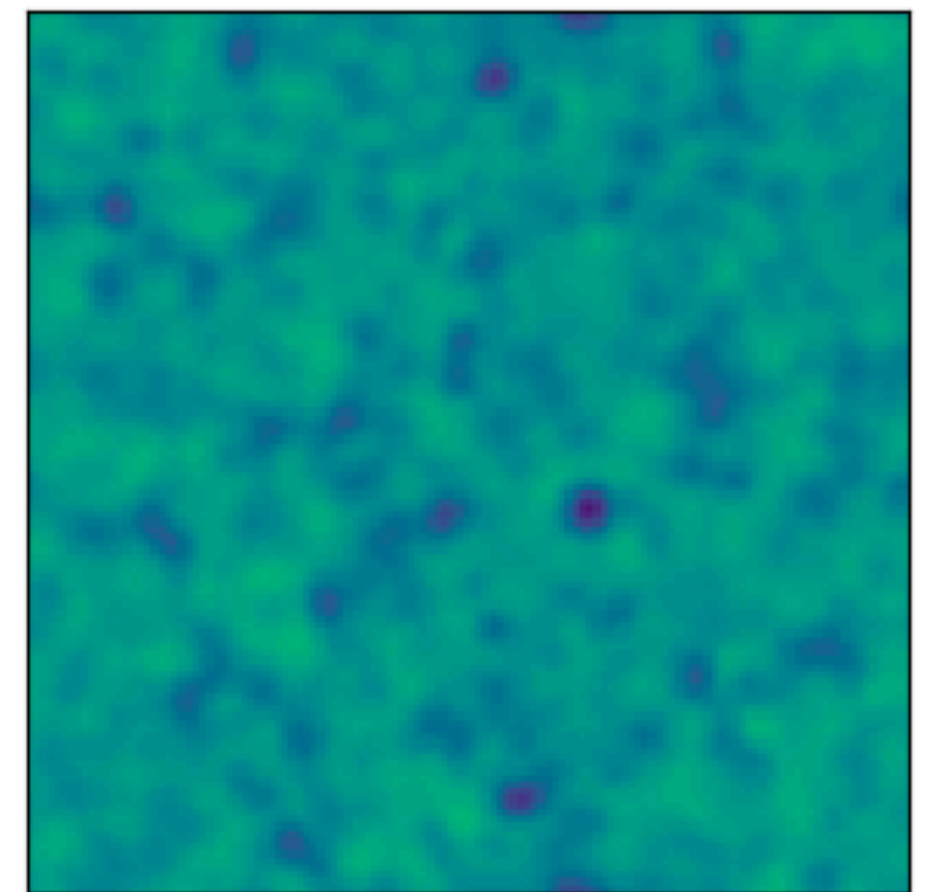
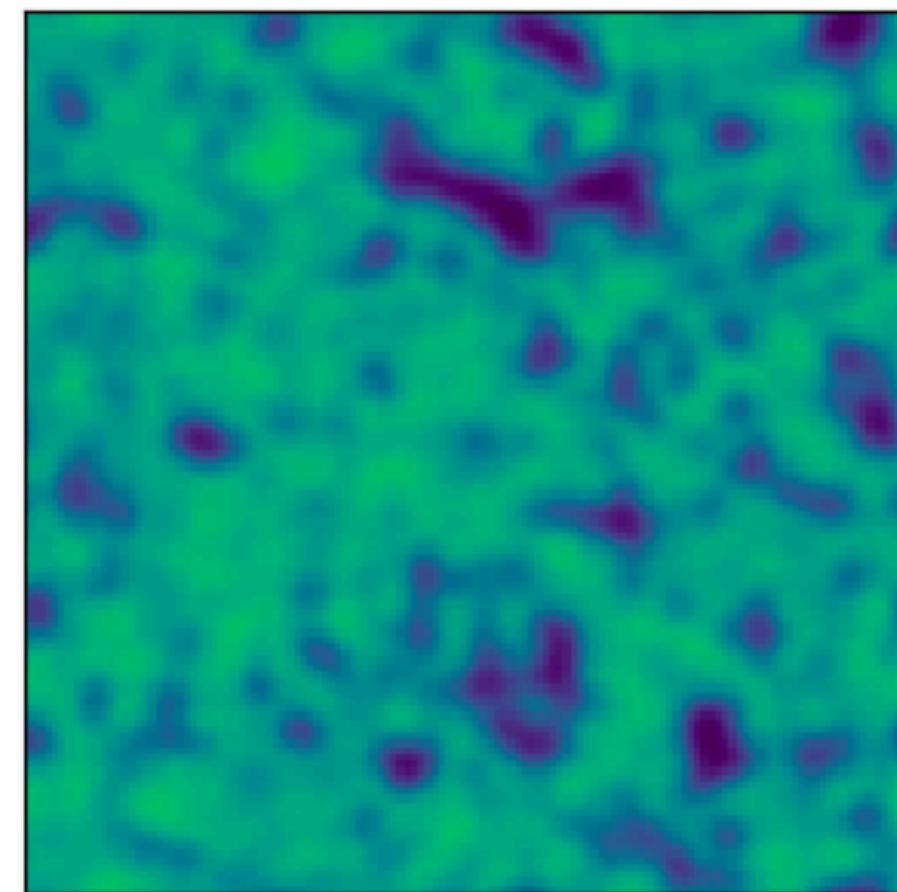
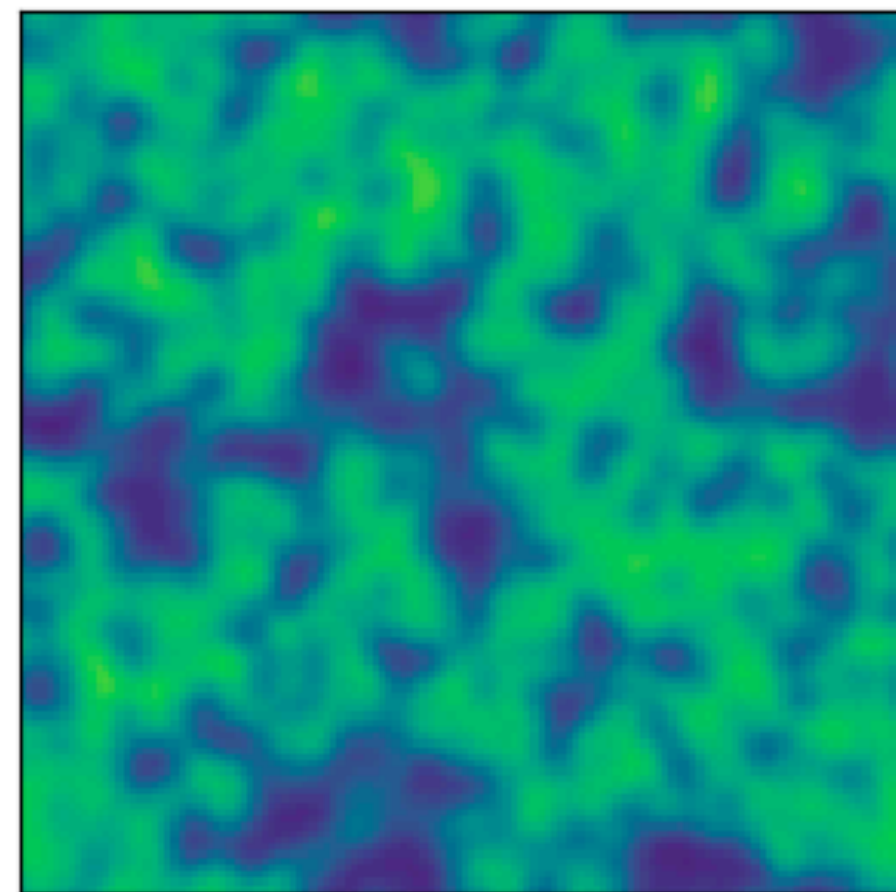
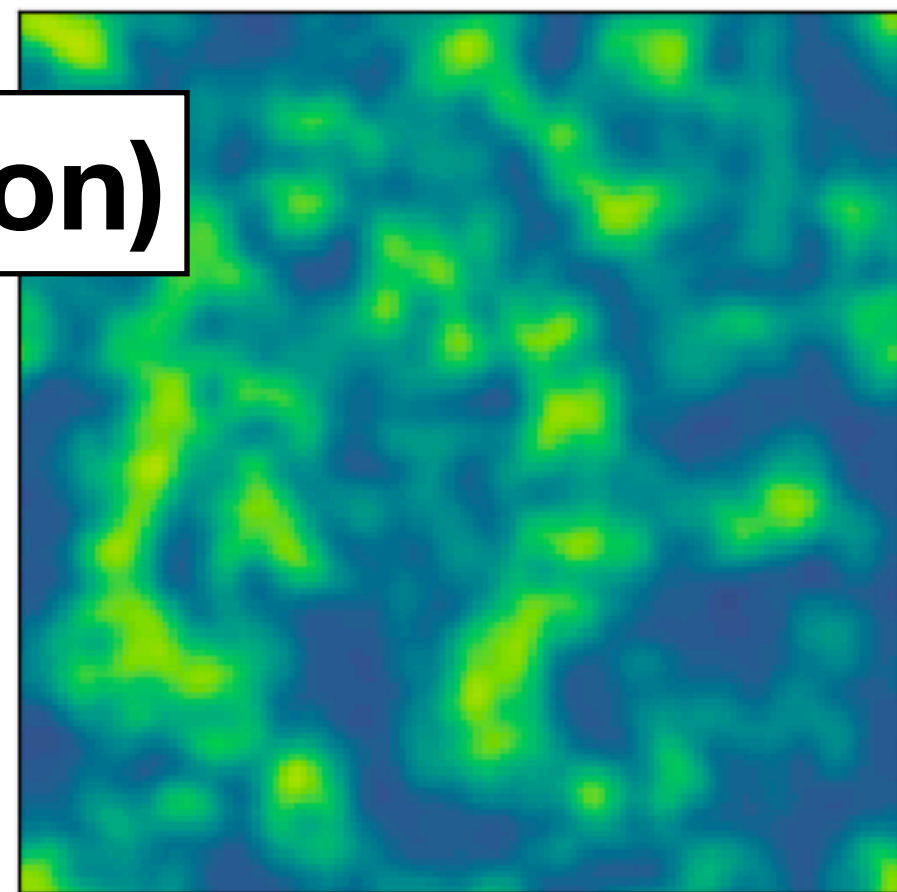
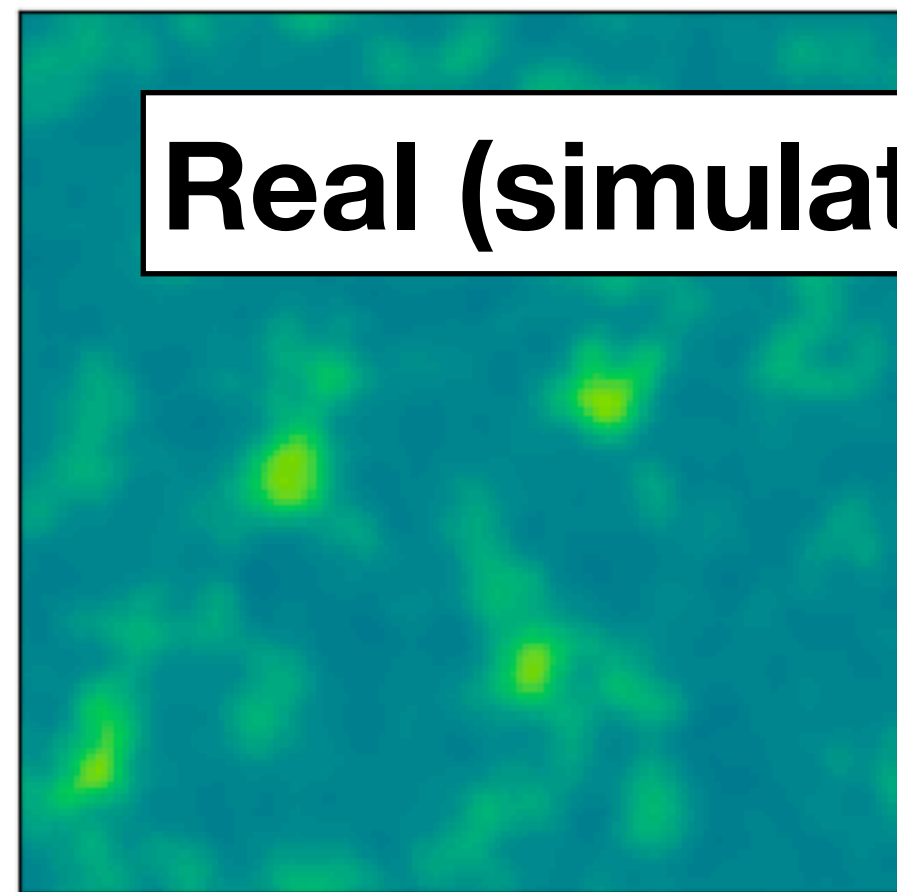
Generated

Generated

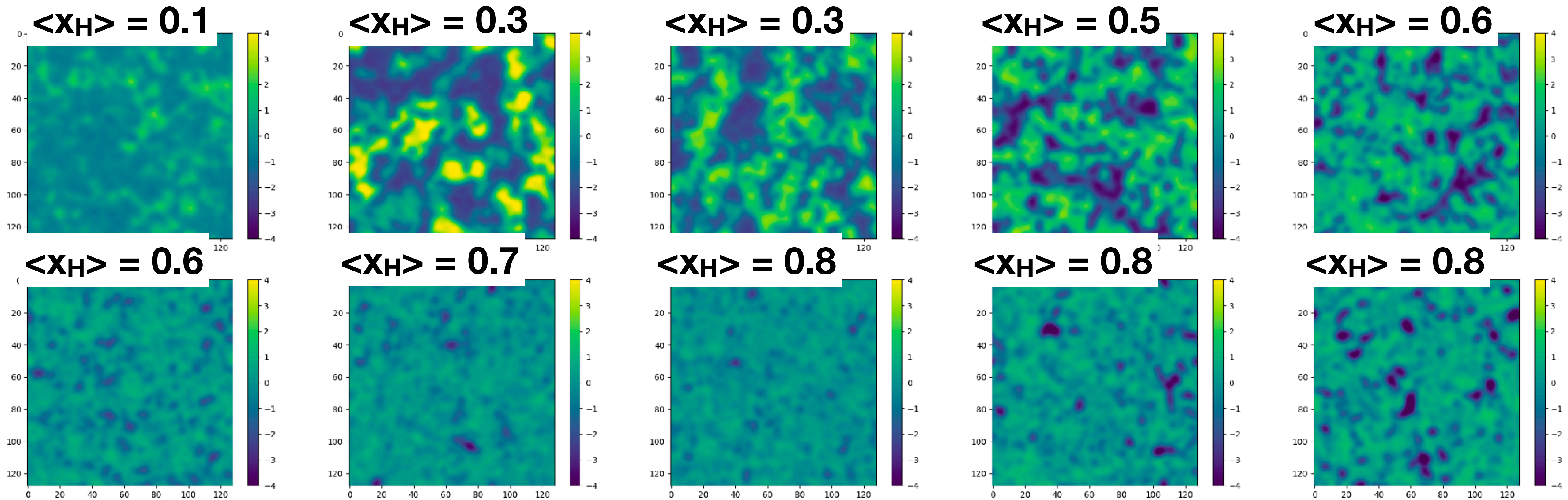


Real

Real (simulation)

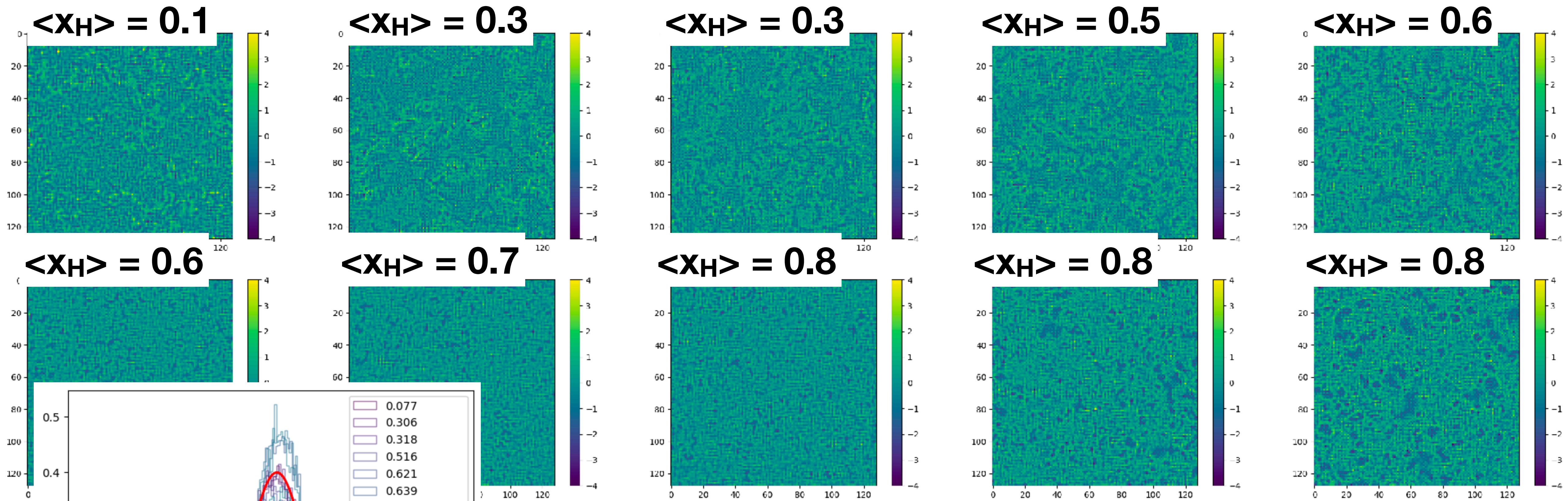


# Result: transformation into latent space



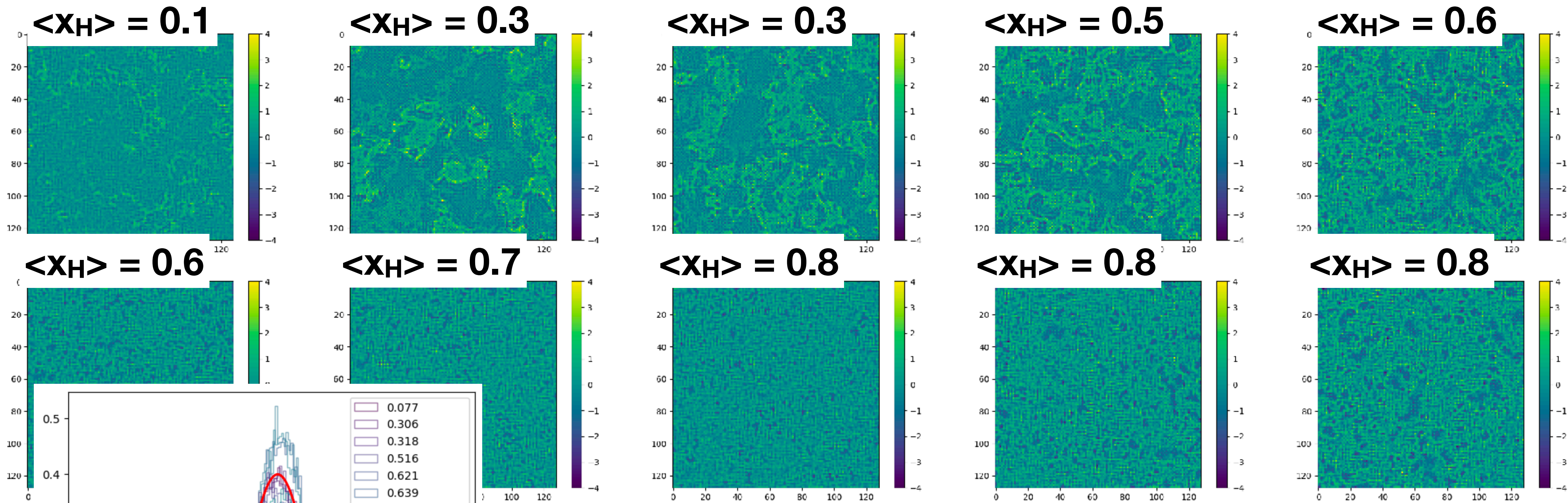
Randomly sampled simulation data (test data)

# Result: transformation into latent space



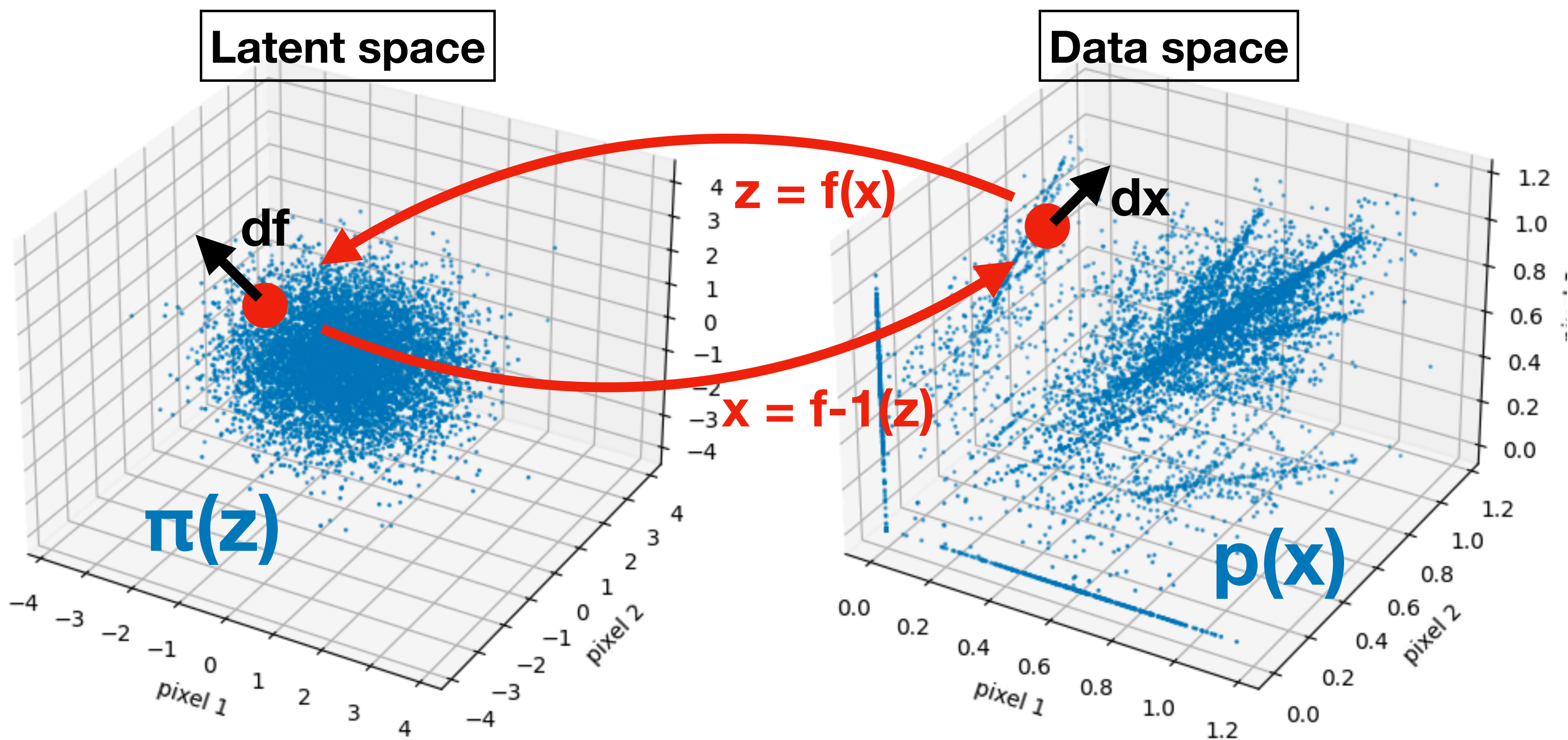
**Transformation with correct parameters**

# Result: transformation into latent space



**Transformation with incorrect parameters  
( $x_{HI} = 0.8$  for all cases)**

# Computation of the probability density



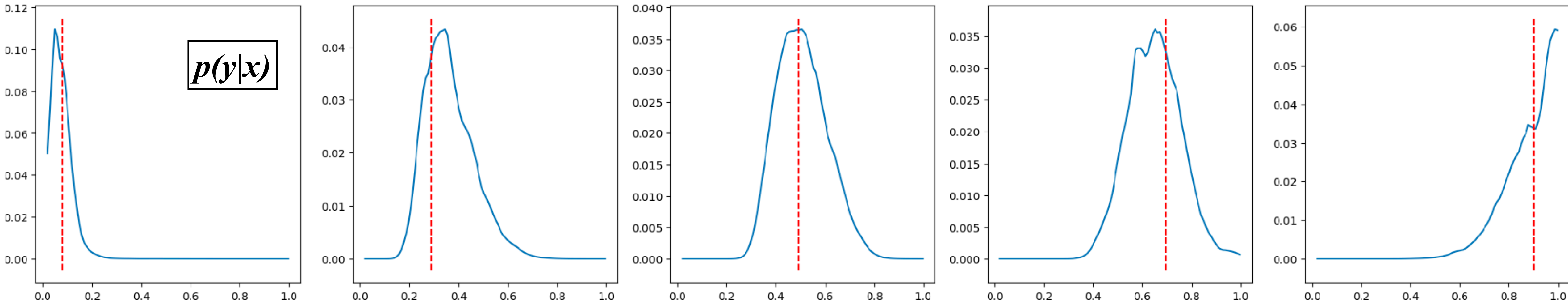
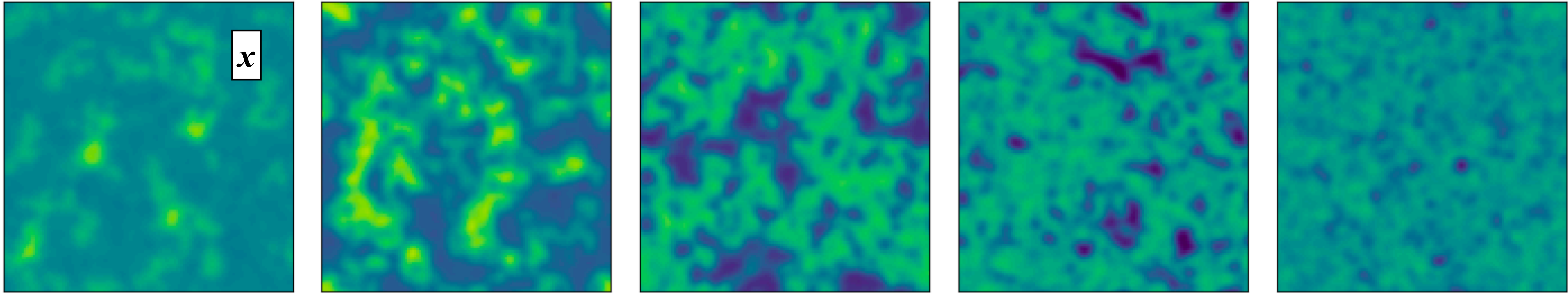
**Assumption: latent space distribution  $\pi(z)$  is known**

$$p(x) = \pi(f(x)) \left| \det \left( \frac{\partial f(x)}{\partial x} \right) \right| = \pi(f(x)) \prod_{l=1}^n \left| \det \left( \frac{\partial f_l(x)}{\partial x} \right) \right|$$

**When the model is conditional:**

$$p(x|y) = \pi(f_y(x)) \left| \det \left( \frac{\partial f_y(x)}{\partial x} \right) \right| \rightarrow p(y|x) \propto p(x|y)p(y)$$

# Result: parameter inference



$x_H = 0.1$

$x_H = 0.3$

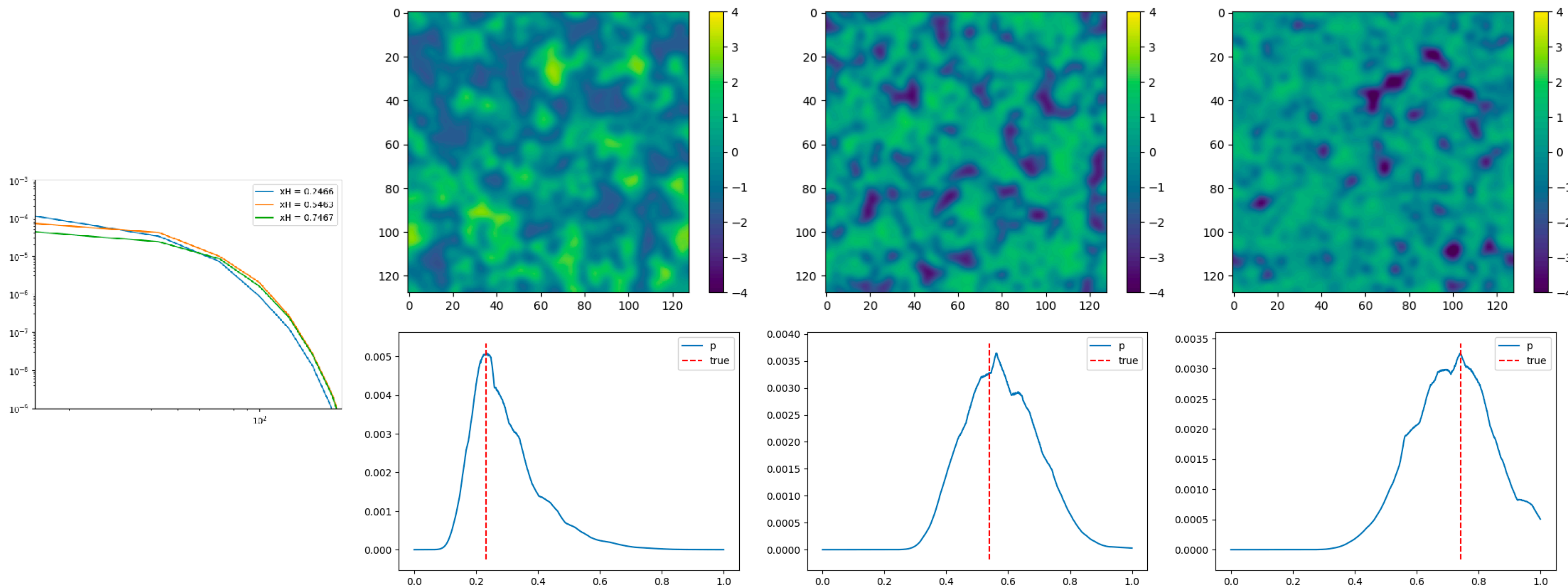
$x_H = 0.5$

$x_H = 0.7$

$x_H = 0.9$

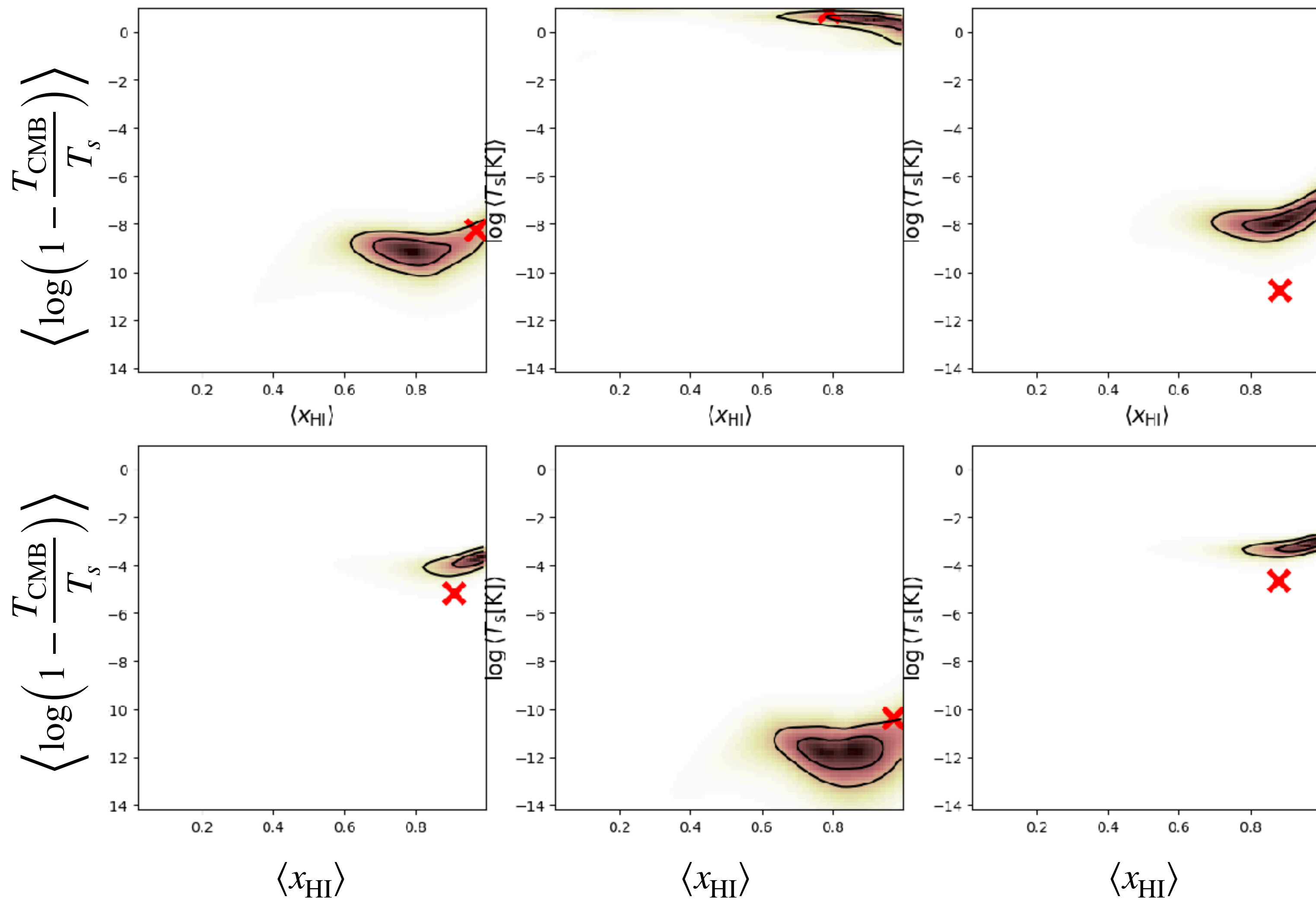
# Is it learning something beyond power spectrum?

## Test with three samples with similar power





# Inferring both ionization and heating state



**21cm signal**

$$\delta T_{21} = T_0 x_{\text{HI}} (1 + \delta_\rho) \left( 1 - \frac{T_{\text{CMB}}}{T_s} \right) \frac{H(z)}{H(z) + \frac{dv_{\parallel}}{dr_{\parallel}}}$$

# Summary

- **Normalizing flow could be a good tool for fully extracting information from EoR 21 cm maps**
- **Issues**
  - **Noise and smoothing are too simple**
  - **Foregrounds are not included**
  - **Tested only one specific simulation**
  - **Is it better to directly use uv-plane than images?**
  - **Could deep-learning methods extract more information than using summary statistics?**
  - **Is NF better than the other CNN-based models? In what aspect?**

**Questions, comments, and suggestions are welcome!**