Inference of Reionization Parameters and Reconstruction of Cosmological Initial Density Field with Observations from the Epoch of Reionization

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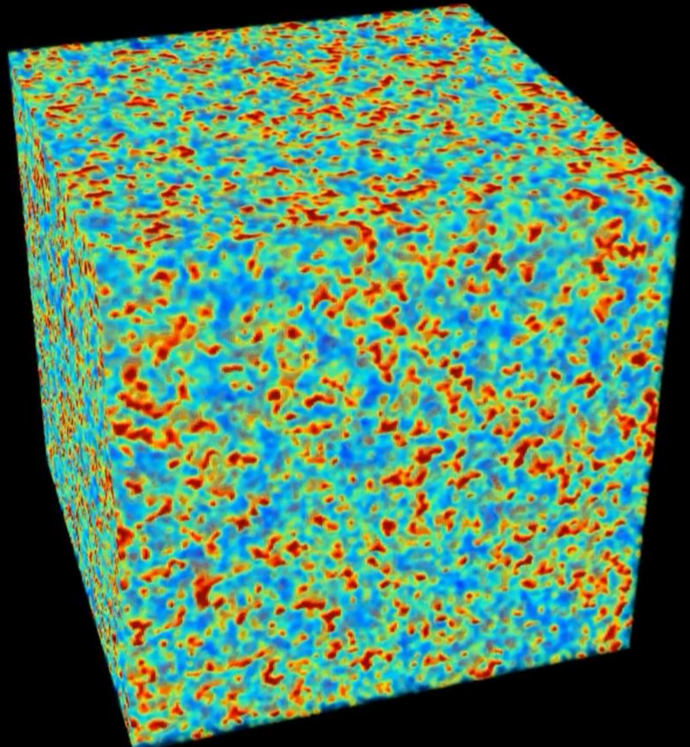
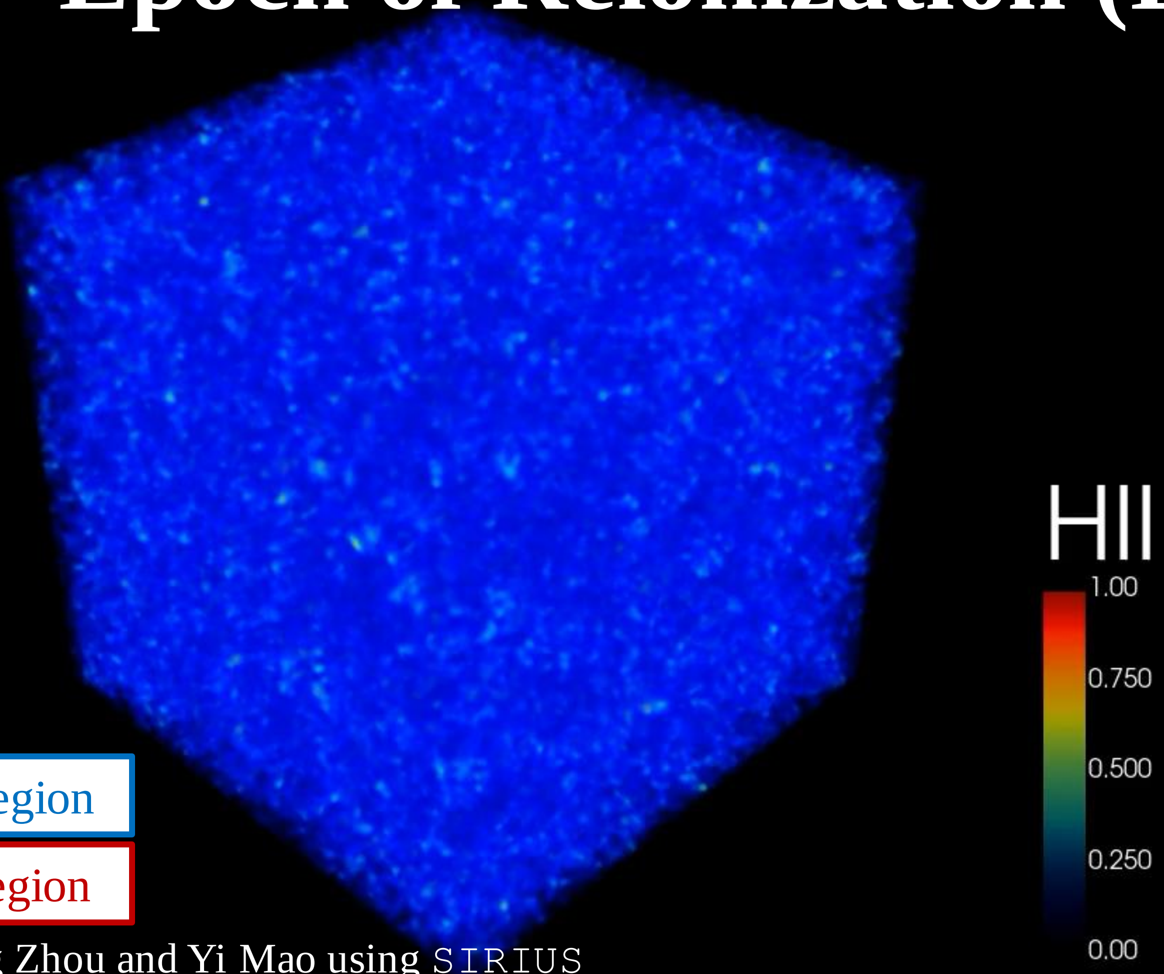
Hongo 21 cm Workshop
University of Tokyo
October 3 – 4, 2024



清華大學天文系
Department of Astronomy, Tsinghua University

z=15.062

Epoch of Reionization (EoR)



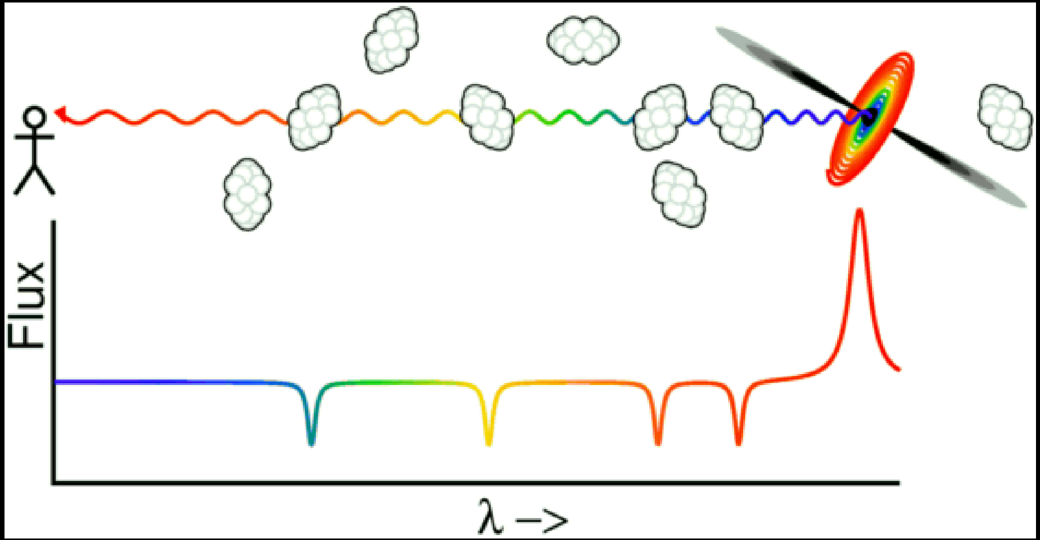
Blue: H I region

Red: H II region

Movie: Meng Zhou and Yi Mao using SIRIUS

Indirect Probes of Cosmic Reionization

Cosmic Probe of EoR 1: Lyα Forest

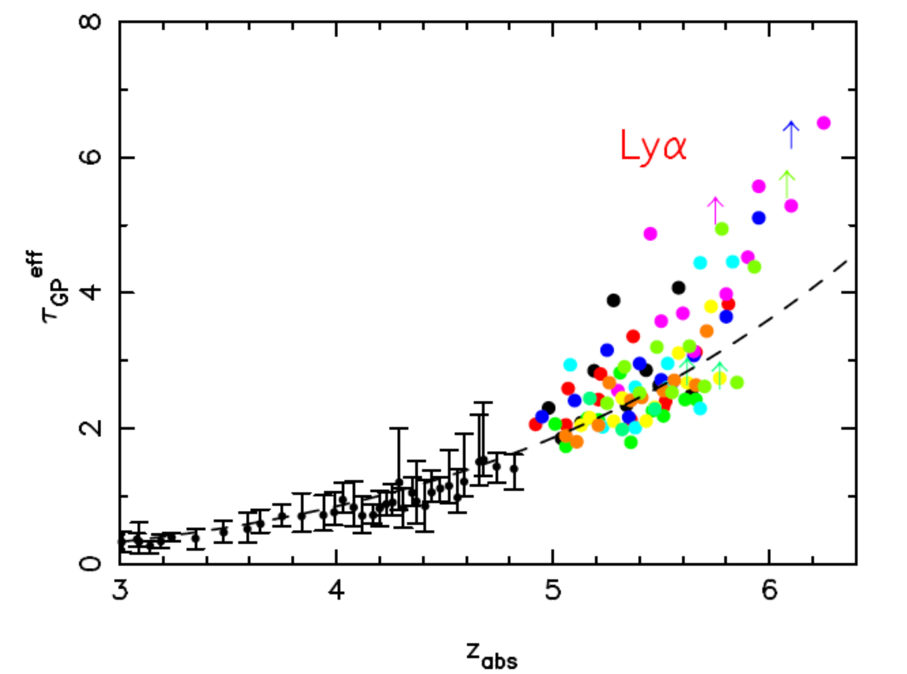


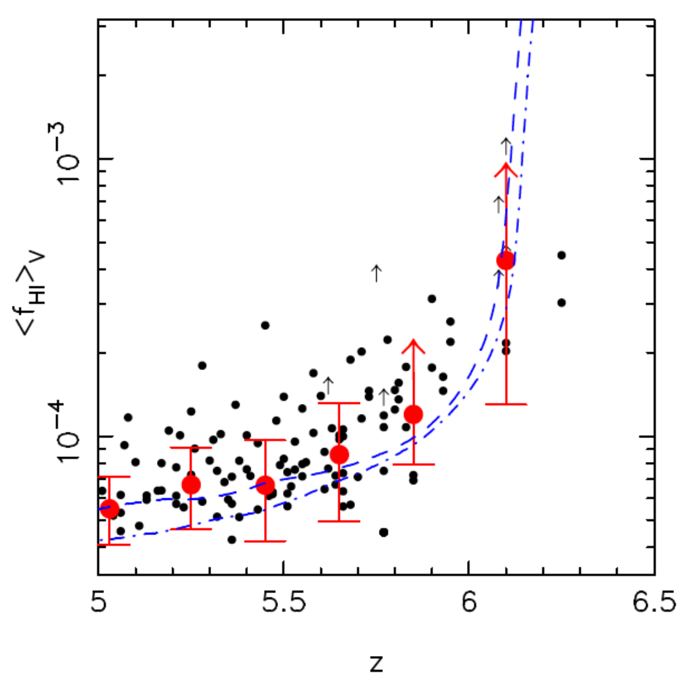
$$\lambda_{\rm obs} = \lambda_0 (1+z)$$

$$\tau_{\rm GP}(z) = 1.8 \times 10^5 h^{-1} \Omega_m^{-1/2} \left(\frac{\Omega_b h^2}{0.02}\right) \left(\frac{1+z}{7}\right)^{3/2} \left(\frac{n_{\rm HI}}{\langle n_{\rm H} \rangle}\right)$$

Cosmic Probe of EoR 1: Lyα Forest

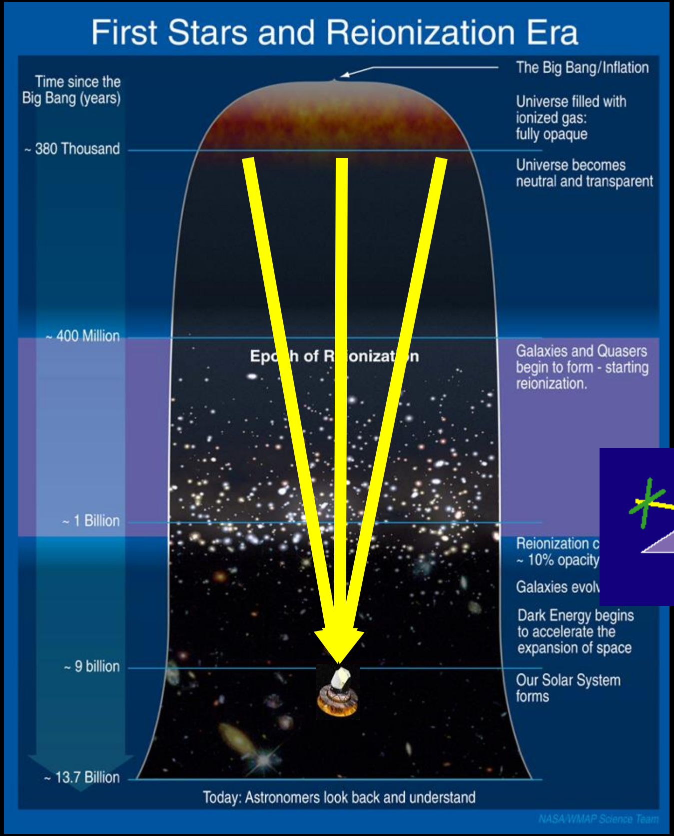
Reionization completed at z～6





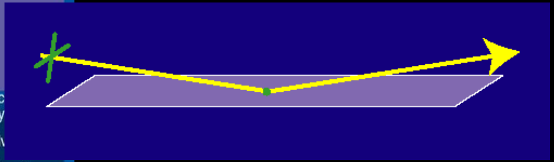
Fan et al (2006)

Cosmic Probe of EoR 2: Cosmic Microwave Background

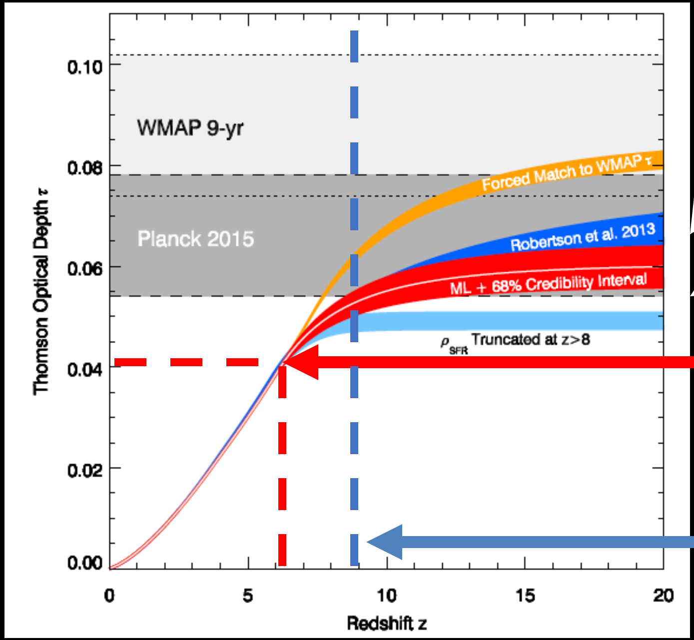


Credit NASA/WMAP Science Team

$$\tau_{\rm es} = \int \sigma_T \, n_e \, d\ell$$



Reionization is an extended process.



Robertson et al. (2015)

$\tau = 0.066 \pm 0.013$

Planck 2015 (TT + lowP/LFI + lensing + BAO)

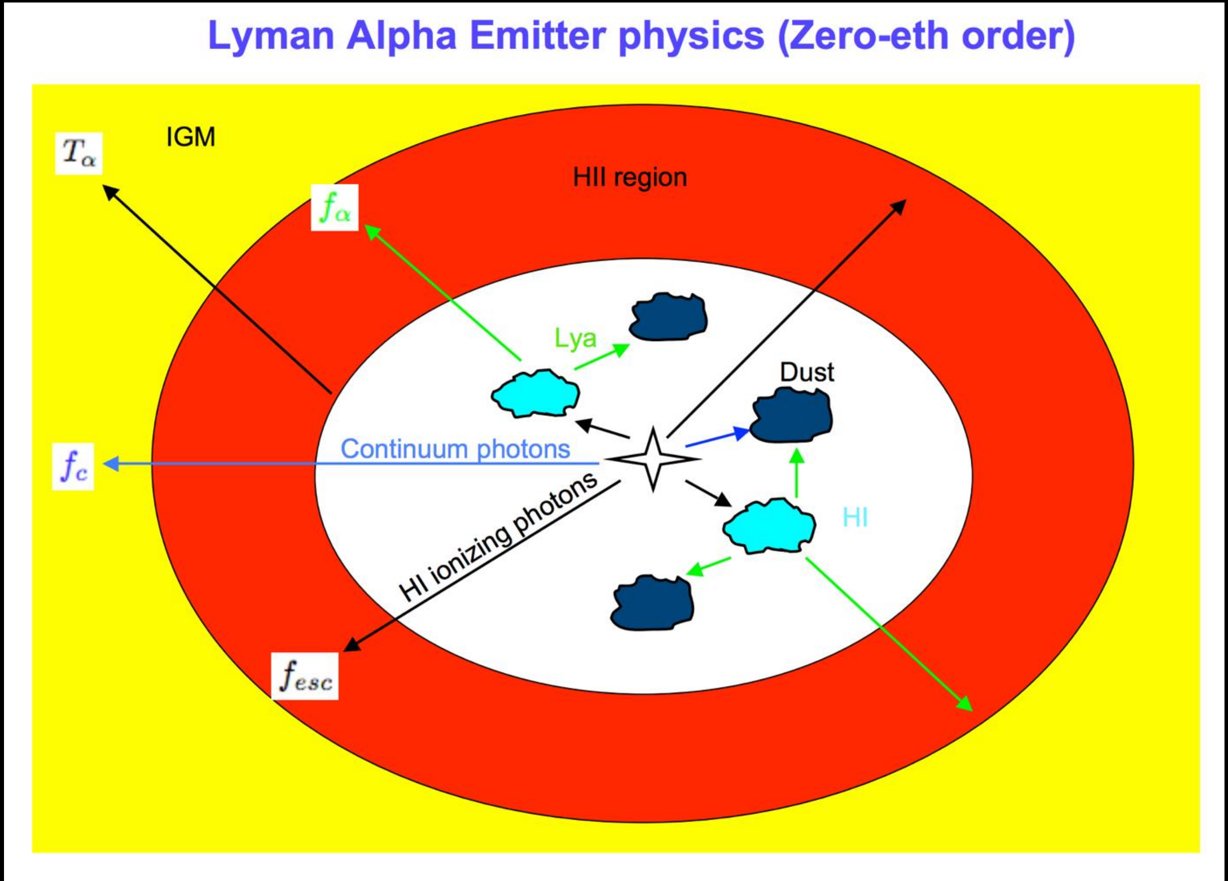
$\tau = 0.058 \pm 0.012$

Planck 2016 (TT + lowP/HFI)

4% CMB photons could have been scattered by the fully ionized IGM at z<6

If reionization would be instantaneous, then $z_{\rm re} = 8.8^{+1.3}_{-1.2}$ (Planck 2015), or $z_{\rm re} = 8.5^{+1.0}_{-1.1}$ (Planck 2016).

Cosmic Probe of EoR 3: Lyα Emitters



Reionization is inhomogeneous and patchy.

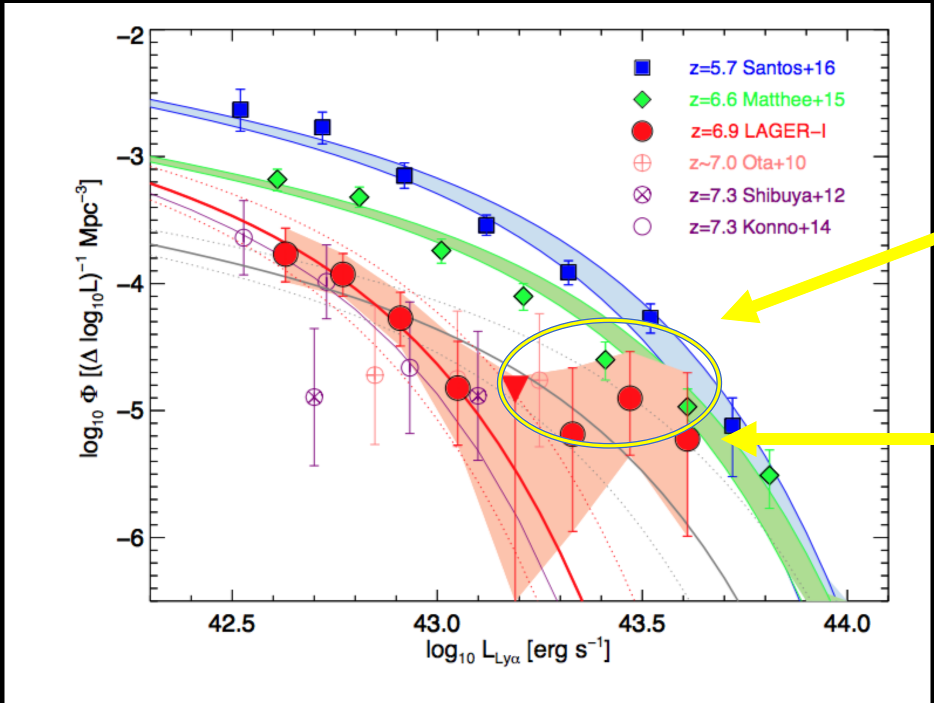


Figure 2. The Lyα LFs of LAE surveys at $z \sim [5.7, 6.6, 7, 7.3]$.

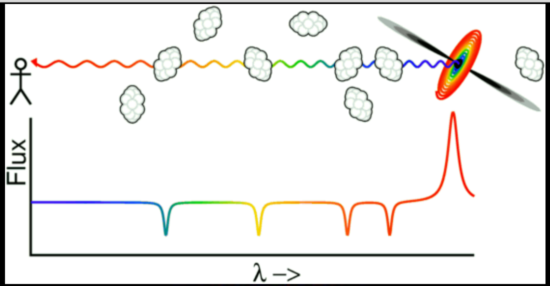
Evolution of LF indicates $x_{HI} \sim 0.4 - 0.6$ at $z = 6.9$ (however, model dependent).

Bump at bright end indicates large HII bubbles, where Hubble flow, galactic inflow/outflow can bring Lyα photons out of resonance.

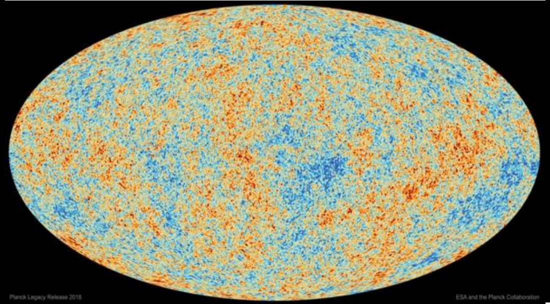
LAGER collaboration (Zheng et al. 2017)

Cosmic Probes of EoR

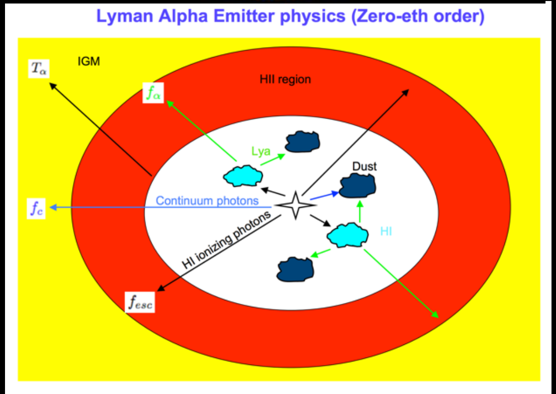
☑ **Lyα forest optical depth：Reionization completed at z～6**



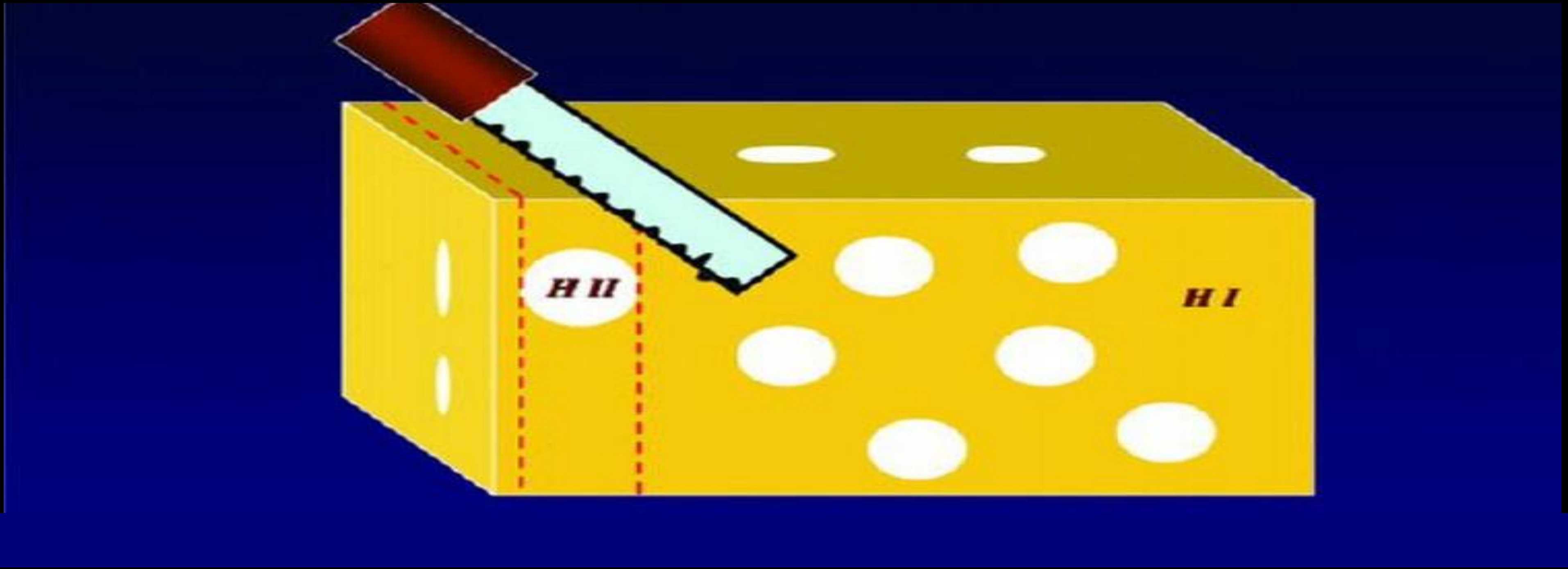
☑ **CMB: Reionization is an extended process**



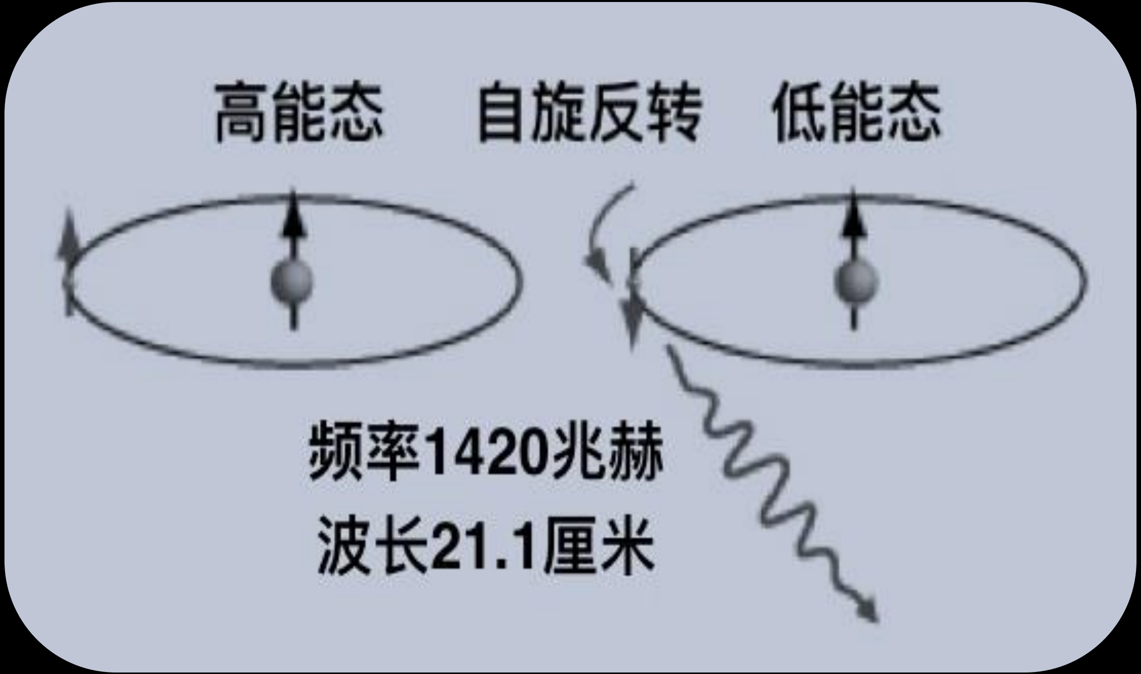
☑ **Lyα emitter：Reionization is inhomogeneous and patchy**



Ionized and Neutral Hydrogen



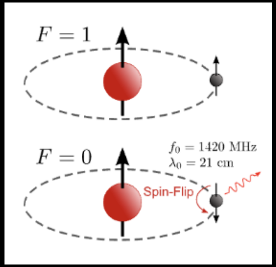
H I 21 cm Line

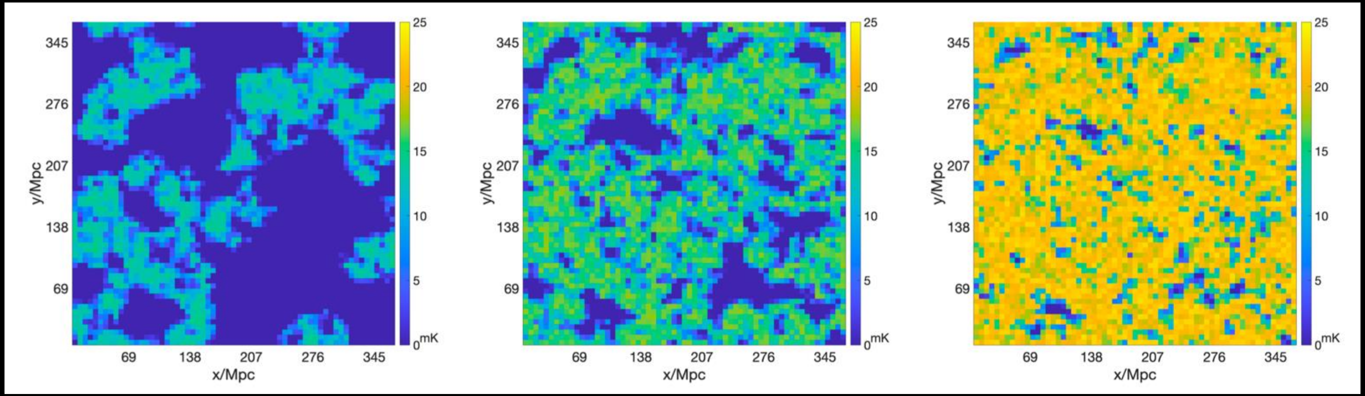


21 cm line is optically thin!

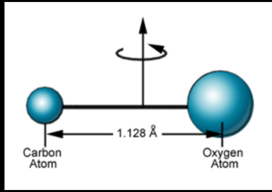
Future Probes of EoR

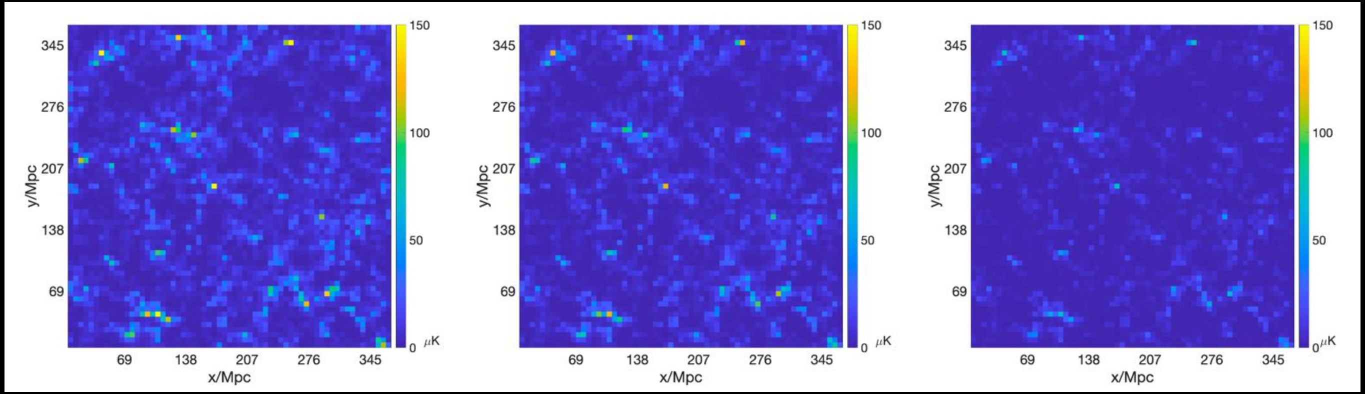
- **H I 21 cm intensity mapping**





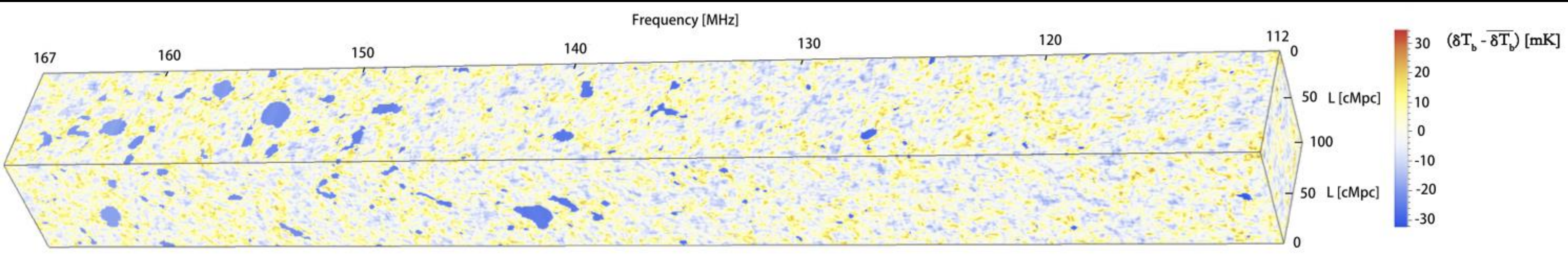
- **Molecular line intensity mapping**

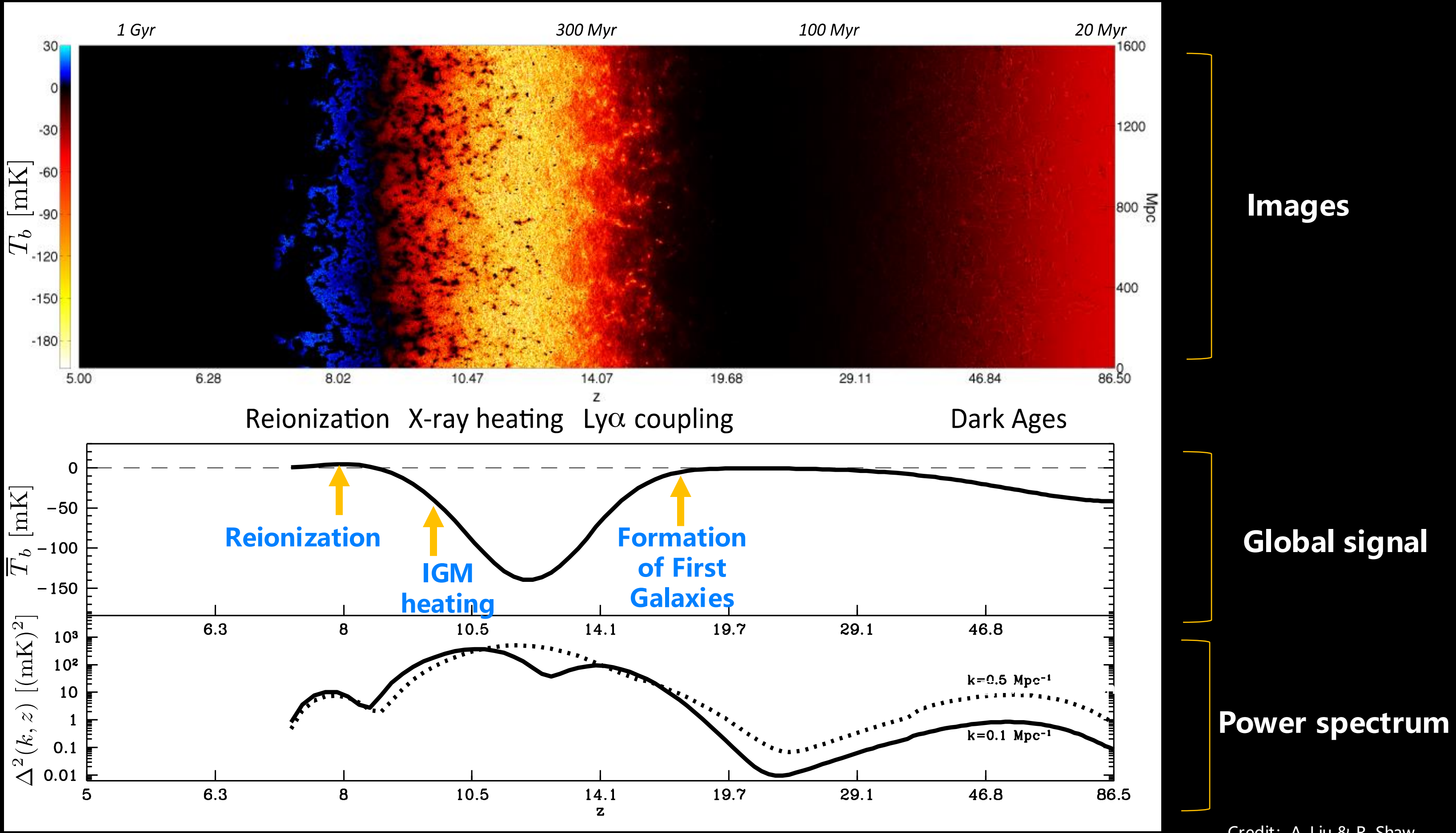




H I 21 cm intensity mapping

Mock Universe on the Lightcone





Credit: A. Liu & R. Shaw

Single antennae measure the global signal

EDGES
高频天线
美国/澳洲

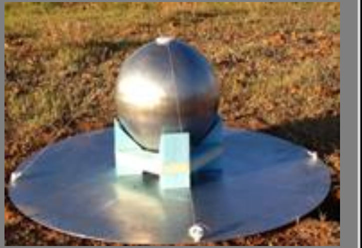
EDGES
低频天线
美国/澳洲

PRIZM
低频天线
南非

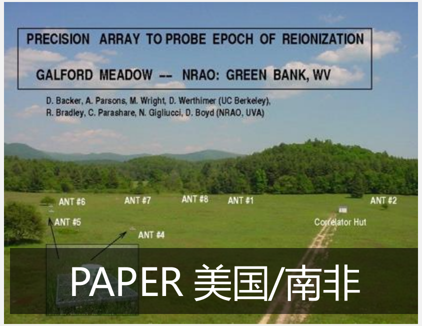
PRIZM
高频天线
南非

LEDA
美国

SARAS-3
印度

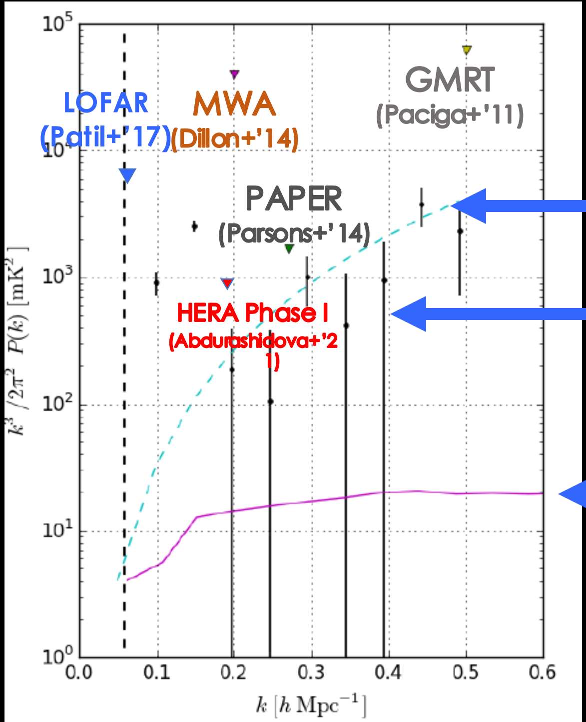


Pathfinder Interferometers measure the power spectrum



Direct Constraints on Reionization: 21-cm

PAPER, MWA, LOFAR and GMRT measure the **power spectrum** of 21-cm brightness temperature fluctuations: $\langle \delta T_b(\mathbf{k})\, \delta T_b^{\star}(\mathbf{k}') \rangle = (2\pi)^3 P(\mathbf{k}) \delta^{(3)}(\mathbf{k} - \mathbf{k}')$



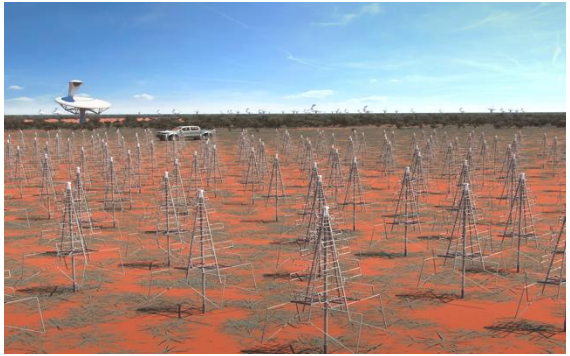
Ali et al. (2015)

Predicted noise 2σ level

PAPER 2015 observation (black dots with 2σ error bars)

Predicted signal from a theoretical model at 50% ionization

Interferometers *will* measure power spectrum *and* images



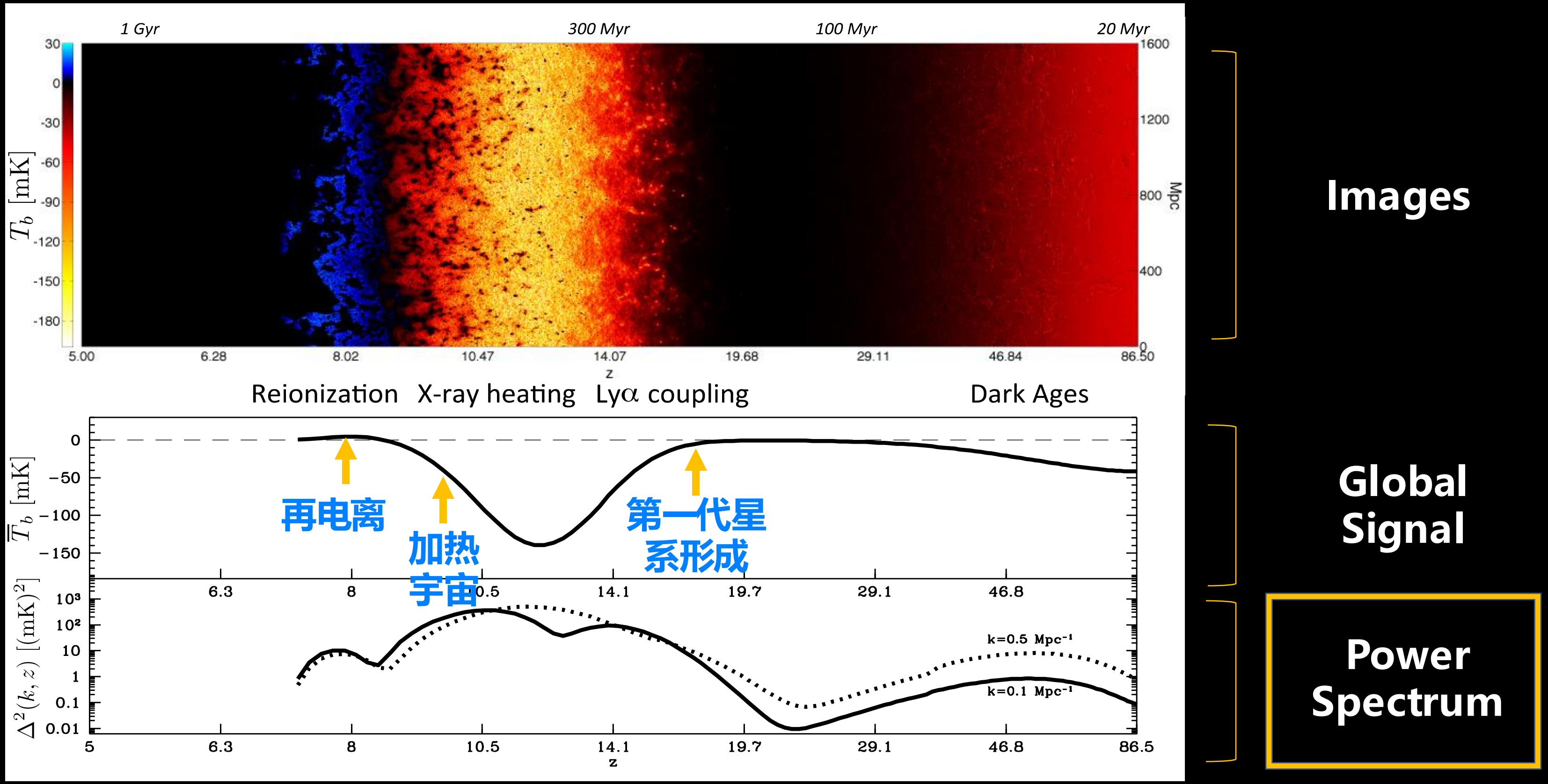
SKA (Square Kilometre Array)
Phase I ~1 Billion euros

HERA

Data Analysis in 21 cm observations

- Calibration
- RFI flagging (in visibility measurement)
- Image making
- Foreground subtraction
- Scientific interpretation

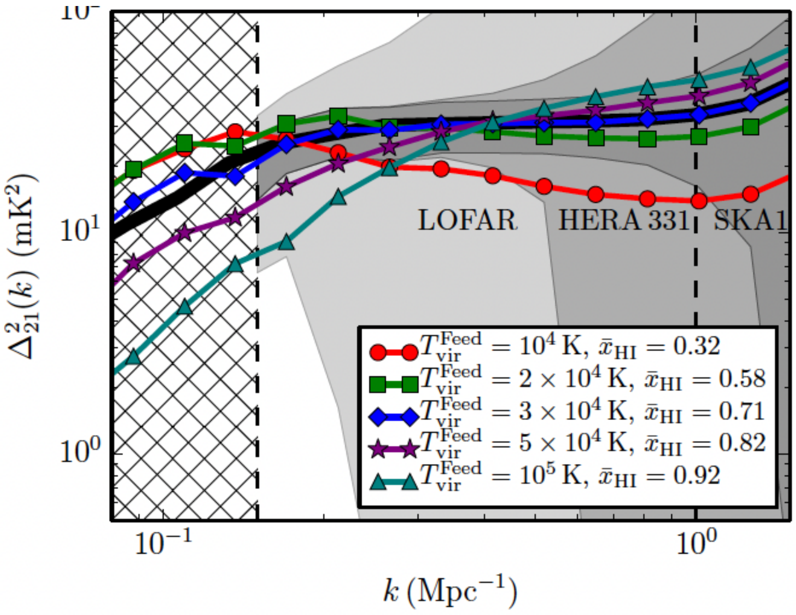
Extract astrophysical information from cleaned data

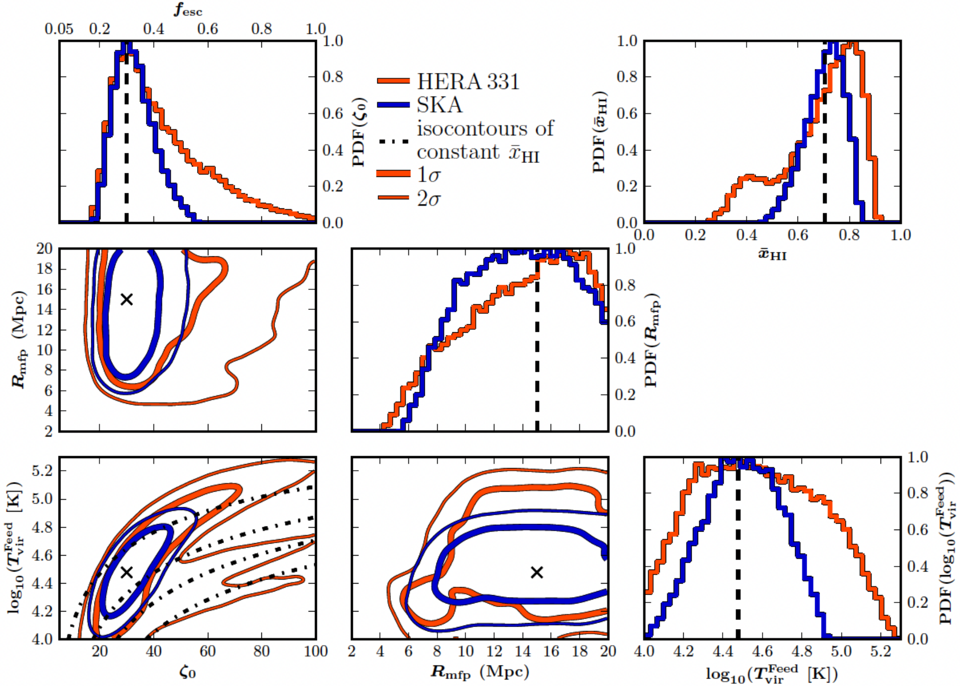


图：A. Liu & R. Shaw

Parameter Estimation using 21 cm Power Spectrum

21cm Power Spectrum





Reionization Parameters

ζ the ionizing efficiency

T_{vir} the minimum virial temperature of halos that host ionizing sources

Bayesian inference of reionization model parameters with conventional MCMC method (21CMMC code)

Greig & Mesinger, 2015, MNRAS

Parameter Estimation using 21 cm Power Spectrum

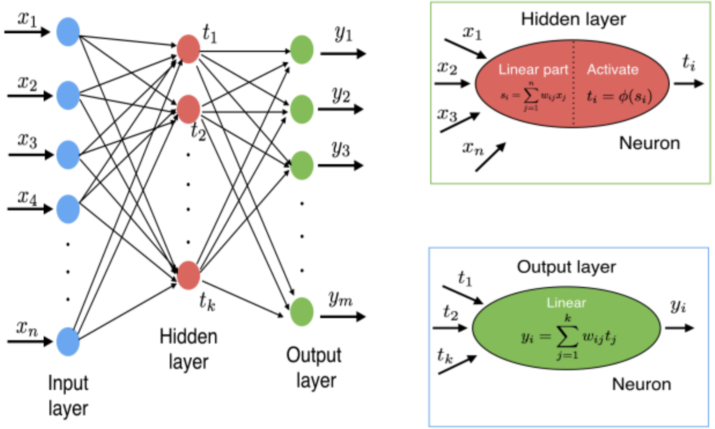


Figure 1. Typical architecture of an artificial neural network. The architecture of the ANN consists of an input layer, a hidden and an output layer of neurons. Each neuron connects the neurons in the next layer.

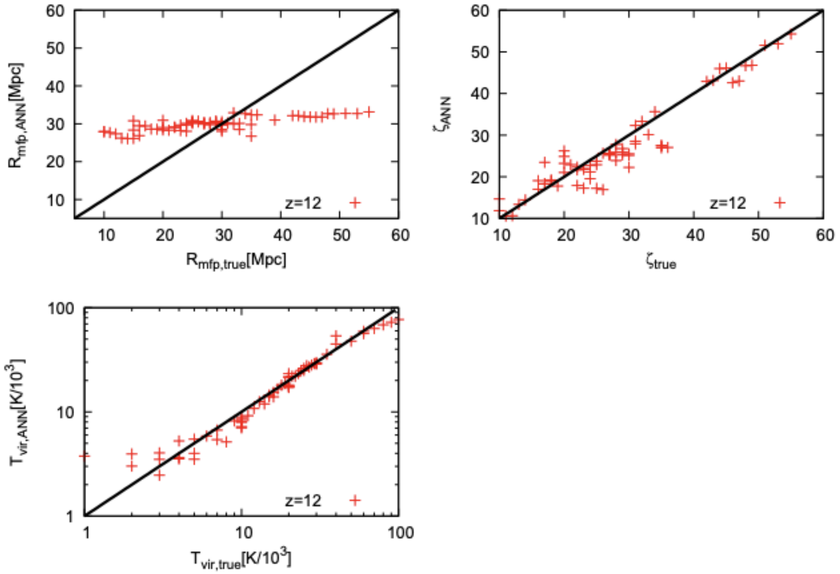


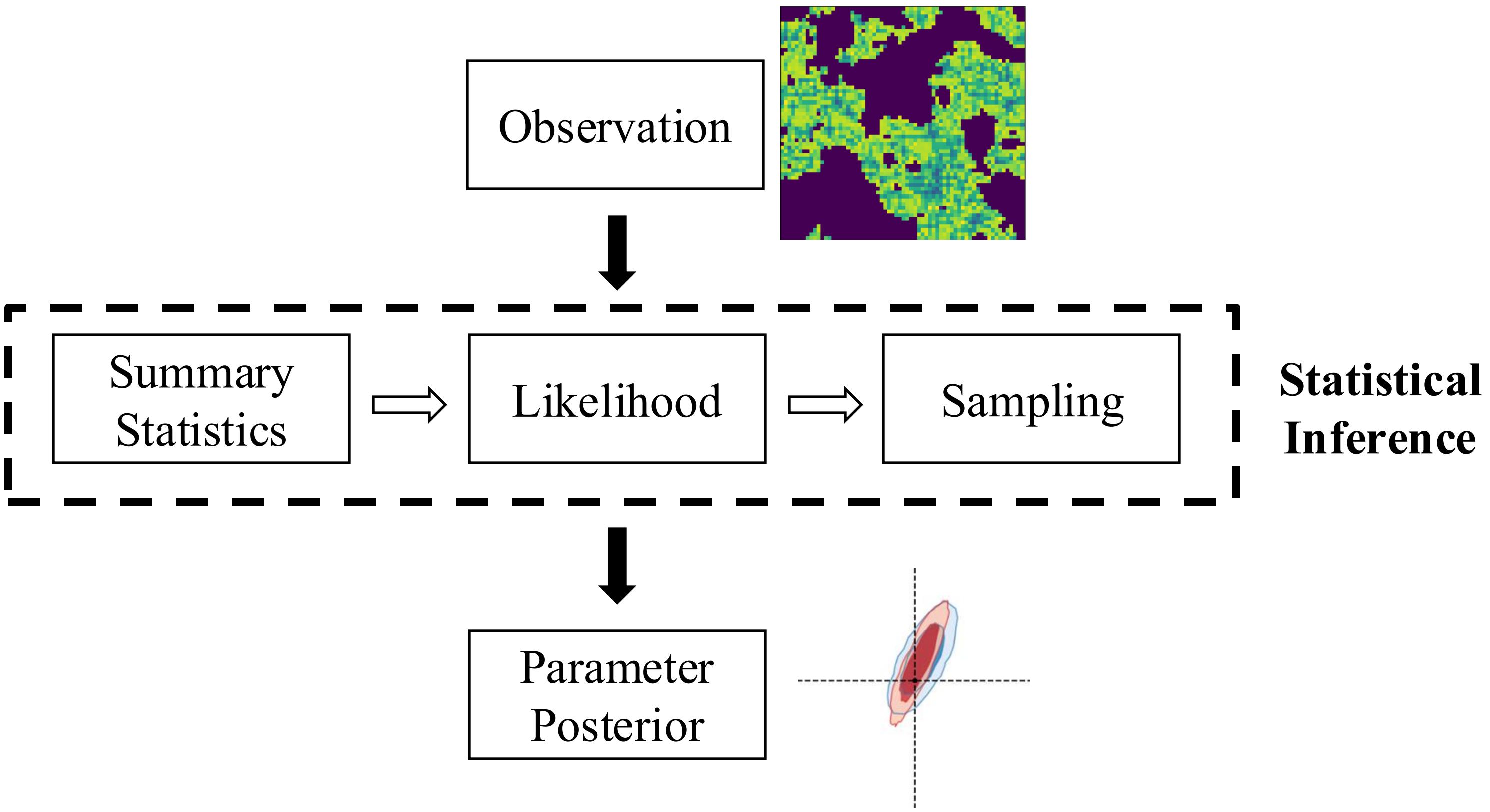
Figure 4. The EoR model parameter values computed by the ANN from the PS against the values used in the simulation at z=12. Note that the result for the Virial temperature is plotted

Estimation of reionization model parameters with artificial neural networks

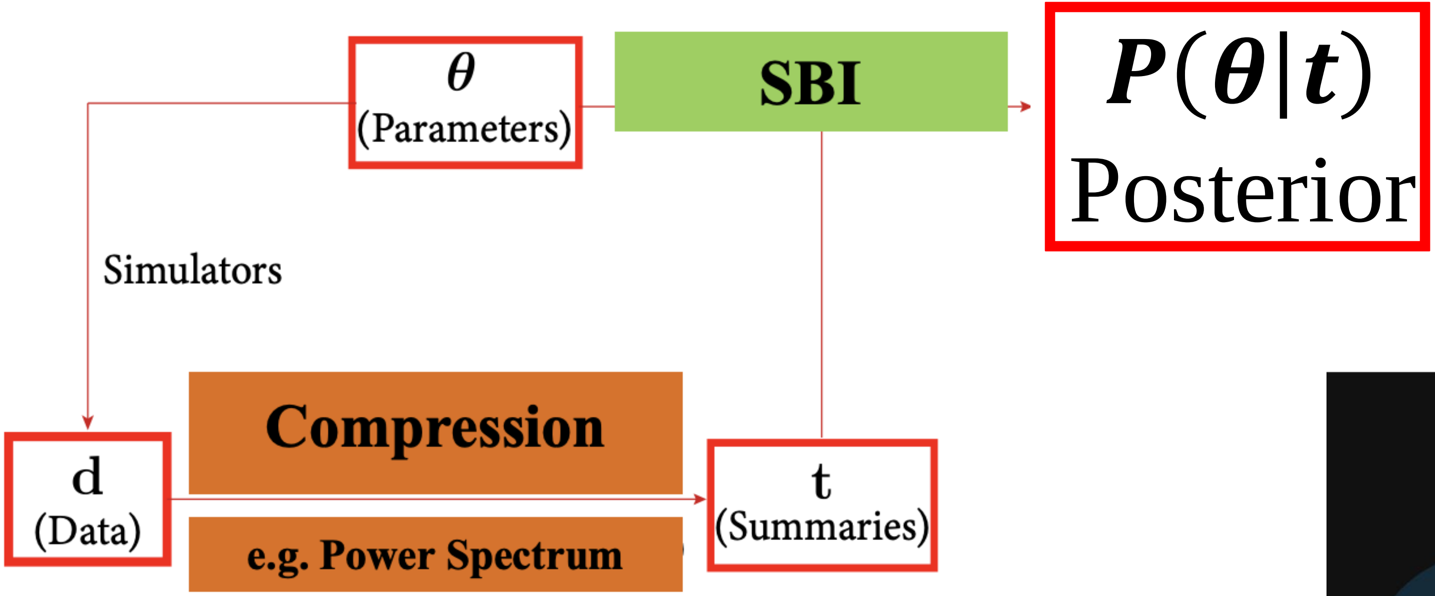
(note: point estimate, not posterior inference)

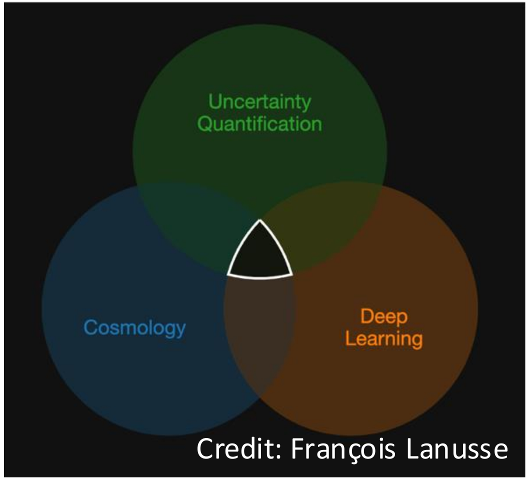
Shimabukuro & Semelin, 2017, MNRAS

Statistical Inference in Cosmology

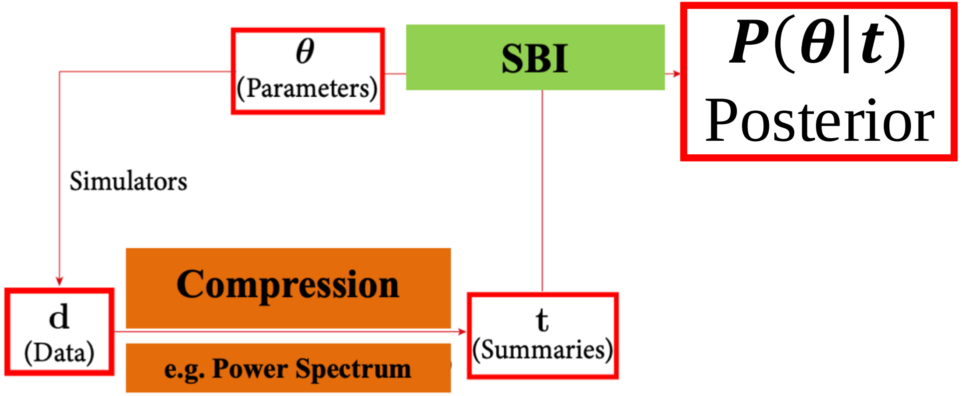


Simulation-Based Inference (SBI)





Simulation-Based Inference



1. Generative Way (density estimation likelihood-free inference, DELFI):

Zhao, **YM**, et al
2022a,2022b, 2023

$$\{\theta, t\} \xrightarrow{\text{Density Estimator}} P(t|\theta) \xrightarrow{\text{Prior}} \boldsymbol{P(\theta|t) \propto P(t|\theta)P(\theta)}$$

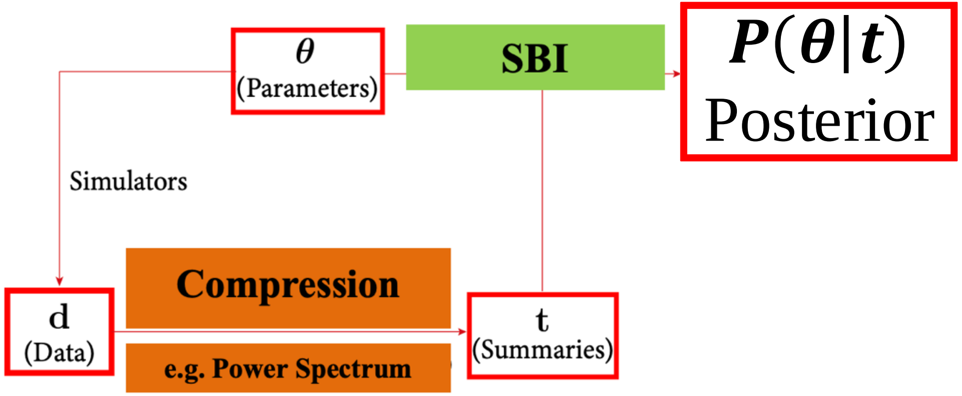
Bayes' Theorem

2. Discriminative Way (Neural Ration Estimation, NRE):

Ce, **YM**, et al
In prep

$$\{\theta, t\} \xrightarrow{\text{Ratio estimation}} r(\theta, t) = \frac{P(t|\theta)}{P(t)} \xrightarrow{\text{Prior}} \boldsymbol{P(\theta|t) = r(\theta, t)P(\theta)}$$

Simulation-Based Inference



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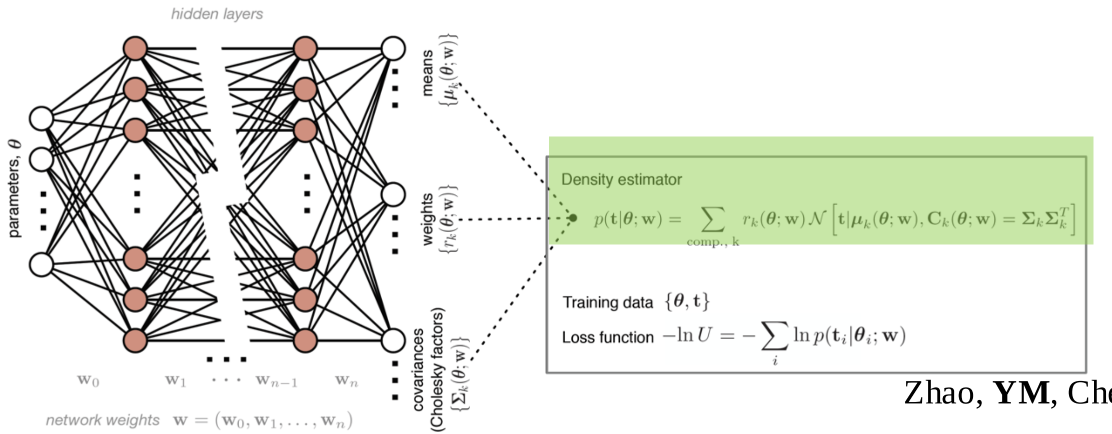
Bayes' Theorem

Mixture Density Network (MDN)

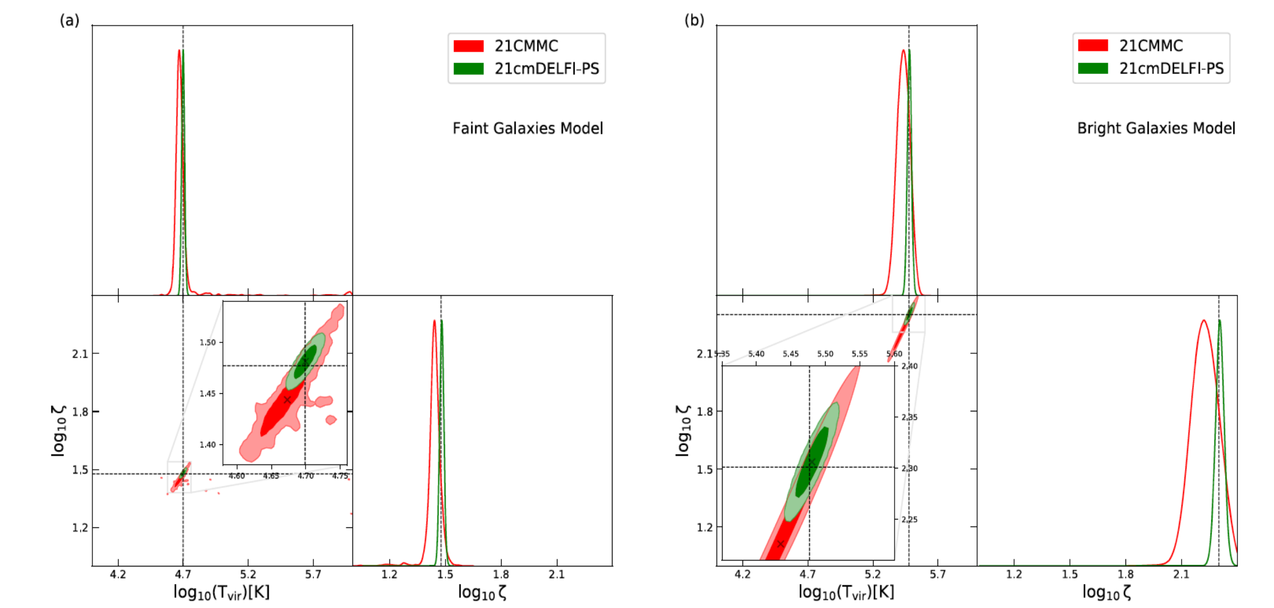
An example of Neural Density Estimators (NDEs)

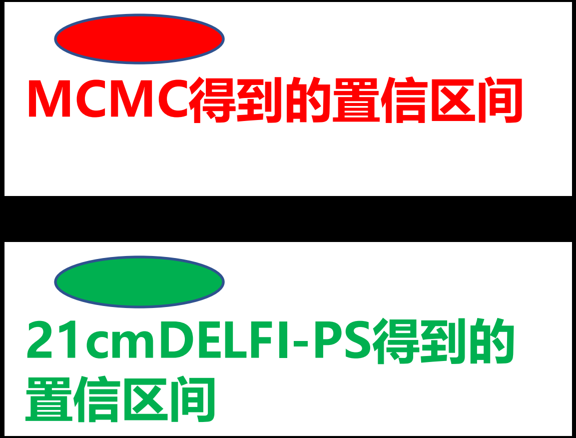


Zhao, **YM**, Cheng & Wandelt, 2022a, ApJ (arXiv:arXiv:2105.03344)

从21厘米功率谱测量出发限制宇宙再电离理论模型

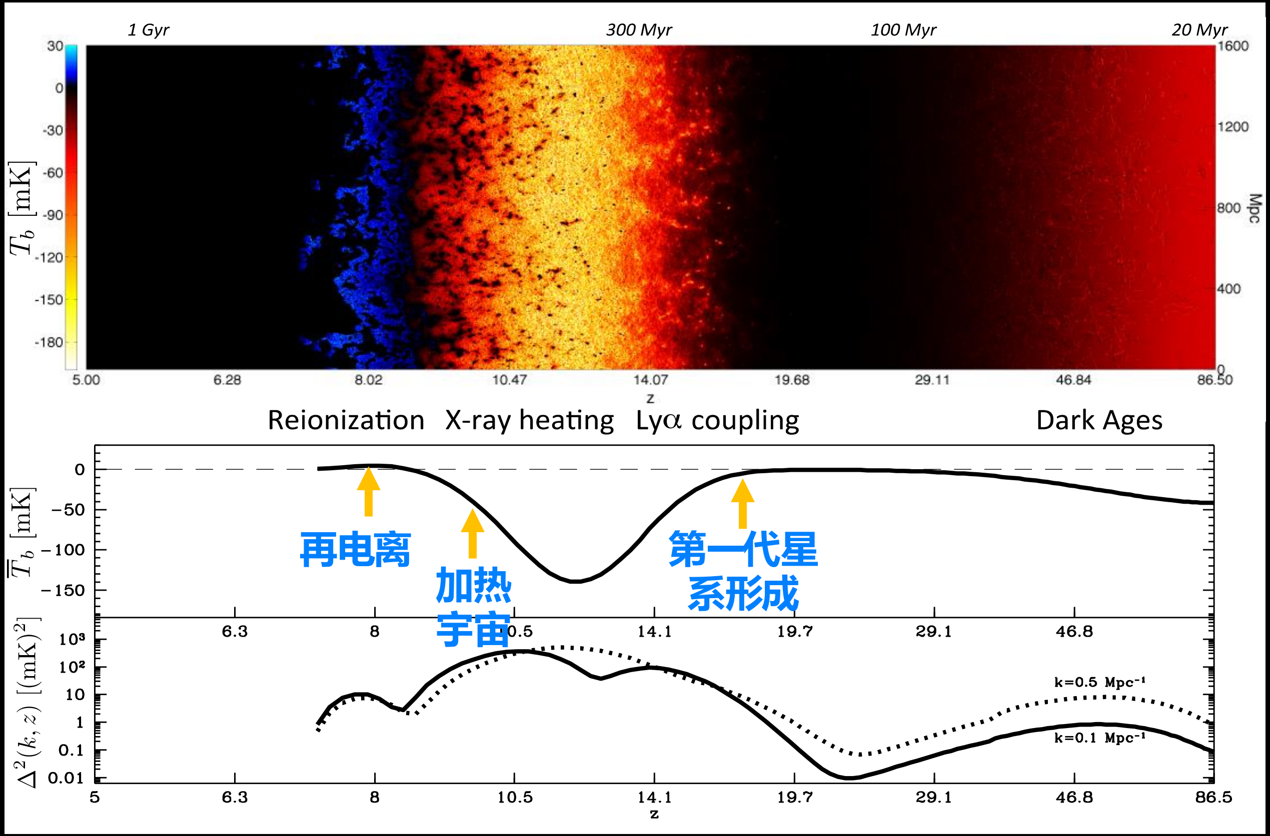
- 传统使用经典的马尔可夫链蒙特卡罗算法（MCMC），需要做特定假设。
- 发展了基于深度学习的贝叶斯统计推断的新方法，开发了新软件21cmDELFI-PS





Zhao, **YM** & Wandelt, 2022b, ApJ (arXiv:2203.15734)

Extract astrophysical information from cleaned data



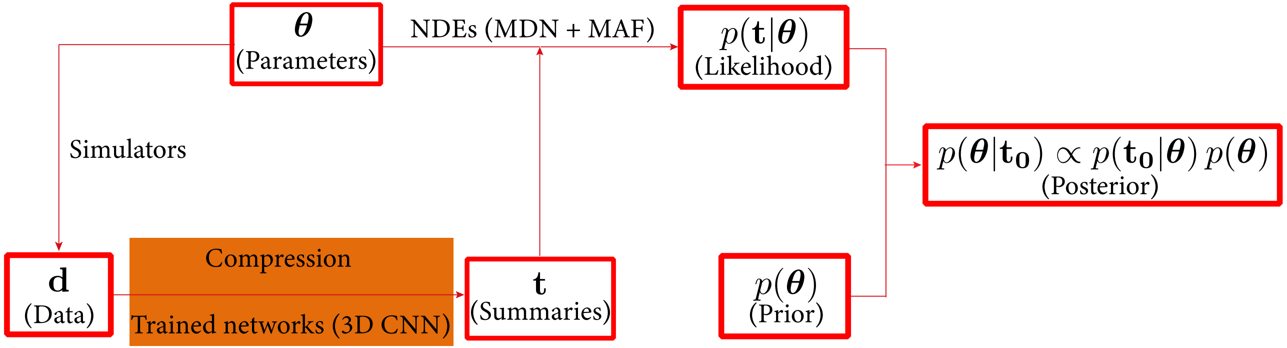
Images

Global Signal

Power Spectrum

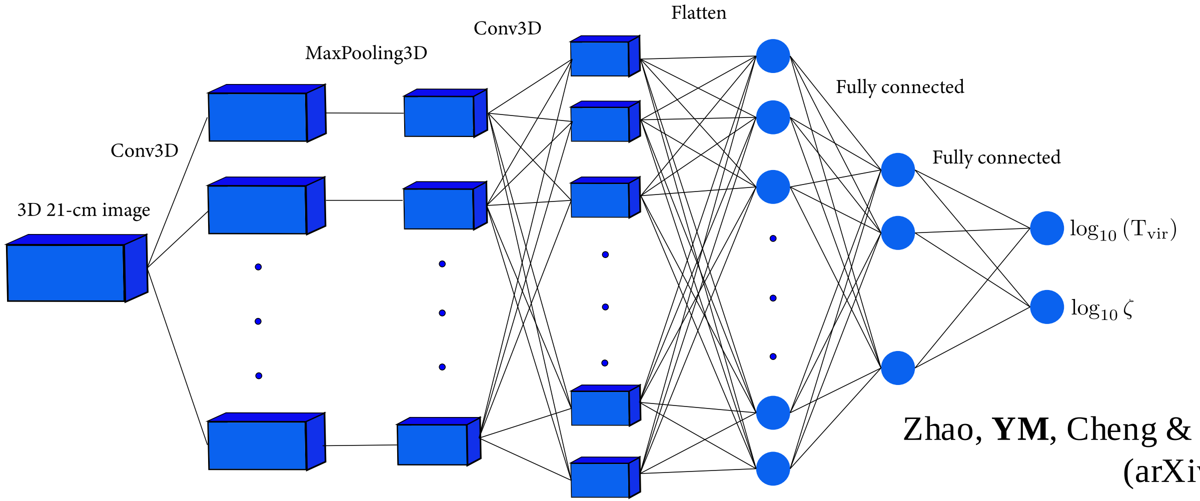
图：A. Liu & R. Shaw

Likelihood-free Bayesian inference



Xiaosheng Zhao
(now postdoc at JHU)

Structure of Convolutional Neural Networks (CNN)



Zhao, **YM**, Cheng & Wandelt, 2022a, ApJ (arXiv:arXiv:2105.03344)

T_{vir} Minimum virial temperature of haloes that host star-forming galaxies

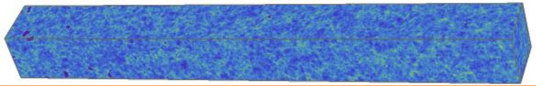
ζ UV Ionizing efficiency of galaxies.

Likelihood-free Bayesian inference

Xiaosheng Zhao (now postdoc at JHU)

Parameters

Simulator



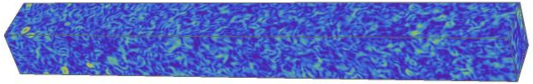
Data

Compression

Convolve with wavelet at scale : $_1$ = * $_j$

Modulus: $_1$ = | $_1$|

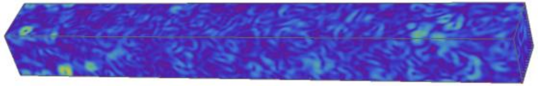
Integration: $_1$ = ∫ $_1{}^q$

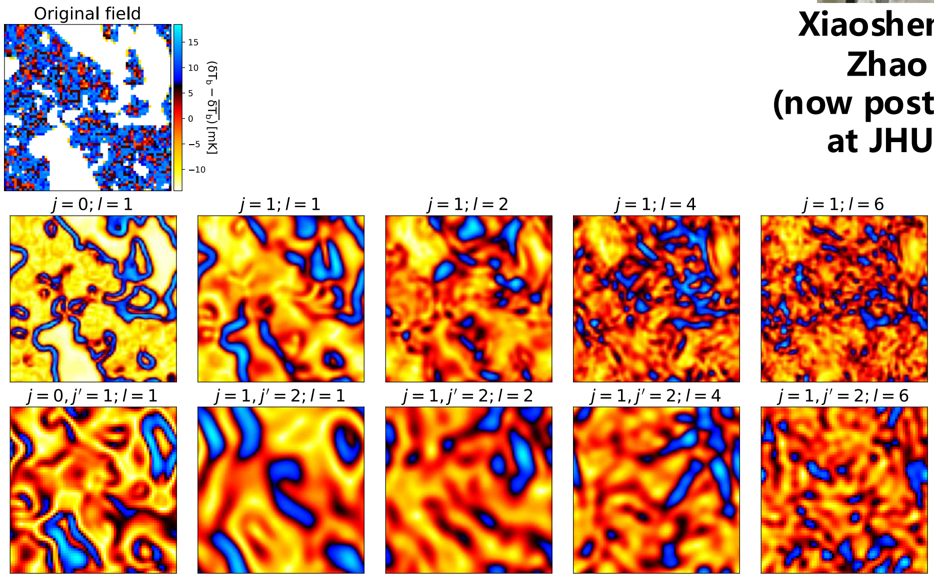


Convolve with wavelet at ′ > : $_2$ = $_1$ * $_{j'}$

Modulus: $_2$ = | $_2$|

Integration: $_2$ = ∫ $_2{}^q$

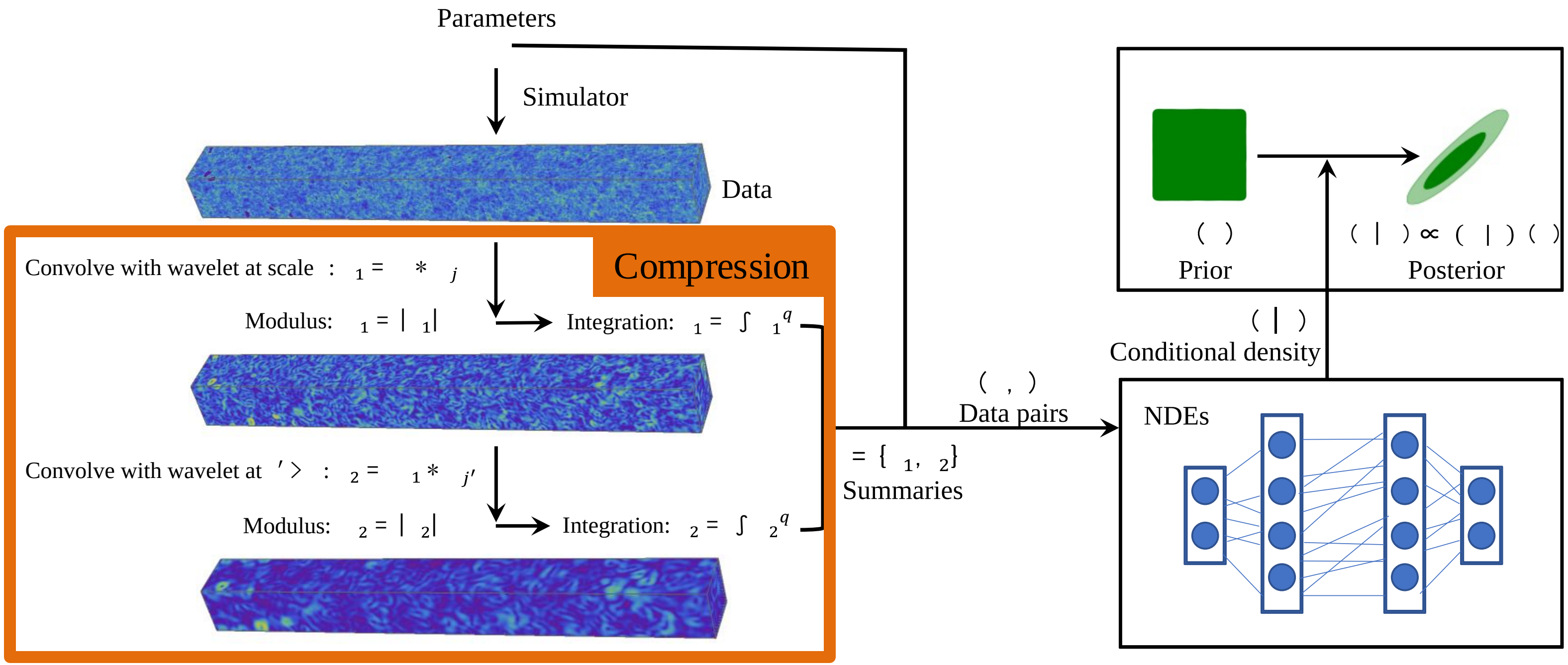




Solid harmonic wavelet scattering transform (**SHWST**)

Zhao, **YM**, Zuo & Wandelt, ApJ 2024 (arXiv: 2310.17602)

Likelihood-free Bayesian inference



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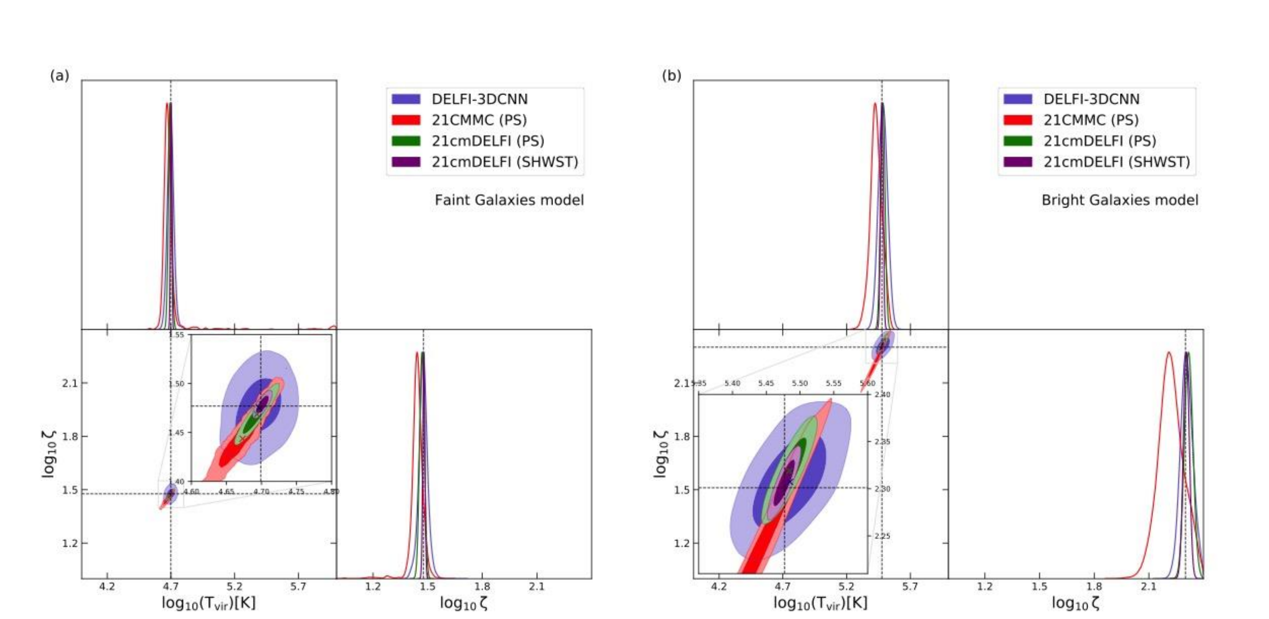
Xiaosheng Zhao (now postdoc at JHU)

Zhao, **YM**, Zuo & Wandelt, ApJ 2024 (arXiv: 2310.17602)

从21厘米图像测量出发限制宇宙再电离理论模型

Xiaosheng Zhao (now postdoc at JHU)

- 对21厘米图像信号进行降维（球谐小波散射变换），发展了基于深度学习进行贝叶斯统计推断的新方法，开发了新软件3D ScatterNet



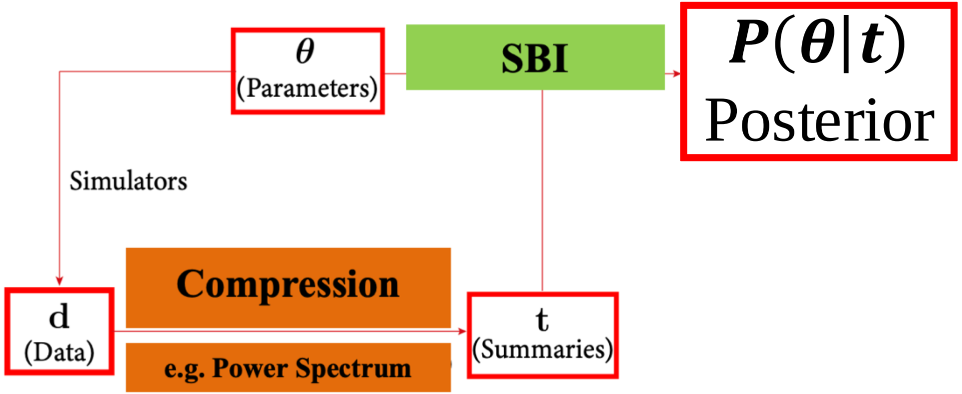
Zhao, **YM**, Zuo & Wandelt, ApJ 2024 (arXiv: 2310.17602)

MCMC得到的置信区间

21cmDELFI-PS得到的置信区间

3D ScatterNet得到的置信区间

Simulation-Based Inference



2. Discriminative Way (Neural Ration Estimation, NRE):

Ce, **YM**, et al
In prep

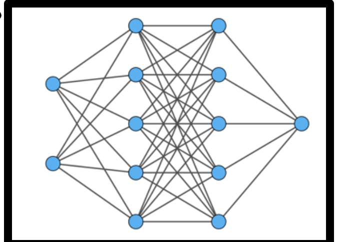
$\{\theta, t\}$ — Ratio estimation → $r(\theta,t) = \frac{P(t|\theta)}{P(t)}$ — Prior → $\boldsymbol{P(\theta|t) = r(\theta,t)P(\theta)}$

$P(\theta,t) = P(t|\theta)P(\theta)$

$P(\theta)P(t)$

$\{\theta, t\}$ — Classifier → $\max_{D} E_{p(\theta,t)}[\log(D(t,\theta)] + E_{p(\theta)p(t)}[\log(1 - D(t,\theta))]$

$D(t,\theta)$



$r(\theta,t) = \frac{P(x|t)}{P(t)}$

$$D^*(t,\theta) = \frac{r(\theta,t)}{r(\theta,t)+1}$$

Comparison between two SBI methods

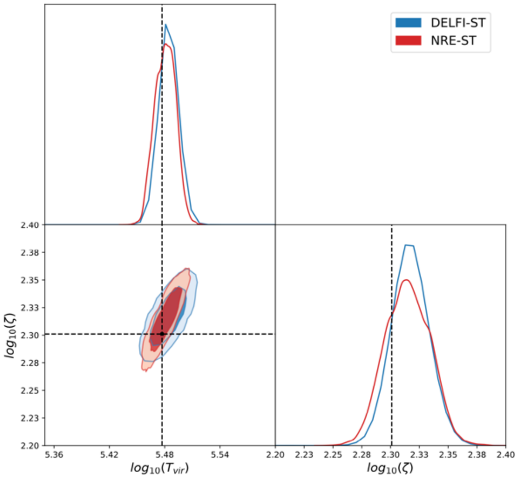
Ce Sui

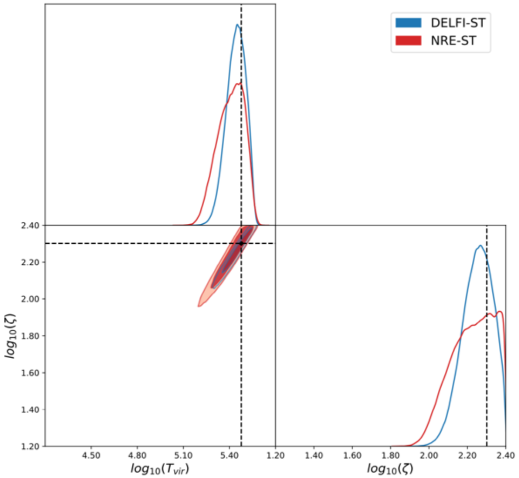
Pure Signal

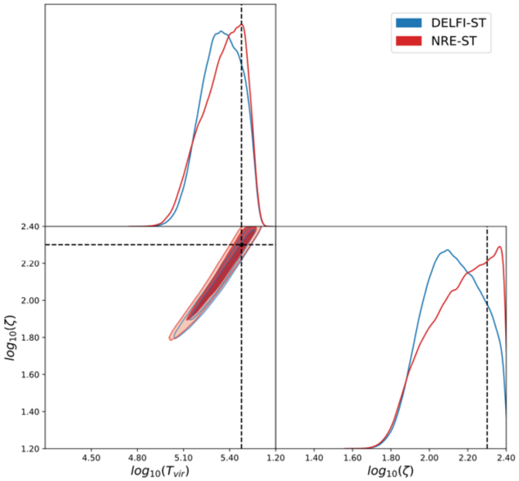
+ Thermal Noise

+Residual Foreground

Bright Galaxy Model







Ce, **YM**, et al. in prep

What else can we learn about the EoR than just constraining reionization *model* parameters?

We can reconstruct the initial density fields

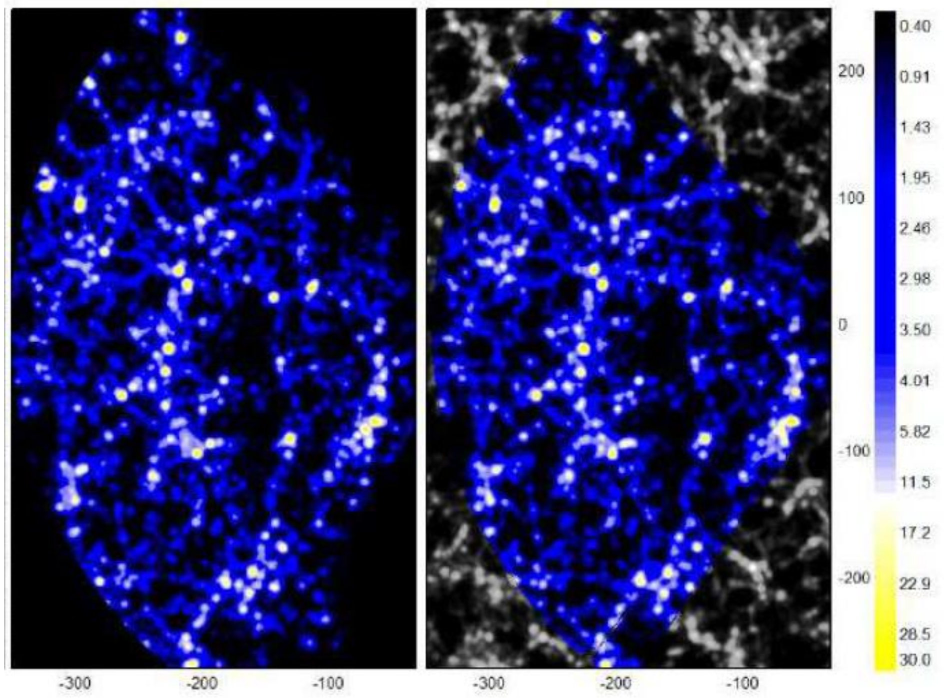
Local Universe:

ELUCID (Wang et al 2013)

BORG (Jasche & Wandelt 2013)

….

$P(\delta^{\mathrm{ini}}|\underline{\delta^{\mathrm{galaxy}}})$ + Hamiltonian Monte Carlo

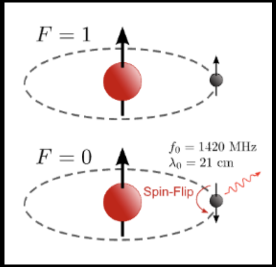


“observed” “resimulated”

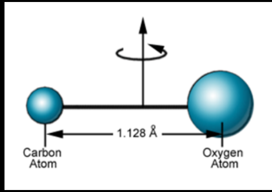
Wang et al 2013

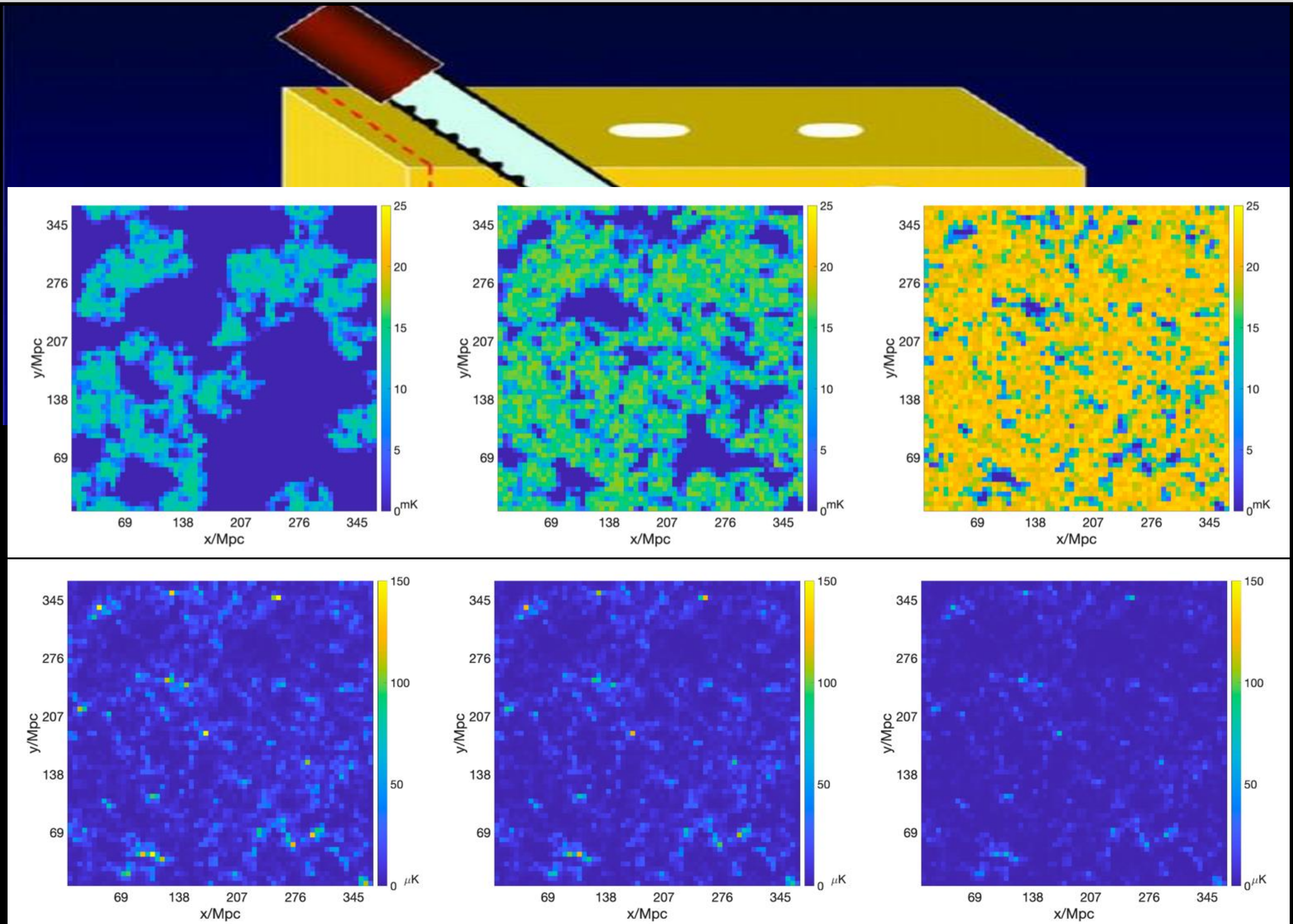
We can reconstruct the initial density fields *from high-redshift observations*

- **H I 21 cm intensity mapping**



- **Molecular line intensity mapping**





We can use line intensity mappings

Minimize Cost Function ~ Maximize Likelihood

Meng Zhou (now postdoc at NAOC)

$$\phi \to 0 \qquad \{\delta^{\mathrm{ini,rec}}\} \to \{\delta^{\mathrm{ini,true}}\}$$

Resimulated LIM

Mock (input) LIM

$$\phi(\delta^{\mathrm{ini}}) = C_{\mathrm{cost}} \sum_{j,\alpha} \frac{\left(T^{\mathrm{mod}}_{j,\alpha} - T^{\mathrm{inp}}_{j,\alpha}\right)^2}{2\left(\sigma^{\mathrm{N}}_j\right)^2} w_j$$

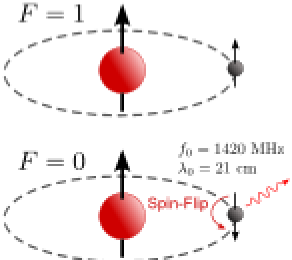
Weight

j: 21cm and CO

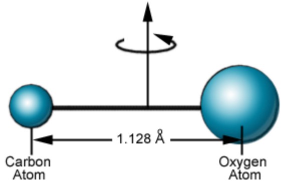
α: pixels

Uncertainty

21cm:
neutral region



CO (1→0):
ionized region



Zhou and **YM**, ApJ 2024 (arXiv:2311.14940)

We employ the *conjugate gradient* to minimize the cost function.

Meng Zhou (now postdoc at NAOC)

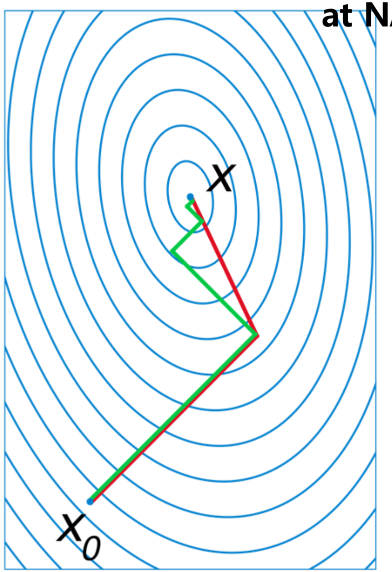
Here is the overall algorithm:

Starting from an arbitrary point $\boldsymbol{x}_0$.

Calculate the gradient, and set $\boldsymbol{g}_0 = \boldsymbol{h}_0 = -\nabla f(\boldsymbol{x}_0)$.

Start iteration until convergence:

- Perform line minimization along $\boldsymbol{h}_i$ to obtain $\boldsymbol{x}_{i+1}$.
- Calculate the gradient vector $\boldsymbol{g}_{i+1} = -\nabla f(\boldsymbol{x}_{i+1})$.
- Determine the new direction $\boldsymbol{h}_{i+1} = \boldsymbol{g}_{i+1} + \gamma_i \boldsymbol{h}_i$.



Zhou and **YM**, ApJ 2024 (arXiv:2311.14940)

Analytical gradients agree with numerical results.

Meng Zhou

Excursion Set model of reionization (ESMR):

$$T_{21\mathrm{cm}}(\mathbf{x}) \sim x_{\mathrm{HI}}(\mathbf{x})[1+\delta(\mathbf{x})]$$

$$x_{\mathrm{HI}}(\mathbf{x}) = \begin{cases} 0, & \text{if } \zeta f_{\mathrm{coll},R}(\mathbf{x}) > 1 \\ 1.0 - \zeta f_{\mathrm{coll},R_{\mathrm{min}}}(\mathbf{x}), & \text{otherwise} \end{cases}$$

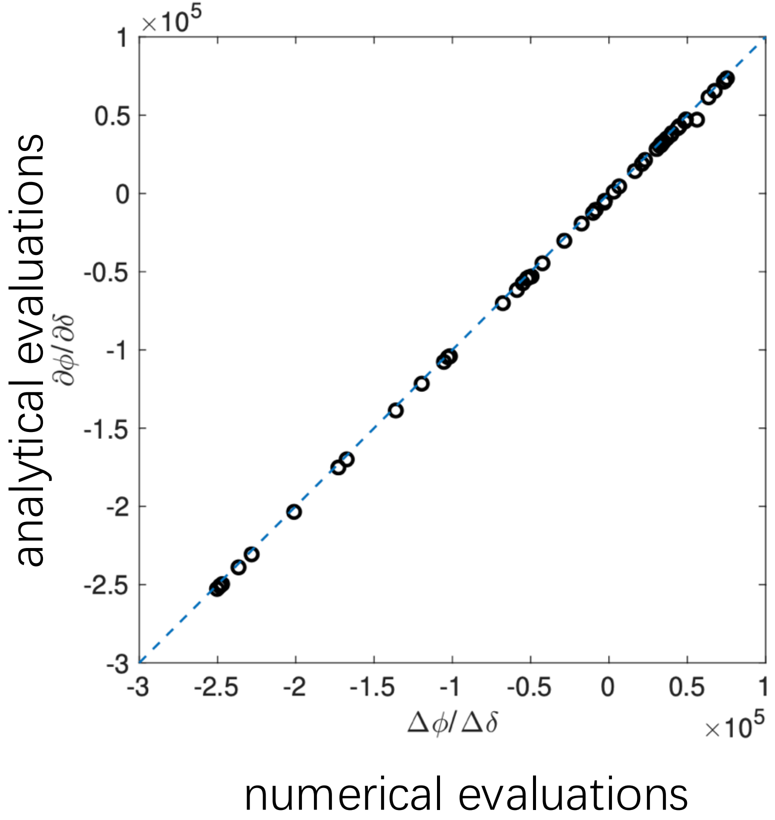
Empirical L~SFR model:

$$T_{\mathrm{CO}}(\mathbf{x}) \sim f_{\mathrm{coll},R_{\mathrm{min}}}(\mathbf{x})[1+\delta(\mathbf{x})]$$

$$\frac{\partial \phi}{\partial \delta^{\mathrm{ini}}} \sim \frac{\partial \phi}{\partial T_j} * \frac{\partial T_j}{\partial \delta} * \frac{\partial \delta}{\partial \delta^{\mathrm{ini}}}$$

ESMR

Zel'dovich approximation



Zhou and **YM**, ApJ 2024 (arXiv:2311.14940)

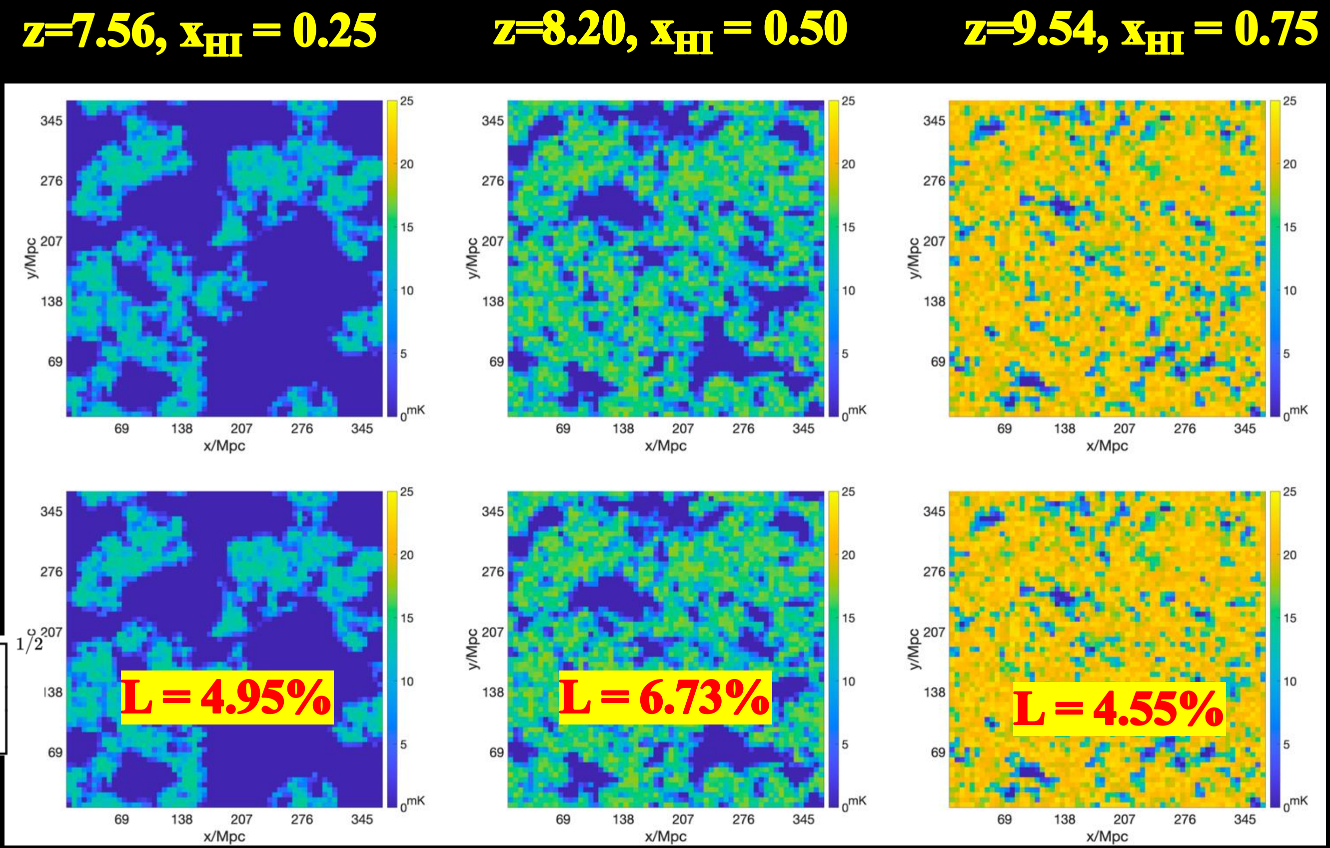
We can reconstruct the initial density fields *from high-redshift observations*

Meng Zhou (now postdoc at NAOC)

中性氢21厘米谱线强度映射

Input maps

Resimulated maps



$$L_{\rm tot} = \left[\frac{1}{2N_p} \sum_{j,\alpha} \frac{\left(T^{\rm mod}_{j,\alpha} - T^{\rm inp}_{j,\alpha}\right)^2}{\left(\sigma^{\rm S}_j\right)^2} \right]^{1/2}$$

Zhou and **YM**, ApJ 2024 (arXiv:2311.14940)

We can reconstruct the initial density fields *from high-redshift observations*

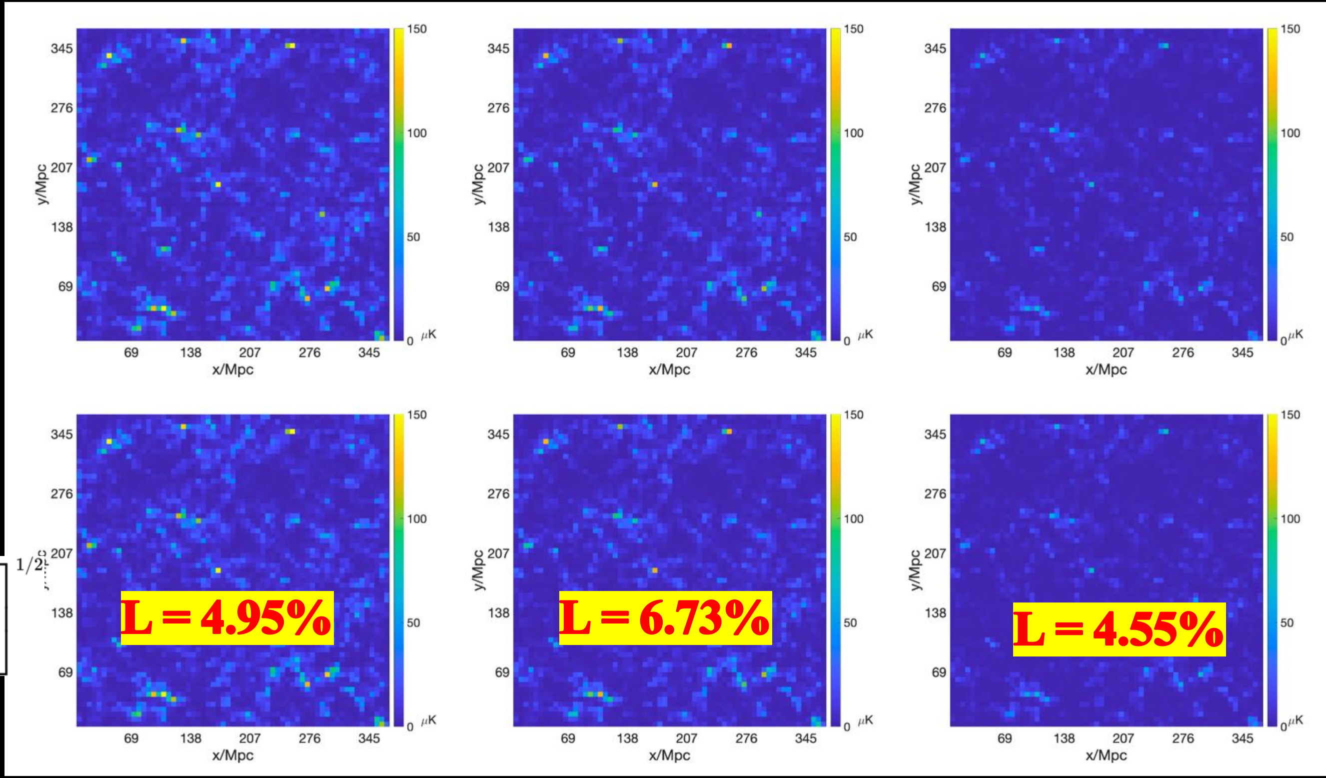
Meng Zhou (now postdoc at NAOC)

CO分子谱线强度映射

z=7.56, x_{HI} = 0.25 | z=8.20, x_{HI} = 0.50 | z=9.54, x_{HI} = 0.75

Input maps

Resimulated maps

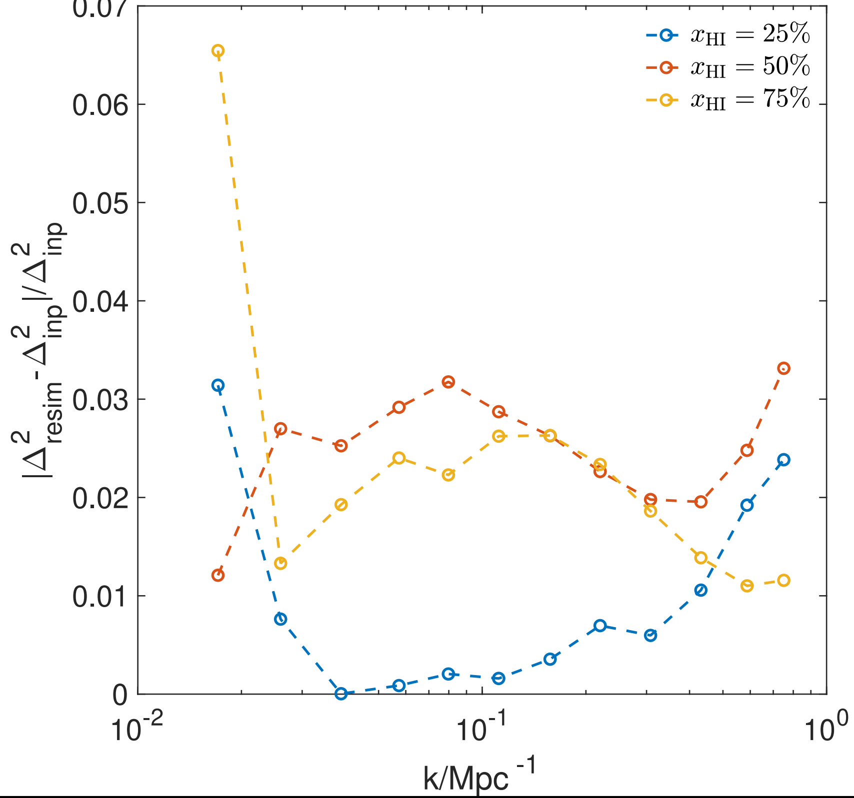


$$L_{\rm tot} = \left[\frac{1}{2N_p} \sum_{j,\alpha} \frac{\left(T^{\rm mod}_{j,\alpha} - T^{\rm inp}_{j,\alpha}\right)^2}{\left(\sigma^{\rm S}_j\right)^2} \right]^{1/2}$$

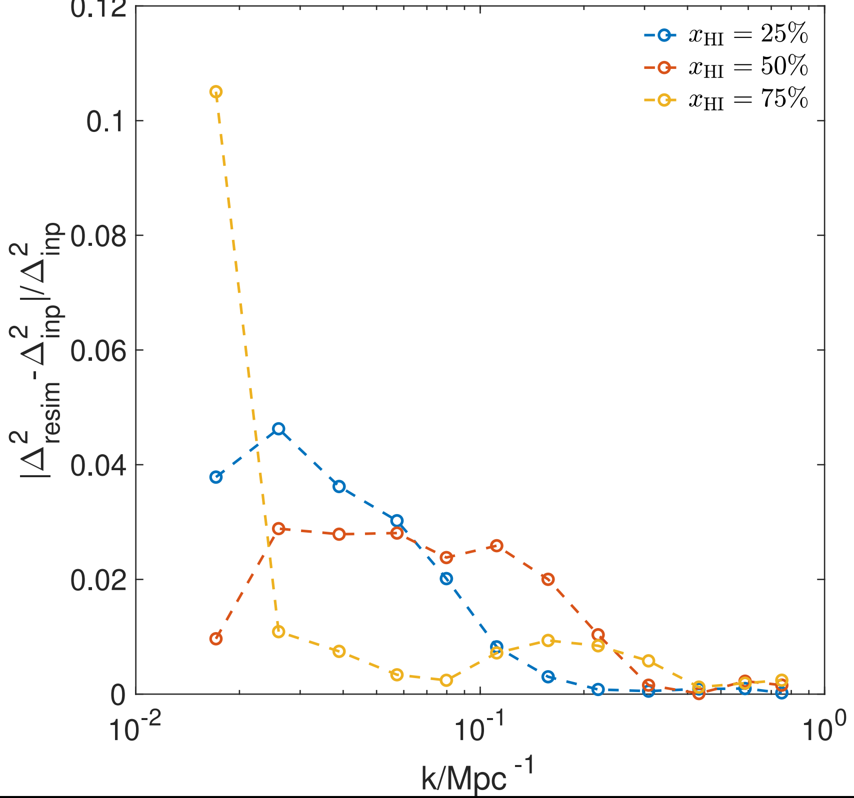
Zhou and **YM**, ApJ 2024
(arXiv:2311.14940)

We can reconstruct the initial density fields *from high-redshift observations*

Relative error of 21cm power spectrum



Relative error of CO power spectrum



Zhou and **YM**, ApJ 2024
(arXiv:2311.14940)

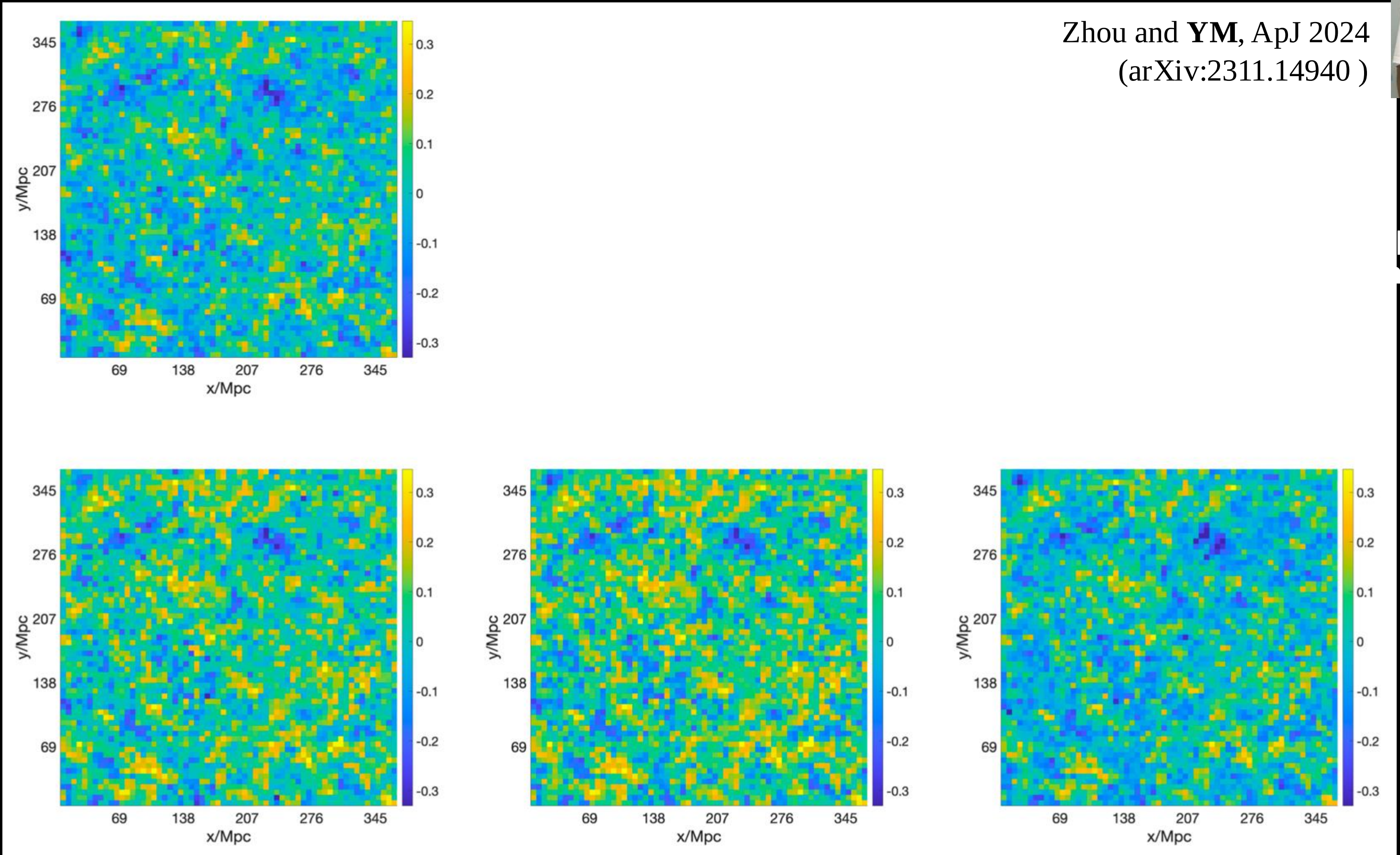
The reconstructed power spectrum is <4% error in most cases.

We can reconstruct the initial density fields *from high-redshift observations*

Zhou and **YM**, ApJ 2024
(arXiv:2311.14940)

True initial overdensity field

Reconstructed initial overdensity field

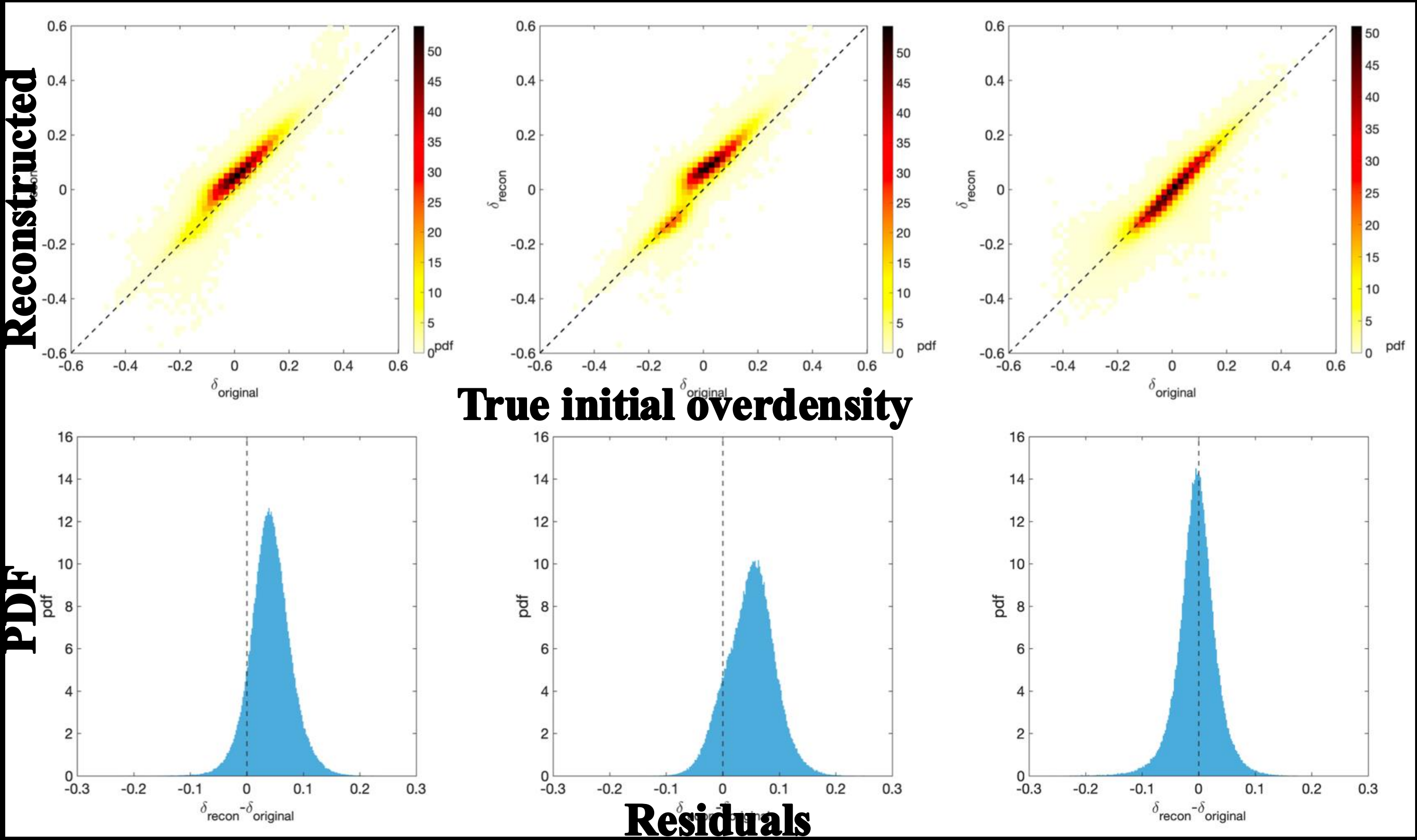


From observations at: z=7.56, x_{HI} = 0.25 z=8.20, x_{HI} = 0.50 z=9.54, x_{HI} = 0.75

Meng Zhou (now postdoc at NAOC)

We can reconstruct the initial density fields *from high-redshift observations*

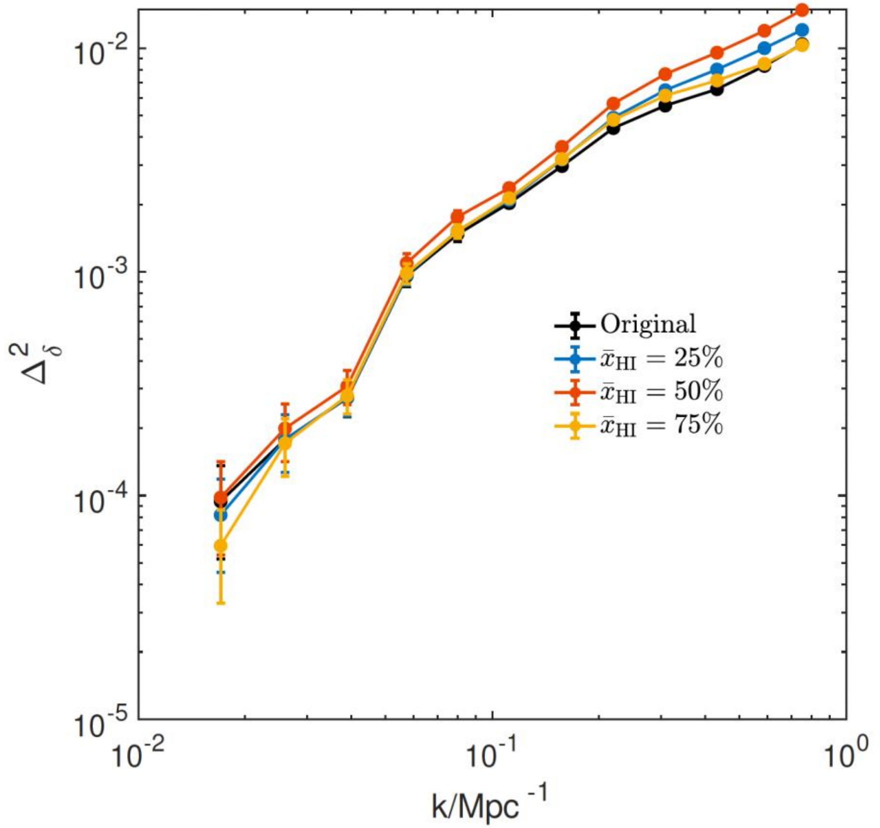
From observations at: $z=7.56$, $x_{HI}=0.25$ $z=8.20$, $x_{HI}=0.50$ $z=9.54$, $x_{HI}=0.75$



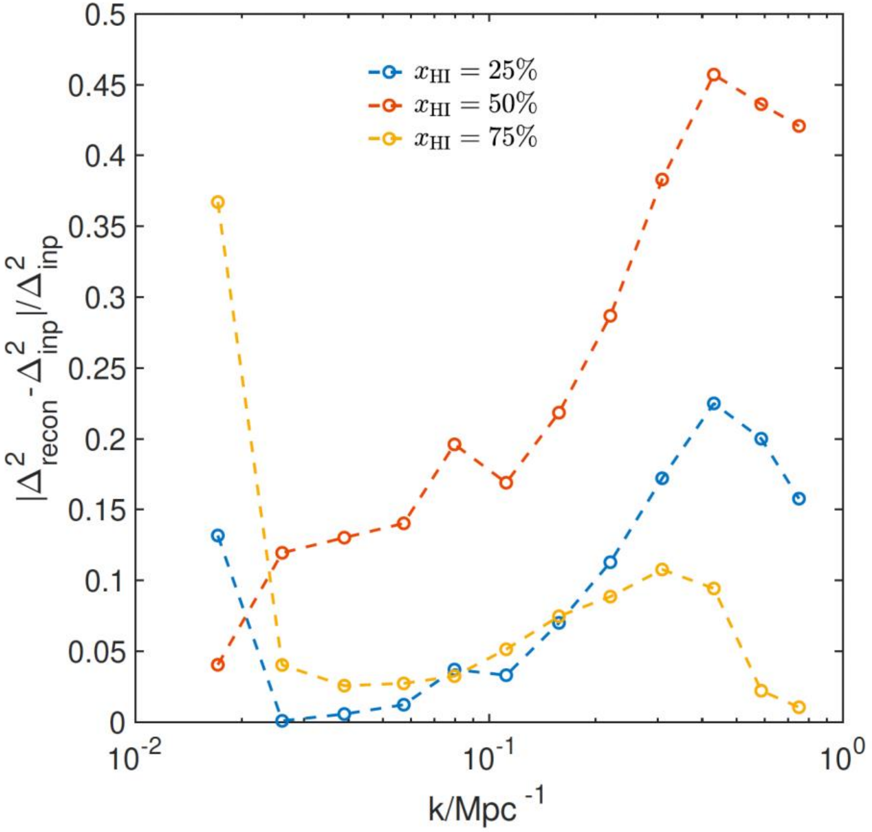
Zhou and **YM**, ApJ 2024
(arXiv:2311.14940)

We can reconstruct the initial density fields *from high-redshift observations*

Power spectrum of initial overdensity



Relative error of initial overdensity power spectrum



Zhou and **YM**, ApJ 2024 (arXiv:2311.14940)

Summary

- We exploit the Simulation-based Inference (SBI) to perform the posterior inference of reionization parameters, both from 21 cm power spectrum and from 21 cm lightcone images.

- We can reconstruct the cosmological initial density fields from the 21 cm and CO intensity mapping during the epoch of reionization. We develop a method based on conjugate gradients, which can successfully reconstruct the initial density fields with a goodness less than 7% at all stages of reionization.